

# Cryptographic software engineering, part 2

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Last time:

- General software engineering.
- Using const-time instructions.
- Comparing time to lower bound.

Example: Adding 1000 integers  
on Cortex-M4F. Lower bound:  
 $2n + 1$  cycles for  $n$  LDR +  $n$  ADD.  
Imagine not knowing this . . .

## Reference implementation:

```
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; ++i)
        result += x[i];
    return result;
}
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Try -01: 8012 cycles.

Try -02: 8012 cycles.

Try -03: 8012 cycles.

Try moving the pointer:

```
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; ++i)
        result += *x++;
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```

8010 cycles.



Try counting down:

```
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 1000; i > 0; --i)
        result += *x++;
    return result;
}
```

Try counting down:

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    int result = 0;
    int i;
    for (i = 1000; i > 0; --i)
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    return result;
}
```

8010 cycles.

Try using an end pointer:

```
int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    while (x != y)
        result += *x++;
    return result;
}
```

Try using an end pointer:

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int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    while (x != y)
        result += *x++;
    return result;
}
```

8010 cycles.

Back to original. Try unrolling:

```
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 2) {
        result += x[i];
        result += x[i + 1];
    }
    return result;
}
```

Back to original. Try unrolling:

```
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 2) {
        result += x[i];
        result += x[i + 1];
    }
    return result;
}
```

5016 cycles.

```
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 5) {
        result += x[i];
        result += x[i + 1];
        result += x[i + 2];
        result += x[i + 3];
        result += x[i + 4];
    }
    return result;
}
```

```
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 5) {
        result += x[i];
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    }
    return result;
}
```

4016 cycles. “Are we done yet?”



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    int i;
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        result += x[i + 2];
        result += x[i + 3];
        result += x[i + 4];
    }
    return result;
}
```

4016 cycles. “Are we done yet?”

No. Use the lower bound ...

```
int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    int x0,x1,x2,x3,x4,
        x5,x6,x7,x8,x9;

    while (x != y) {
        x0 = 0[(volatile int *)x];
        x1 = 1[(volatile int *)x];
        x2 = 2[(volatile int *)x];
        x3 = 3[(volatile int *)x];
        x4 = 4[(volatile int *)x];
        x5 = 5[(volatile int *)x];
        x6 = 6[(volatile int *)x];
```

```
x7 = 7[(volatile int *)x];  
x8 = 8[(volatile int *)x];  
x9 = 9[(volatile int *)x];  
  
result += x0;  
  
result += x1;  
  
result += x2;  
  
result += x3;  
  
result += x4;  
  
result += x5;  
  
result += x6;  
  
result += x7;  
  
result += x8;  
  
result += x9;  
  
x0 = 10[(volatile int *)x];  
x1 = 11[(volatile int *)x];
```

```
x2 = 12[(volatile int *)x];  
x3 = 13[(volatile int *)x];  
x4 = 14[(volatile int *)x];  
x5 = 15[(volatile int *)x];  
x6 = 16[(volatile int *)x];  
x7 = 17[(volatile int *)x];  
x8 = 18[(volatile int *)x];  
x9 = 19[(volatile int *)x];  
x += 20;  
  
result += x0;  
result += x1;  
result += x2;  
result += x3;  
result += x4;  
result += x5;
```

```
result += x6;
```

```
result += x7;
```

```
result += x8;
```

```
result += x9;
```

```
}
```

```
return result;
```

```
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```
    result += x6;  
    result += x7;  
    result += x8;  
    result += x9;  
}  
  
return result;  
}
```

2526 cycles. Even better in asm.

```
    result += x6;  
    result += x7;  
    result += x8;  
    result += x9;  
}  
  
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— [citation needed]



## A real example

Salsa20 reference software:

30.25 cycles/byte on this CPU.

Lower bound for arithmetic:

64 bytes require

21 · 16 1-cycle ADDs,

20 · 16 1-cycle XORs,

so at least 10.25 cycles/byte.

Also many rotations, but

ARMv7-M instruction set

includes free rotation

as part of XOR instruction.

(Compiler knows this.)

Detailed benchmarks show several cycles/byte spent on `load_littleendian` and `store_littleendian`.

Can replace with `LDR` and `STR`.  
(Compiler doesn't see this.)

Then observe 23 cycles/byte:  
18 cycles/byte for rounds,  
plus 5 cycles/byte overhead.  
Still far above 10.25 cycles/byte.

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Still far above 10.25 cycles/byte.

Gap is mostly loads, stores.  
Minimize load/store cost by  
choosing “spills” carefully.

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On bigger CPUs,  
selecting vector instructions  
is critical for performance.

<https://bench.cr.yp.to>

includes 2392 implementations  
of 614 cryptographic primitives.

>20 implementations of Salsa20.

Haswell: Reasonably simple ref  
implementation compiled with  
`gcc -O3 -fomit-frame-pointer`  
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merged implementation  
with “machine-independent”  
optimizations and best of 121  
compiler options:  $4.52\times$  slower.

## Fast random permutations

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McEliece encryption example:

Randomly order 6960 bits

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NTRU encryption example:

Randomly order 761 trits

$(\pm 1, \dots, \pm 1, 0, \dots, 0)$ , wt 286.

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Example:  $n = 6960$  bits;

weight 119; 31-bit  $r_i$ ; no restart.

Any output is produced in  
 $\leq 119!(n - 119)! \binom{2^{31} + n - 1}{n}$  ways;  
 i.e.,  $< 1.02 \cdot 2^{31n} / \binom{n}{119}$  ways.

Factor  $< 1.02$  increase in  
attacker's chance of winning.

Which sorting algorithm?

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But these algorithms rely on secret branches and secret indices.

Exercise: convert mergesort into constant-time mergesort using  $\Theta(n^2)$  operations.

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Sorting network on next slide:  
Batcher’s merge-exchange sort.  
 $\Theta(n(\log n)^2)$  minmax operations;  
 $(1/4)(e^2 - e + 4)n - 1$  for  $n = 2^e$ .

```
void sort(int32 *x, long long n)
{ long long t, p, q, i;
  t = 1; if (n < 2) return;
  while (t < n-t) t += t;
  for (p = t; p > 0; p >>= 1) {
    for (i = 0; i < n-p; ++i)
      if (!(i & p))
        minmax(x+i, x+i+p);
    for (q = t; q > p; q >>= 1) {
      for (i = 0; i < n-q; ++i)
        if (!(i & p))
          minmax(x+i+p, x+i+q);
    }
  }
}
```



How many cycles on, e.g.,  
Intel Haswell CPU core?

Every cycle: a vector of 8 32-bit  
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$\geq 3008$  cycles for  $n = 1024$ .

Current software (from 2017  
Bernstein–Chuengsatiansup–  
Lange–van Vredendaal “NTRU  
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Some gap, but already  $5\times$   
faster than Intel’s Integrated  
Performance Primitives library.

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CPUs are evolving farther and farther away from this naive model.

Fundamental hardware costs of constant-time arithmetic are much lower than random access.

## Modular arithmetic

Basic ECC operations:

add, sub, mul of, e.g.,  
integers mod  $2^{255} - 19$ .

(Basic NTRU operations:

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polynomials mod  $x^{761} - x - 1$ .)

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Typical “big-integer library”:

a variable-length uint32 string

$(f_0, f_1, \dots, f_{\ell-1})$  represents

the nonnegative integer

$$f_0 + 2^{32} f_1 + \dots + 2^{32(\ell-1)} f_{\ell-1}.$$

Uniqueness:  $\ell = 0$  or  $f_{\ell-1} \neq 0$ .



Library provides functions acting on this representation: (1)  $f, g \mapsto fg$ ; (2)  $f, g \mapsto f \bmod g$ ; etc.

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Need a different representation for constant-time arithmetic.

Can also gain speed this way.

Constant-time bigint library:

a constant-length `uint32` string

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Adding two  $\ell$ -limb integers:

always allocate  $\ell + 1$  limbs.

Don't remove top zero limb.

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refined than  $2^0, 2^{32}, 2^{64}, 2^{96}, \dots$ ;

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$f \bmod p$  is as short as  $p$ .

Usually faster representation:

`uint32` string  $(f_0, f_1, \dots, f_9)$

represents  $f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9$ .

Constant bound on each  $f_i$ .

More limbs than before,  
but save time by avoiding  
overflows and delaying carries.

After multiplication,  
replace  $2^{255}$  with 19.



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After multiplication,  
replace  $2^{255}$  with 19.

Slightly faster on some CPUs:

`int32` string  $(f_0, f_1, \dots, f_9)$ .

```
int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
    f7_2 * (int64) g7_19;
...
int64 h4 = f0g4 + f1g3_2
          + f2g2 + f3g1_2
          + f4g0 + f5g9_38
          + f6g8_19 + f7g7_38
          + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```

Initial computation of  $h_0, \dots, h_9$   
is polynomial multiplication  
modulo  $x^{10} - 19$ .

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At end of computation:  
**freeze** representation  
into unique representation  
suitable for network transmission.

Much more about ECC speed:  
see, e.g., [2015 Chou](#).

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Progress in deploying proven  
fast software: see, e.g., 2015  
Bernstein–Schwabe “[gfverif](#)”;  
2017 [HAACL\\*](#) [X25519](#) in Firefox.

gfverif has verified ref10  
 implementation of X25519,  
 plus occasional annotations,  
 against the following specification:

$p = 2^{255} - 19$

$A = 486662$

$x_2, z_2, x_3, z_3 = 1, 0, x_1, 1$

for  $i$  in reversed(range(255)):

$n_i = \text{bit}(n, i)$

$x_2, x_3 = \text{cswap}(x_2, x_3, n_i)$

$z_2, z_3 = \text{cswap}(z_2, z_3, n_i)$

$x_3, z_3 = (4 * (x_2 * x_3 - z_2 * z_3))^{**2},$

$4 * x_1 * (x_2 * z_3 - z_2 * x_3)^{**2})$

$x_2, z_2 = ((x_2^{**2} - z_2^{**2})^{**2},$

$4 * x_2 * z_2 * (x_2^{**2} + A * x_2 * z_2 + z_2^{**2}))$

```
x3, z3 = (x3%p, z3%p)
```

```
x2, z2 = (x2%p, z2%p)
```

```
cut(x2)
```

```
cut(x3)
```

```
cut(z2)
```

```
cut(z3)
```

```
x2, x3 = cswap(x2, x3, ni)
```

```
z2, z3 = cswap(z2, z3, ni)
```

```
cut(x2)
```

```
cut(z2)
```

```
return x2*pow(z2, p-2, p)
```

What's verified: output of ref10  
is the same as spec mod  $p$ ,  
and is between 0 and  $p - 1$ .

## “What a difference a prime makes”

NIST P-256 prime  $p$  is

$$2^{256} - 2^{224} + 2^{192} + 2^{96} - 1.$$

ECDSA standard specifies  
reduction procedure given  
an integer “ $A$  less than  $p^2$ ”:

Write  $A$  as

$$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, \\ A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0),$$

meaning  $\sum_i A_i 2^{32i}$ .

Define

$$T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$$

as

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0);$   
 $(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0);$   
 $(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0);$   
 $(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8);$   
 $(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9);$   
 $(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11});$   
 $(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12});$   
 $(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13});$   
 $(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14}).$

Compute  $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4.$

Reduce modulo  $p$  “by adding or subtracting a few copies” of  $p.$

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Variable-time loop is unsafe.

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Correct but quite slow:

conditionally add  $4p$ ,

conditionally add  $2p$ ,

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Even worse: what about platforms where  $2^{32}$  isn't best radix?

There are many more ways that cryptographic design choices affect difficulty of building fast correct constant-time software.

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What's better use of time:  
implementing ECDSA, or  
upgrading protocol to EdDSA?