Sorting integer arrays: security, speed, and verification

D. J. Bernstein

Bob's laptop screen:

```
From: Alice


Thank you for your
submission. We received
many interesting papers,
and unfortunately your
```

Bob assumes this message is something Alice actually sent.

But today's "security" systems fail to guarantee this property. Attacker could have modified or forged the message.

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If TCB works correctly, then message is guaranteed to be from Alice, no matter what the rest of the system does.

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Massive TCB has many bugs, including many security holes.
Any hope of fixing this?

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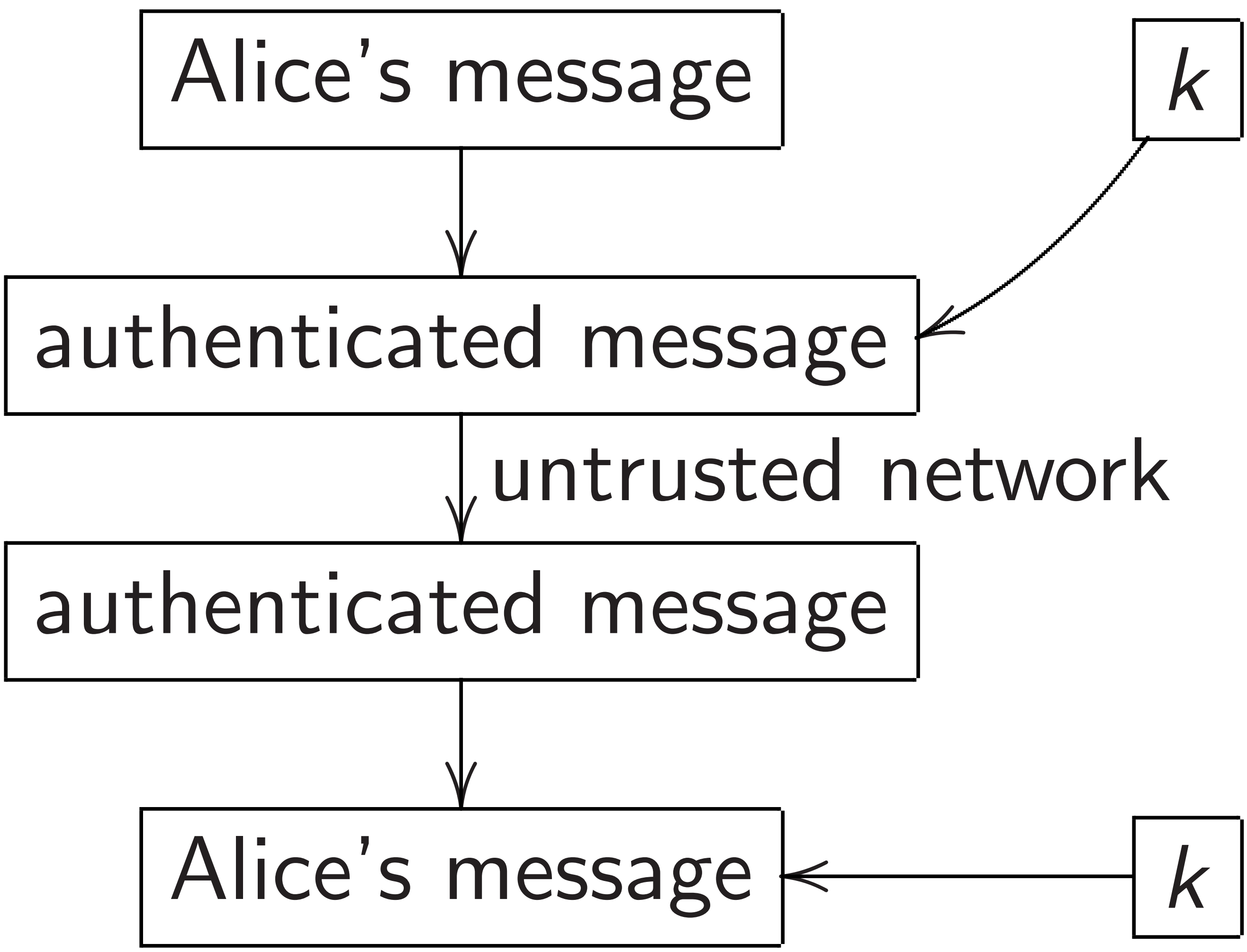
Alice also runs many VMs.

Cryptography

How does Bob's laptop know that incoming network data is from Alice's laptop?

Cryptographic solution: Message-authentication codes.



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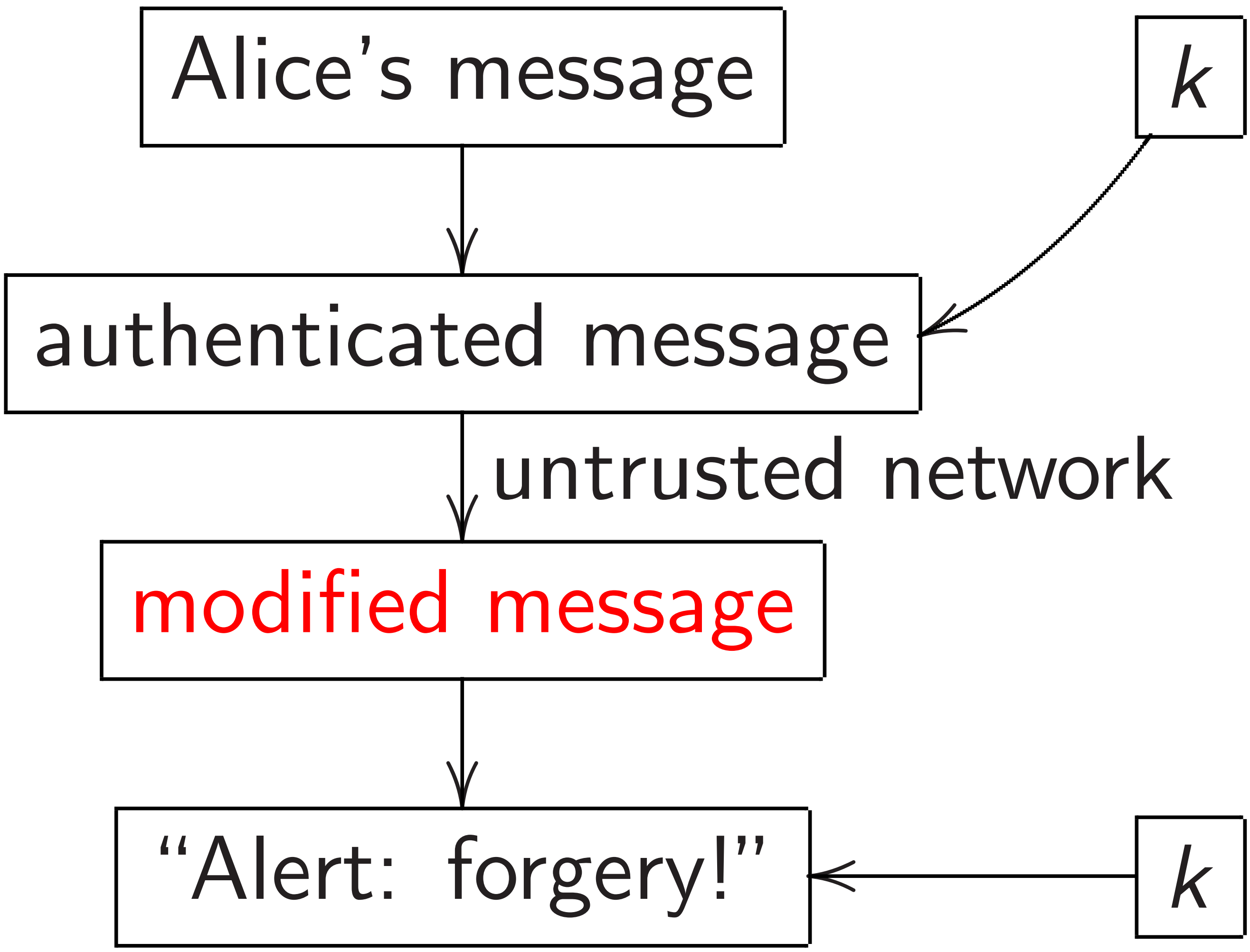

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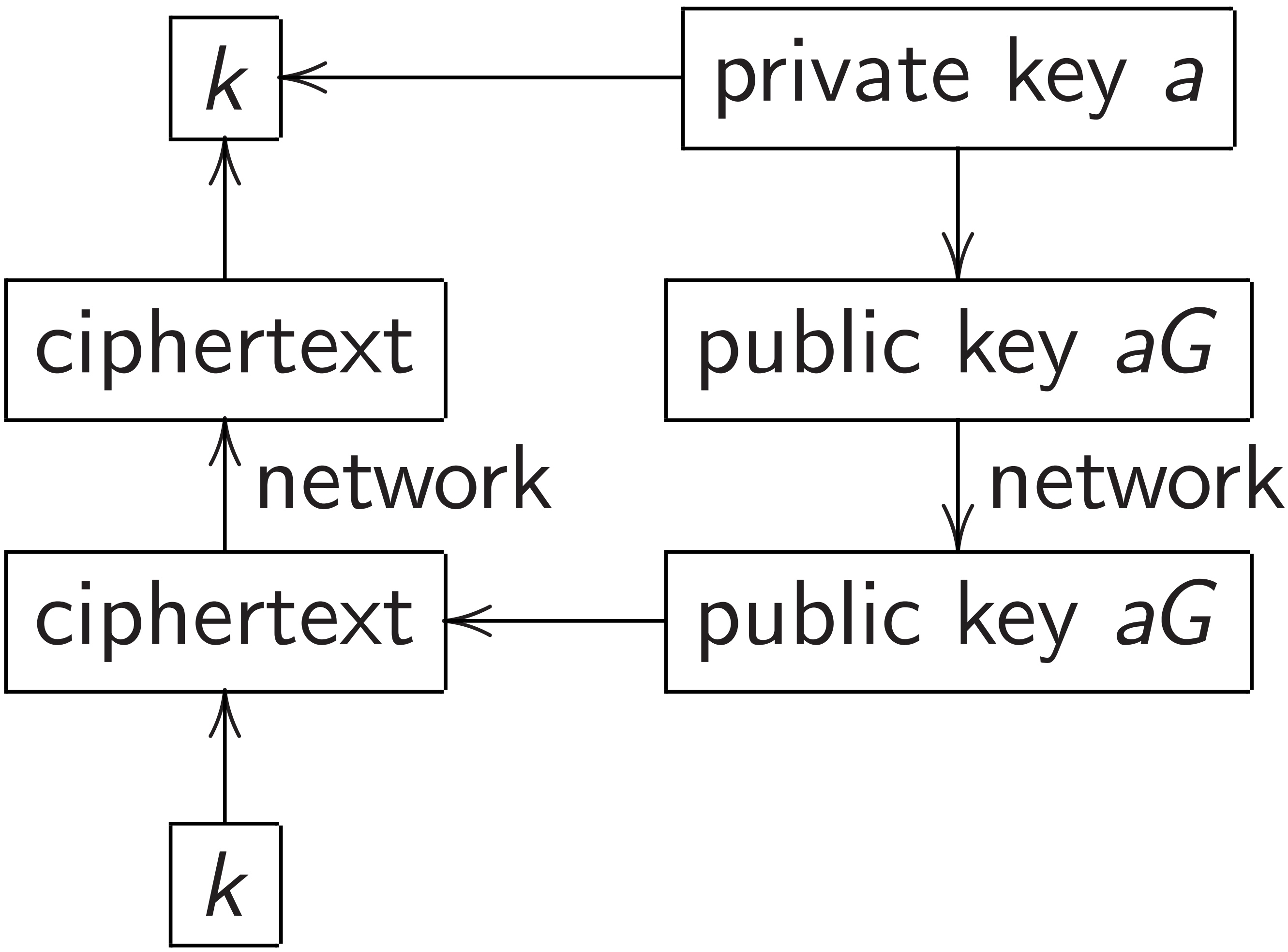
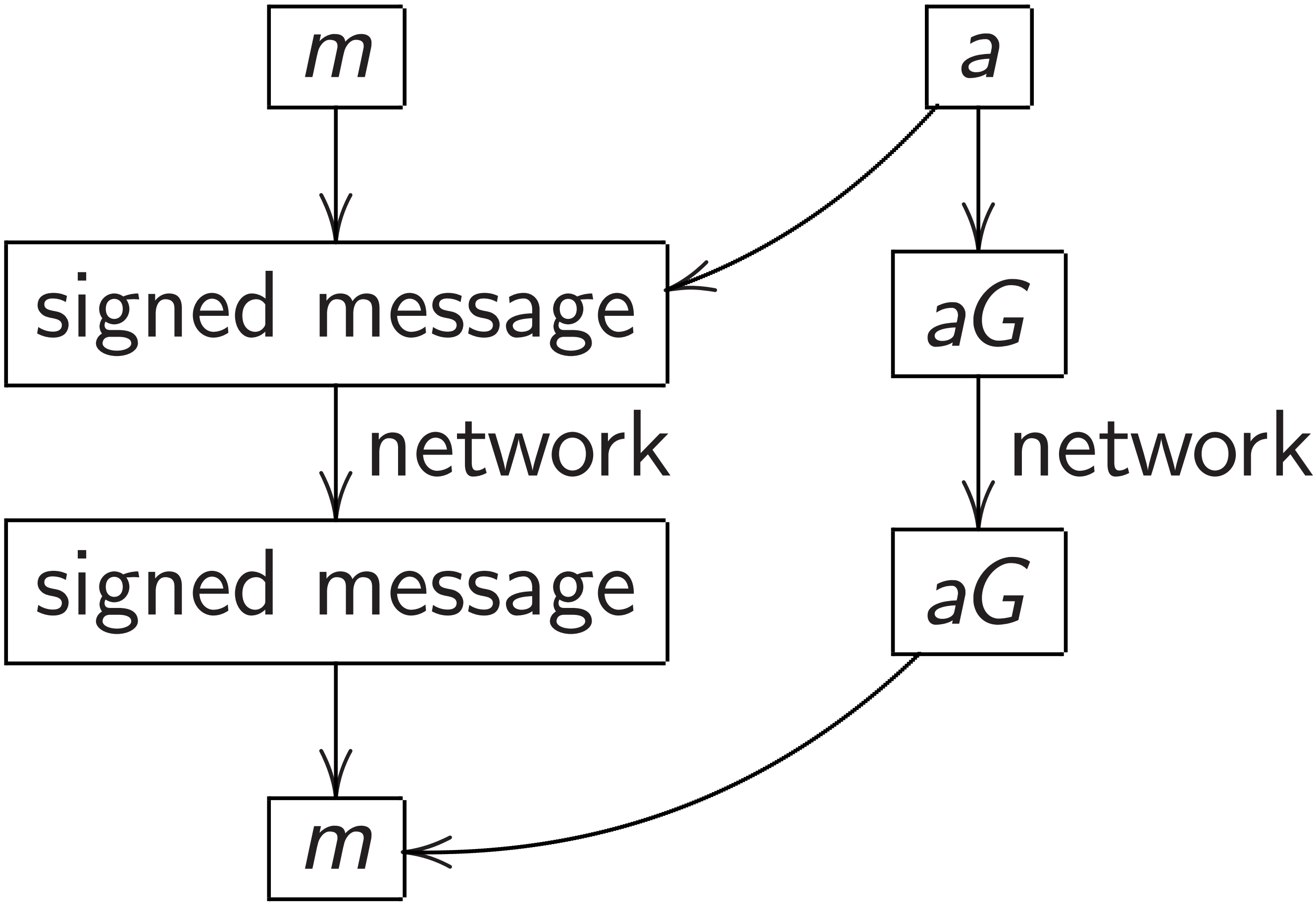
Solution 1:
Public-key encryption.



Solution 2:

Public-key signatures.



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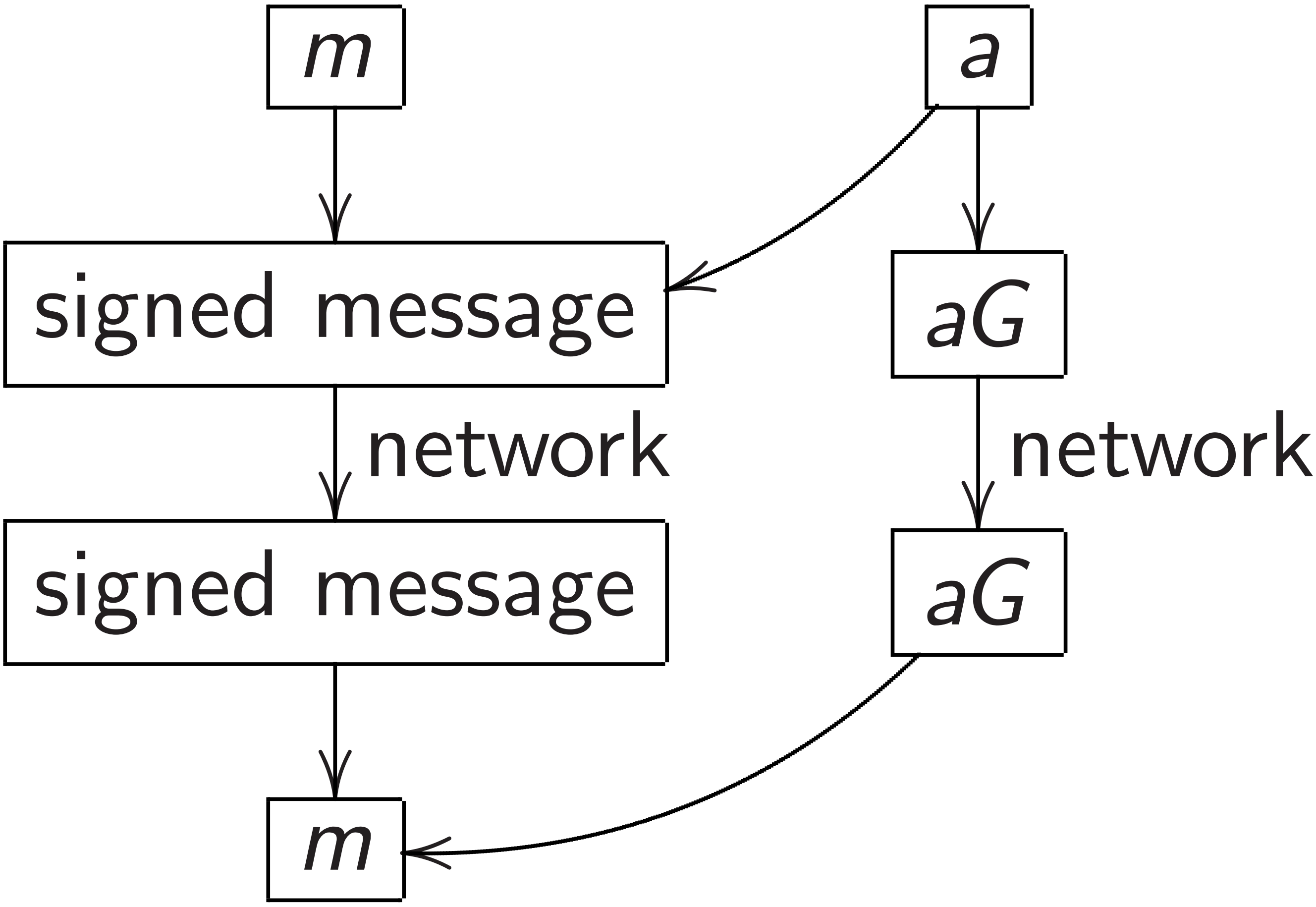

No more shared secret k

but Alice still has secret a.

Cryptography requires TCB

to protect secrecy of keys,

even if user has no other secrets.

Constant-time software

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Many attacks (e.g. TLBleed from 2018 Gras–Razavi–Bos–Giuffrida) show that this portion of the CPU has trouble keeping secrets.

Typical literature on this topic:

Understand this portion of CPU. But details are often proprietary, not exposed to security review.

Try to push attacks further. This becomes very complicated.

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For researchers: This is great!

For auditors: This is a nightmare.
Many years of security failures.
No confidence in future security.

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Good match for attitude and
experience of CPU designers: e.g.,
Intel issues errata for correctness
bugs, not for information leaks.

Case study: Constant-time sorting

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Subroutine in some submissions: sort array of secret integers. e.g. sort 768 32-bit integers.

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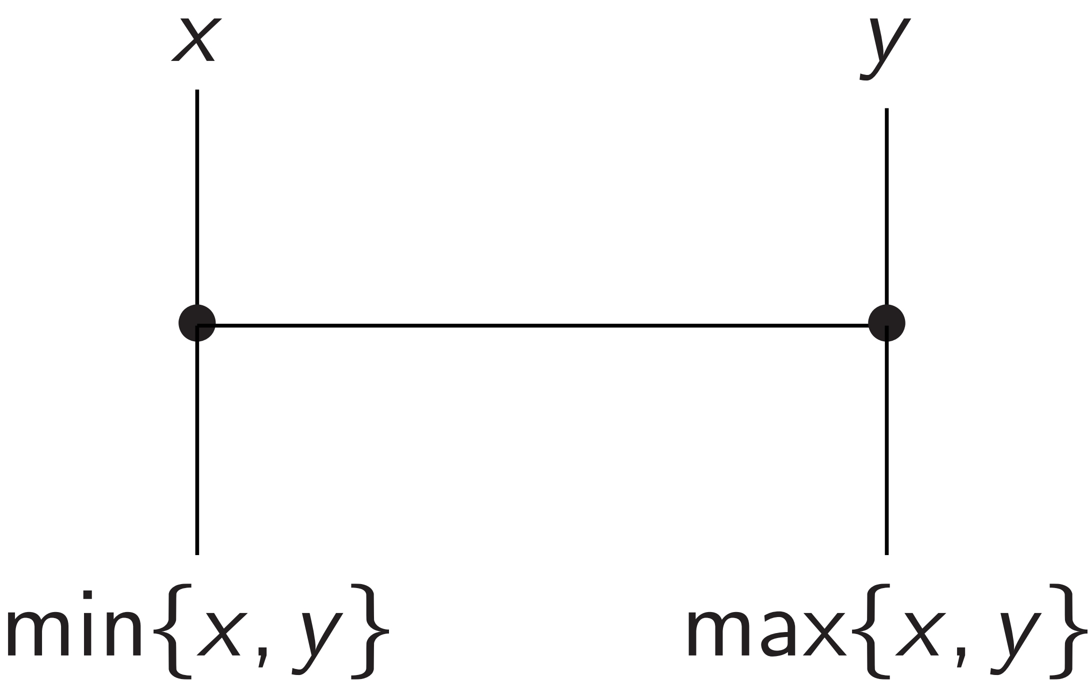
One submission to competition: "Radix sort is used as constant-time sorting algorithm." Some versions of radix sort avoid secret branches. But data addresses in radix sort still depend on secrets.

Foundation of solution:
a **comparator** sorting 2 integers.

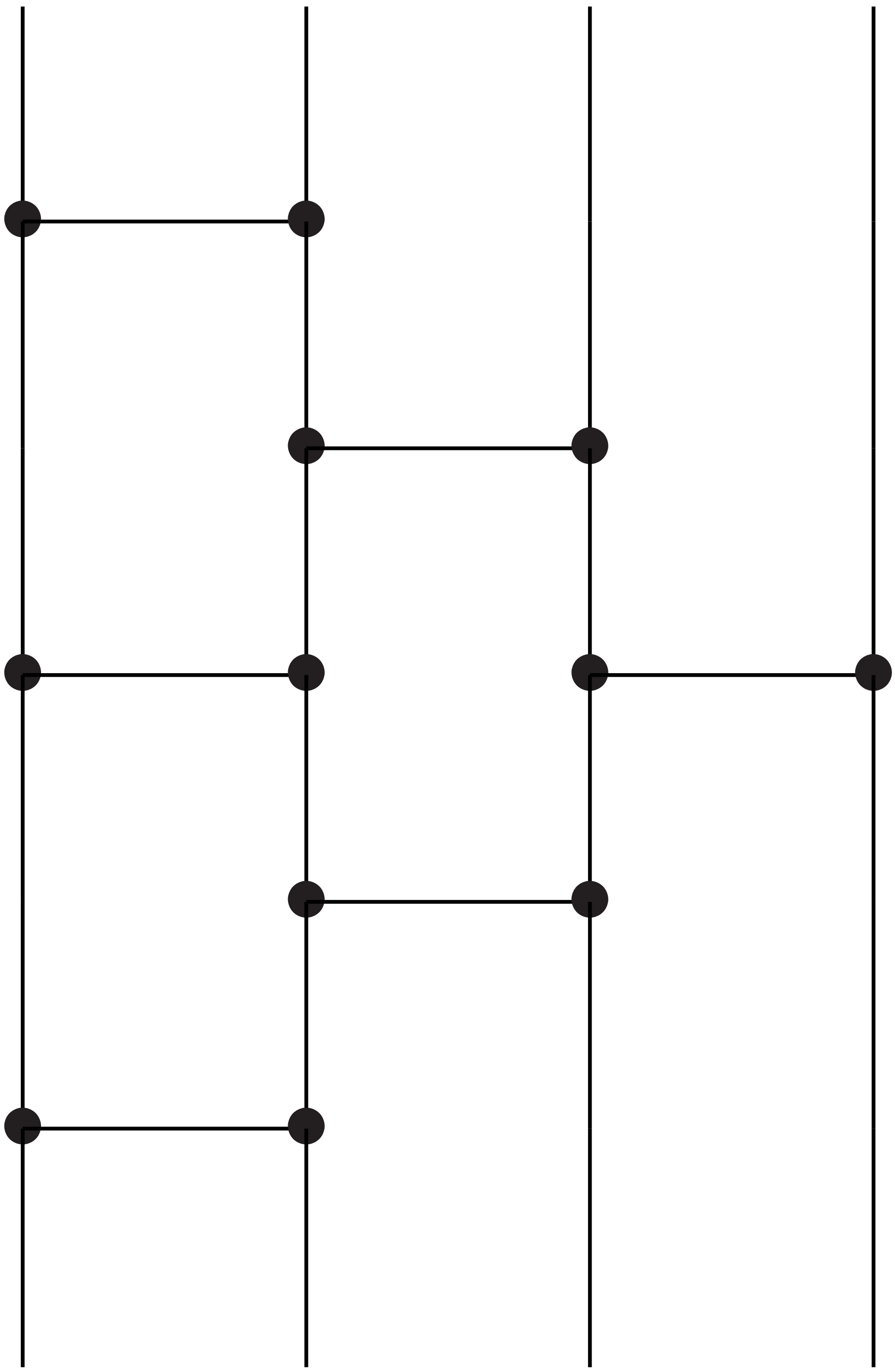


Easy constant-time exercise in C.
Warning: C standard allows compiler to screw this up.

Even easier exercise in asm.

Combine comparators into a **sorting network** for more inputs.

Example of a sorting network:



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Speed is a serious issue in the post-quantum competition. "Cost" is evaluation criterion; "we'd like to stress this once again on the forum that we'd really like to see more platform-optimized implementations"; etc.

```
void int32_sort(int32 *x,int64 n)
{ int64 t,p,q,i;
  if (n < 2) return;
  t = 1;
  while (t < n - t) t += t;
  for (p = t;p > 0;p >>= 1) {
    for (i = 0;i < n - p;++i)
      if (!(i & p))
        minmax(x+i,x+i+p);
    for (q = t;q > p;q >>= 1)
      for (i = 0;i < n - q;++i)
        if (!(i & p))
          minmax(x+i+p,x+i+q);
  }
}
```

Previous slide: C translation of 1973 Knuth "merge exchange", which is a simplified version of 1968 Batcher "odd-even merge" sorting networks.

$\approx n(\log_2 n)^2/4$ comparators.
Much faster than bubble sort.

Warning: many other descriptions of Batcher's sorting networks require n to be a power of 2.
Also, Wikipedia says "Sorting networks . . . are not capable of handling arbitrarily large inputs."

This constant-time sorting code

↓ vectorization (for Haswell)

Constant-time sorting code
included in 2017
Bernstein–Chuengsatiansup–
Lange–van Vredendaal
“NTRU Prime” software release

↓ revamped for higher speed

New: “djbsort”
constant-time sorting code

The slowdown for constant time

Massive fast-sorting literature.

2015 Gueron–Krasnov: AVX and AVX2 (Haswell) optimization of quicksort. For 32-bit integers: $\approx$45 cycles/byte for $n \approx 2^{10}$, $\approx$55 cycles/byte for $n \approx 2^{20}$.

Slower than "the radix sort implemented of IPP, which is the fastest in-memory sort we are aware of": 32, 40 cycles/byte.

IPP: Intel's Integrated Performance Primitives library.

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Warning: Comparison for $n \approx 2^{20}$ involves microarchitecture details beyond Haswell core. Should measure all code on same CPU.

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Loading a 32-bit integer from a random address: much slower.

Conditional branch: much slower.

Verification

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Test the sorting software on many random inputs, increasing inputs, decreasing inputs. Seems to work.

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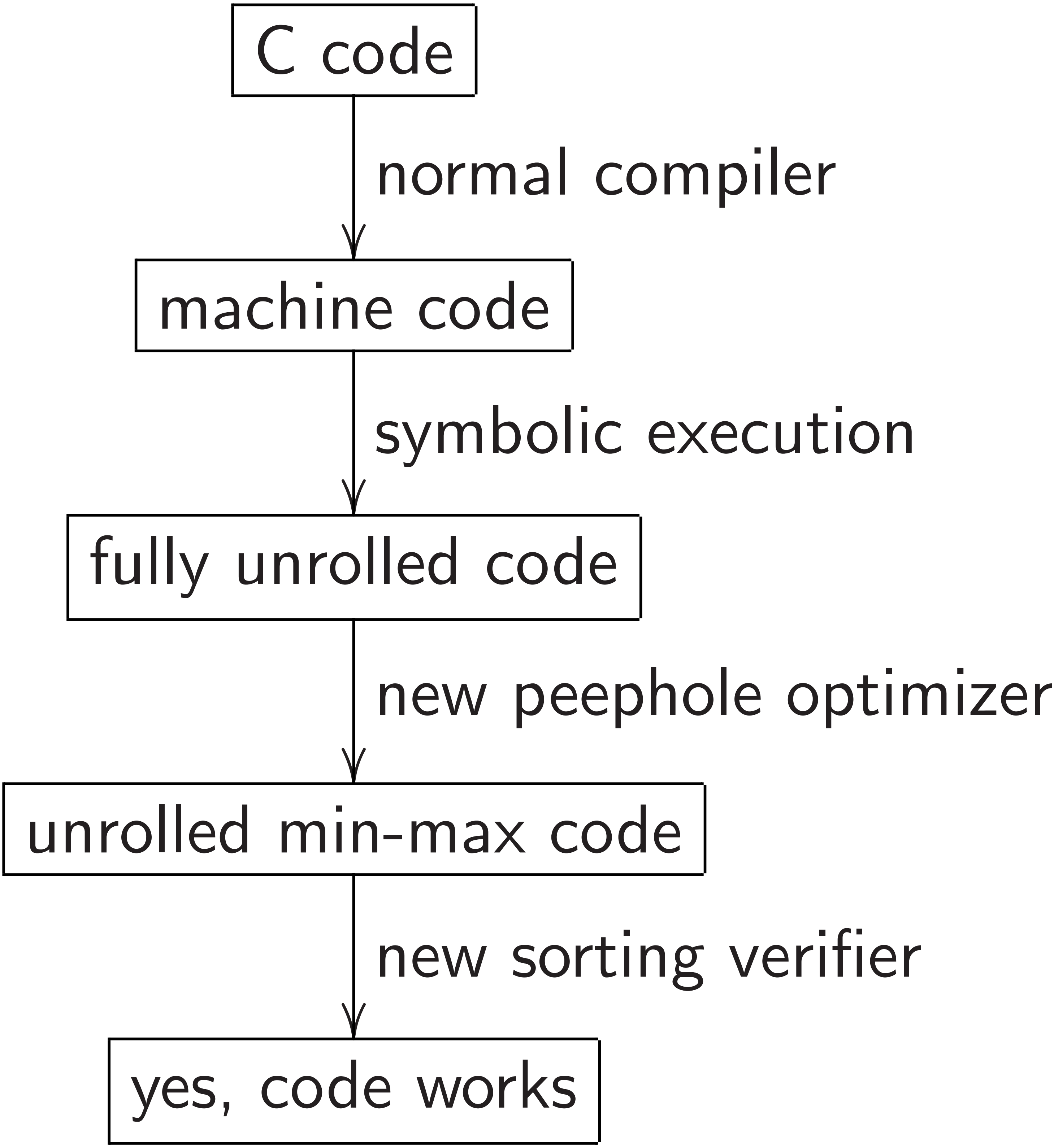
Test the sorting software on many random inputs, increasing inputs, decreasing inputs. Seems to work.

But are there *occasional* inputs where this sorting software fails to sort correctly?

History: Many security problems involve occasional inputs where TCB works incorrectly.

For each used n (e.g., 768):



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Sorting verifier: decompose
DAG into merging networks.
Verify each merging network
using generalization of 2007
Even–Levi–Litman, correction of
1990 Chung–Ravikumar.

First djbsort release,
verified `int32` on AVX2:

https://sorting.cr.yp.to

Includes the sorting code;
automatic build-time tests;
simple benchmarking program;
verification tools.

Web site shows how to
use the verification tools.

Next release planned:
verified ARM NEON code
and verified portable code.