Generating random primes faster

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The standard algorithm to generate random primes:

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proof.arithmetic(False)
while True:
  p = randrange(2^(n-1),2^n)
  p = ZZ(p)
  if p.is_prime(): print p
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Standard speedup using wheels:
e.g., force $p \bmod 6 \in \{1, 5\}$.
Wheel using all primes $q \leq n^{O(1)}$:
$n^{1+o(1)}$ iterations per prime.
Recall $\prod_{q \leq y} \left(1 - \frac{1}{q}\right) \in \Theta\left(\frac{1}{\log y}\right)$.

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Most iterations are much simpler: Fermat test rejects p.

Fast reject by trial division/ECM? Still $n^{3+o(1)}$ bit ops per prime.

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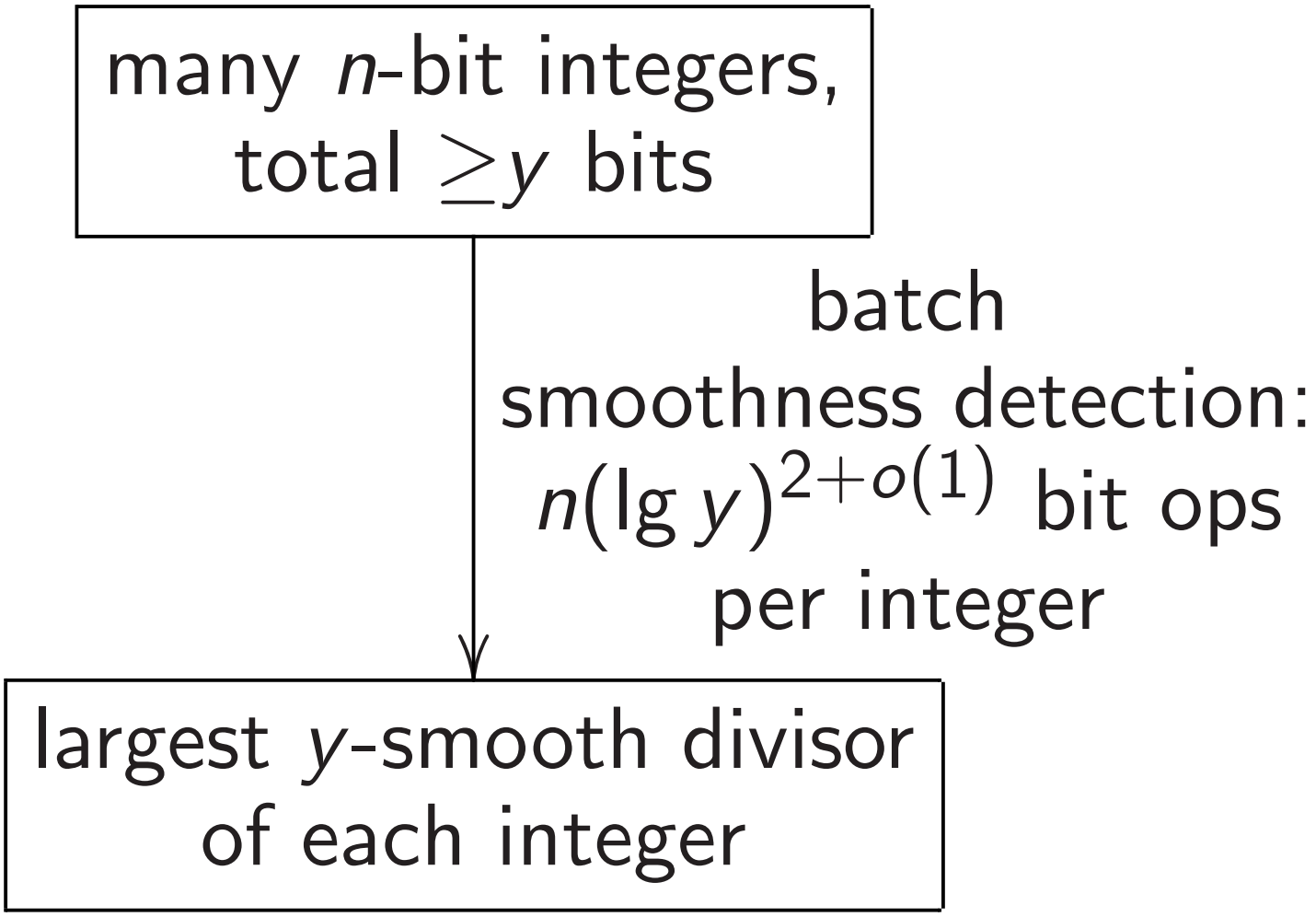

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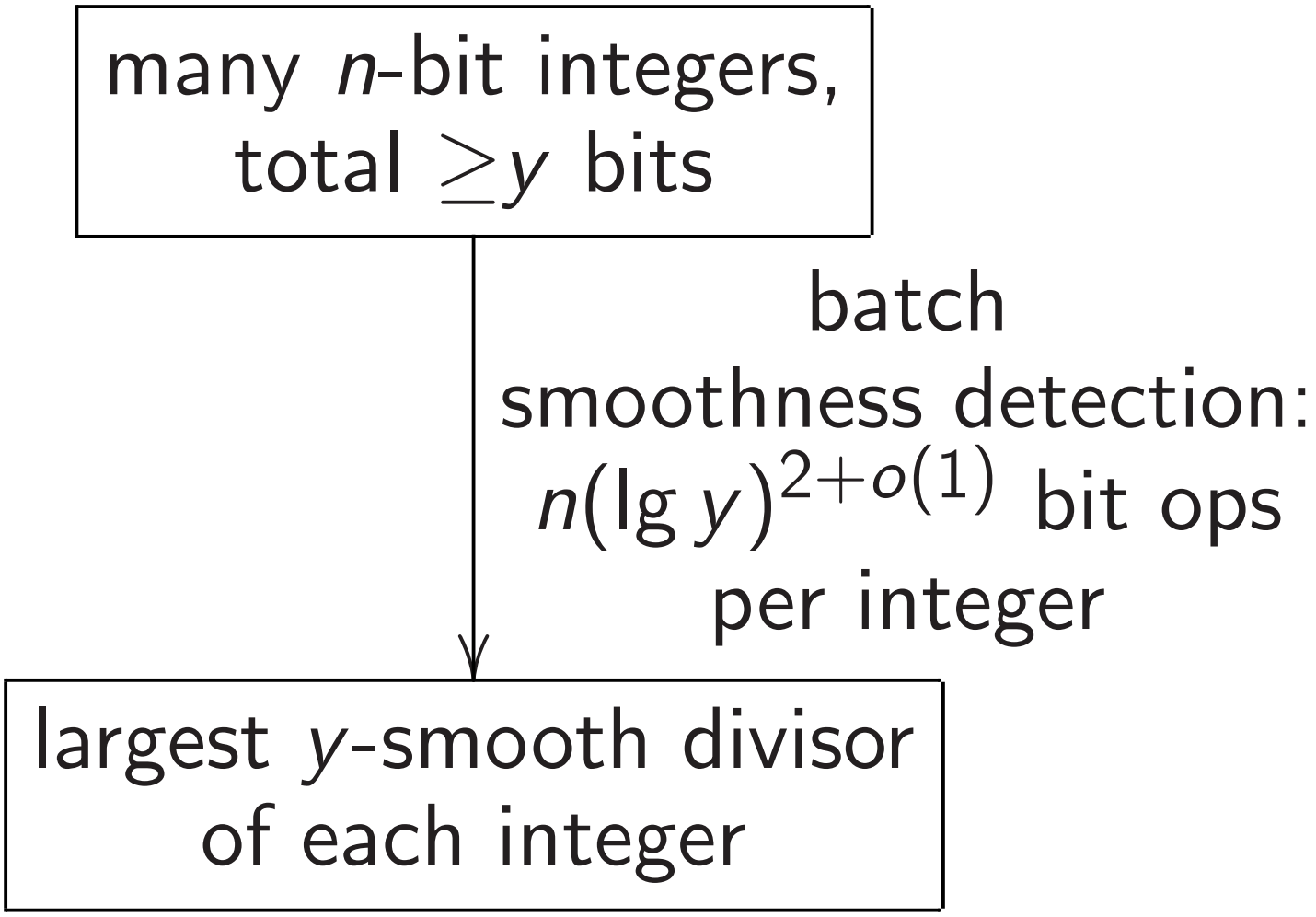

Apply batch smoothness detection for $y = 2^{2^0}$, then $y = 2^{2^1}$, then $y = 2^{2^2}$, ..., then $y \approx 2^{n^{0.5+o(1)}}$.