Sorting integer arrays: security, speed, and verification

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Bob’s laptop screen:

From: Alice

Thank you for your submission. We received many interesting papers, and unfortunately your

Bob assumes this message is something Alice actually sent.

But today’s “security” systems fail to guarantee this property. Attacker could have modified or forged the message.
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Bob’s security policy for this talk: If message is displayed on Bob’s screen as “From: Alice” then message is from Alice.

If TCB works correctly, then message is guaranteed to be from Alice, no matter what the rest of the system does.
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Massive TCB has many bugs, including many security holes. Any hope of fixing this?
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e.g. Bob runs many VMs:

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<tr>
<th>VM A</th>
<th>VM C</th>
<th>...</th>
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<tbody>
<tr>
<td>Alice data</td>
<td>Charlie data</td>
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Alice also runs many VMs.
How does Bob’s laptop know that incoming network data is from Alice’s laptop?

Cryptographic solution: Message-authentication codes.
Cryptography

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Alice’s message

authenticated message

untrusted network

modified message

“Alert: forgery!”
Important for Alice and Bob to share the same secret $k$.

What if attacker was spying on their communication of $k$?
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Solution 1: Public-key encryption.
Solution 2:
Public-key signatures.

\[ m \]

\[ \text{signed message} \]

\[ \text{network} \]

\[ \text{signed message} \]

\[ a \]

\[ aG \]

\[ \text{network} \]

\[ m \]
Solution 2:
Public-key signatures.

No more shared secret $k$ but Alice still has secret $a$.

**Cryptography requires TCB to protect secrecy of keys,**
even if user has no other secrets.
Constant-time software

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Many attacks show that this portion of the CPU has trouble keeping secrets. e.g. RIDL: 2019 Schaik–Milburn–Österlund–Frigo–Maisuradze–Razavi–Bos–Giuffrida.
Typical literature on this topic:
Understand this portion of CPU. But details are often proprietary, not exposed to security review.

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For researchers: This is great!

For auditors: This is a nightmare. Many years of security failures. No confidence in future security.
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TCB analysis: Need this portion of the CPU to be correct, but don’t need it to keep secrets. Makes auditing much easier.

Good match for attitude and experience of CPU designers: e.g., Intel issues errata for correctness bugs, not for information leaks.
Case study: Constant-time sorting

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Subroutine in some submissions: sort array of secret integers. e.g. sort 768 32-bit integers.
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One submission to competition: “Radix sort is used as constant-time sorting algorithm.”

Some versions of radix sort avoid secret branches. But data addresses in radix sort still depend on secrets.
Foundation of solution:
a comparator sorting 2 integers.

\[
\begin{align*}
x & \quad y \\
\min\{x, y\} & \quad \max\{x, y\}
\end{align*}
\]

Easy constant-time exercise in C. Warning: C standard allows compiler to screw this up.

Even easier exercise in asm.
Combine comparators into a **sorting network** for more inputs.

Example of a sorting network:
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Speed is a serious issue in the post-quantum competition. “Cost” is evaluation criterion; “we’d like to stress this once again on the forum that we’d really like to see more platform-optimized implementations”; etc.
void int32_sort(int32 *x,int64 n)
{
    int64 t,p,q,i;
    if (n < 2) return;
    t = 1;
    while (t < n - t) t += t;
    for (p = t;p > 0;p >>= 1) {
        for (i = 0;i < n - p;++i)
            if (!(i & p))
                minmax(x+i,x+i+p);
        for (q = t;q > p;q >>= 1)
            for (i = 0;i < n - q;++i)
                if (!(i & p))
                    minmax(x+i+p,x+i+q);
    }
}
Previous slide: C translation of 1973 Knuth “merge exchange”, which is a simplified version of 1968 Batcher “odd-even merge” sorting networks.

$$\approx n (\log_2 n)^2 / 4$$ comparators. Much faster than bubble sort.

Warning: many other descriptions of Batcher’s sorting networks require $n$ to be a power of 2. Also, Wikipedia says “Sorting networks . . . are not capable of handling arbitrarily large inputs.”
This constant-time sorting code

vectorization (for Haswell)

Constant-time sorting code included in 2017
Bernstein–Chuengsatiansup–Lange–van Vredendaal
“NTRU Prime” software release

revamped for higher speed

New: djbsort constant-time sorting code
The slowdown for constant time

How much speed did we lose by refusing to use variable-time quicksort, radix sort, etc.?

Cycles on Intel Haswell CPU core to sort $n = 768$ 32-bit integers:

26948 stdsort (variable-time)
22812 herf (variable-time)
17748 krasnov (variable-time)
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No slowdown. New speed records!
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Answer: well-known trends in CPU design, reflecting fundamental hardware costs of various operations.
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Every cycle, Haswell core can do 8 “min” ops on 32-bit integers + 8 “max” ops on 32-bit integers.
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Loading a 32-bit integer from a random address: much slower.

Conditional branch: much slower.
Verification

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Test the sorting software on many random inputs, increasing inputs, decreasing inputs. Seems to work.
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But are there occasional inputs where this sorting software fails to sort correctly?

History: Many security problems involve occasional inputs where TCB works incorrectly.
For each used $n$ (e.g., 768):

- **C code**
  - normal compiler
  - machine code
    - symbolic execution
    - fully unrolled code
      - new peephole optimizer
      - unrolled min-max code
        - new sorting verifier
        - yes, code works
Symbolic execution: use existing angr.io toolkit, with several tiny new patches for eliminating byte splitting, adding a few missing vector instructions.
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Current djbsort release (verified fast int32 on AVX2, verified portable int32, fast uint32, fast float32):

<sorting.cr yp.to>

Includes the sorting code; automatic build-time tests; simple benchmarking program; verification tools.

Web site shows how to use the verification tools.

Next release planned: verified ARM NEON code.