Sorting integer arrays:
security, speed, and verification
D. J. Bernstein

Bob's laptop screen:
From: Alice

Thank you for your
submission. We received many interesting papers,
and unfortunately your

Bob assumes this message is something Alice actually sent.

But today's "security" systems fail to guarantee this property. Attacker could have modified or forged the message.

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If TCB works correctly,
then message is guaranteed to be from Alice, no matter what the rest of the system does.

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Massive TCB has many bugs, including many security holes.
Any hope of fixing this?

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Alice also runs many VMs.

## Cryptography

How does Bob's laptop know that incoming network data is from Alice's laptop?

Cryptographic solution:
Message-authentication codes.
Alice's message
authenticated message
$\downarrow$ untrusted network
authenticated message

Alice's message

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"Alert: forgery!"

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Solution 1:
Public-key encryption.


Solution 2:
Public-key signatures.


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No more shared secret $k$ but Alice still has secret $a$. Cryptography requires TCB to protect secrecy of keys, even if user has no other secrets.

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optimizations depending on addresses of memory locations.

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Many attacks (e.g. TLBleed from 2018 Gras-Razavi-Bos-Giuffrida) show that this portion of the CPU has trouble keeping secrets.

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 Understand this portion of CPU. But details are often proprietary, not exposed to security review.Try to push attacks further. This becomes very complicated.

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For researchers: This is great!
For auditors: This is a nightmare. Many years of security failures.
No confidence in future security.

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Good match for attitude and experience of CPU designers: e.g., Intel issues errata for correctness bugs, not for information leaks.

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Subroutine in some submissions: sort array of secret integers. e.g. sort 768 32-bit integers.

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One submission to competition: "Radix sort is used as constant-time sorting algorithm." Some versions of radix sort avoid secret branches.
But data addresses in radix sort still depend on secrets.

## Foundation of solution:

 a comparator sorting 2 integers.
$\min \{x, y\}$
$\max \{x, y\}$

Easy constant-time exercise in C .
Warning: C standard allows
compiler to screw this up.
Even easier exercise in asm.

Combine comparators into a sorting network for more inputs.

Example of a sorting network:


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Speed is a serious issue in the post-quantum competition. "Cost" is evaluation criterion; "we'd like to stress this once again on the forum that wed really like to see more platformoptimized implementations" ; etc.
void int32_sort (int32 *x,int64 n)
\{ int64 t,p,q,i;
if ( $n<2$ ) return;
$\mathrm{t}=1$;
while ( $\mathrm{t}<\mathrm{n}-\mathrm{t}) \mathrm{t}+=\mathrm{t}$;
for ( $p=t ; p>0 ; p \gg=1)\{$

$$
\begin{aligned}
& \text { for }(i=0 ; i<n-p ;++i) \\
& \text { if }(!(i \& p)) \\
& \quad \operatorname{minmax}(x+i, x+i+p)
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Previous slide: C translation of 1973 Knuth "merge exchange", which is a simplified version of 1968 Batcher "odd-even merge" sorting networks.
$\approx n\left(\log _{2} n\right)^{2} / 4$ comparators. Much faster than bubble sort.

Warning: many other descriptions of Batcher's sorting networks require $n$ to be a power of 2 . Also, Wikipedia says "Sorting networks . . . are not capable of handling arbitrarily large inputs."

## This constant-time sorting code

vectorization
(for Haswell)
Constant-time sorting code included in 2017
Bernstein-Chuengsatiansup-Lange-van Vredendaal
"NTRU Prime" software release
revamped for
higher speed
New: "djbsort"
constant-time sorting code

## The slowdown for constant time

Massive fast-sorting literature.
2015 Gueron-Krasnov: AVX and AVX2 (Haswell) optimization of quicksort. For 32-bit integers: $\approx 45$ cycles/byte for $n \approx 2^{10}$, $\approx 55$ cycles/byte for $n \approx 2^{20}$.

Slower than "the radix sort implemented of IPP, which is the fastest in-memory sort we are aware of": 32, 40 cycles/byte.

IPP: Intel's Integrated Performance Primitives library.

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No slowdown. New speed records!
Warning: Comparison for $n \approx 2^{20}$ involves microarchitecture details beyond Haswell core. Should measure all code on same CPU.

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Loading a 32-bit integer from a random address: much slower.

Conditional branch: much slower.

## Verification

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Test the sorting software on many random inputs, increasing inputs, decreasing inputs. Seems to work.

But are there occasional inputs where this sorting software fails to sort correctly?

History: Many security problems involve occasional inputs
where TCB works incorrectly.

For each used $n$ (e.g., 768):

## C code

normal compiler

## machine code

symbolic execution
fully unrolled code
new peephole optimizer
unrolled min-max code
new sorting verifier
yes, code works

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Sorting verifier: decompose DAG into merging networks. Verify each merging network using generalization of 2007
Even-Levi-Litman, correction of 1990 Chung-Ravikumar.

First djbsort release,
verified int 32 on $\mathrm{AVX2}$ :
https://sorting.cr.yp.to
Includes the sorting code;
automatic build-time tests;
simple benchmarking program; verification tools.

Web site shows how to use the verification tools.

Next release planned: verified ARM NEON code and verified portable code.

