Cryptographic software engineering, part 2

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Last time:
• General software engineering.
• Using const-time instructions.
• Comparing time to lower bound.

Example: Adding 1000 integers on Cortex-M4F. Lower bound: $2n + 1$ cycles for $n$ LDR + $n$ ADD. Imagine not knowing this . . .
Reference implementation:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; ++i)
        result += x[i];
    return result;
}
```
Reference implementation:

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int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; ++i)
        result += x[i];
    return result;
}
```

Try -O0: 8012 cycles.
Reference implementation:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; ++i)
        result += x[i];
    return result;
}
```

Try `-O0s`: 8012 cycles.
Try `-O1`: 8012 cycles.
Reference implementation:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0;i < 1000;++i)
        result += x[i];
    return result;
}
```

Try `-Os`: 8012 cycles.
Try `-O1`: 8012 cycles.
Try `-O2`: 8012 cycles.
Reference implementation:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; ++i)
        result += x[i];
    return result;
}
```

Try `-Os`: 8012 cycles.
Try `-O1`: 8012 cycles.
Try `-O2`: 8012 cycles.
Try `-O3`: 8012 cycles.
Try moving the pointer:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; ++i)
        result += *x++;
    return result;
}
```
Try moving the pointer:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; ++i)
        result += *x++;
    return result;
}
```

8010 cycles.
Try counting down:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 1000; i > 0; --i)
        result += *x++;
    return result;
}
```
Try counting down:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 1000;i > 0;--i)
        result += *x++;
    return result;
}
```

8010 cycles.
Try using an end pointer:

```c
int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    while (x != y)
    {
        result += *x++;
    }
    return result;
}
```
Try using an end pointer:

```c
int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    while (x != y)
    {
        result += *x++;
    }
    return result;
}
```

8010 cycles.
Back to original. Try unrolling:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 2) {
        result += x[i];
        result += x[i + 1];
    }
    return result;
}
```
Back to original. Try unrolling:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 2) {
        result += x[i];
        result += x[i + 1];
    }
    return result;
}
```

5016 cycles.
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 5) {
        result += x[i];
        result += x[i + 1];
        result += x[i + 2];
        result += x[i + 3];
        result += x[i + 4];
    }
    return result;
}
```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 5) {
        result += x[i];
        result += x[i + 1];
        result += x[i + 2];
        result += x[i + 3];
        result += x[i + 4];
    }
    return result;
}
```

4016 cycles. “Are we done yet?”
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 5) {
        result += x[i];
        result += x[i + 1];
        result += x[i + 2];
        result += x[i + 3];
        result += x[i + 4];
    }
    return result;
}

4016 cycles. “Are we done yet?”
No. Use the lower bound . . .
int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    int x0, x1, x2, x3, x4,  
        x5, x6, x7, x8, x9;

    while (x != y) {
        x0 = 0[&volatile int *x];
        x1 = 1[&volatile int *x];
        x2 = 2[&volatile int *x];
        x3 = 3[&volatile int *x];
        x4 = 4[&volatile int *x];
        x5 = 5[&volatile int *x];
        x6 = 6[&volatile int *x];
x7 = 7[(volatile int *)x];
x8 = 8[(volatile int *)x];
x9 = 9[(volatile int *)x];
result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
result += x5;
result += x6;
result += x7;
result += x8;
result += x9;
x0 = 10[(volatile int *)x];
x1 = 11[(volatile int *)x];
x2 = 12[(volatile int *)x];
x3 = 13[(volatile int *)x];
x4 = 14[(volatile int *)x];
x5 = 15[(volatile int *)x];
x6 = 16[(volatile int *)x];
x7 = 17[(volatile int *)x];
x8 = 18[(volatile int *)x];
x9 = 19[(volatile int *)x];
x += 20;
result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
result += x5;
result += x6;
result += x7;
result += x8;
result += x9;

return result;
}
result += x6;
result += x7;
result += x8;
result += x9;
}

return result;
}

2526 cycles. Even better in asm.
result += x6;
result += x7;
result += x8;
result += x9;
}

return result;
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result += x8;
result += x9;
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return result;
}

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A real example

Salsa20 reference software: 30.25 cycles/byte on this CPU.

Lower bound for arithmetic: 64 bytes require
21 · 16 1-cycle ADDs,
20 · 16 1-cycle XORs,
so at least 10.25 cycles/byte.

Also many rotations, but ARMv7-M instruction set includes free rotation as part of XOR instruction. (Compiler knows this.)
Detailed benchmarks show several cycles/byte spent on load\_littleendian and store\_littleendian.

Can replace with LDR and STR. (Compiler doesn’t see this.)

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Gap is mostly loads, stores. Minimize load/store cost by choosing “spills” carefully.
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Spill to FPU instead of stack?
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On bigger CPUs, selecting vector instructions is critical for performance.
https://bench.cr.yp.to includes 2392 implementations of 614 cryptographic primitives. >20 implementations of Salsa20.

Haswell: Reasonably simple ref implementation compiled with gcc -O3 -fomit-frame-pointer is 6.15× slower than fastest Salsa20 implementation.
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merged implementation with “machine-independent” optimizations and best of 121 compiler options: 4.52× slower.
Fast random permutations

Goal: Put list \((x_1, \ldots, x_n)\) into a random order.
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One textbook strategy:
Sort \((Mr_1 + x_1, \ldots, Mr_n + x_n)\) for random \((r_1, \ldots, r_n)\), suitable \(M\).
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McEliece encryption example:
Randomly order 6960 bits \((1, \ldots, 1, 0, \ldots, 0)\), weight 119.
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McEliece encryption example:
Randomly order 6960 bits \((1, \ldots, 1, 0, \ldots, 0)\), weight 119.

NTRU encryption example:
Randomly order 761 trits \((\pm1, \ldots, \pm1, 0, \ldots, 0)\), wt 286.
Simulate uniform random $r_i$ using RNG: e.g., stream cipher.
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How many bits in $r_i$? Negligible collisions? Occasional collisions?
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Uniform distribution; some cost.
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Restart on collision?
Uniform distribution; some cost.

Example: $n = 6960$ bits; weight 119; 31-bit $r_i$; no restart.
Any output is produced in $\leq 119!(n - 119)!(2^{31} + n - 1) \binom{n}{119}$ ways; i.e., $< 1.02 \cdot 2^{31n}/\binom{n}{119}$ ways.
Factor $< 1.02$ increase in attacker’s chance of winning.
Which sorting algorithm?

Reference bubblesort code does \( n(n - 1)/2 \) minmax operations.
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Many standard algorithms use fewer operations: mergesort, quicksort, heapsort, radixsort, etc.

But these algorithms rely on secret branches and secret indices.
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Many standard algorithms use fewer operations: mergesort, quicksort, heapsort, radixsort, etc.

But these algorithms rely on secret branches and secret indices.

Exercise: convert mergesort into constant-time mergesort using $\Theta(n^2)$ operations.
Converting bubblesort into constant-time bubblesort loses only a constant factor: cost of constant-time minmax.
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Sorting network on next slide: Batcher’s merge-exchange sort. \( \Theta(n(\log n)^2) \) minmax operations; 
\( (1/4)(e^2 - e + 4)n - 1 \) for \( n = 2^e \).
void sort(int32 *x, long long n)
{
    long long t, p, q, i;
    t = 1; if (n < 2) return;
    while (t < n-t) t += t;
    for (p = t; p > 0; p >>= 1) {
        for (i = 0; i < n-p; ++i)
            if (!(i & p))
                minmax(x+i, x+i+p);
        for (q = t; q > p; q >>= 1) {
            for (i = 0; i < n-q; ++i)
                if (!(i & p))
                    minmax(x+i+p, x+i+q);
        }
    }
}
How many cycles on, e.g., Intel Haswell CPU core?

Every cycle: a vector of 8 32-bit “min” operations and a vector of 8 32-bit “max” operations.
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Every cycle: a vector of 8 32-bit “min” operations and a vector of 8 32-bit “max” operations.

\[\geq 3008\] cycles for \(n = 1024\).


Some gap, but already 5× faster than Intel’s Integrated Performance Primitives library.
Constant-time code faster than “optimized” non-constant-time code? How is this possible?
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People optimize algorithms for a naive model of CPUs:
- Branches are fast.
- Random access is fast.
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People optimize algorithms for a naive model of CPUs:
• Branches are fast.
• Random access is fast.

CPUs are evolving farther and farther away from this naive model. Fundamental hardware costs of constant-time arithmetic are much lower than random access.
Modular arithmetic

Basic ECC operations: add, sub, mul of, e.g., integers mod $2^{255} - 19$.

(Basic NTRU operations: add, sub, mul of, e.g., polynomials mod $x^{761} - x - 1$.)
Modular arithmetic

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(Basic NTRU operations: add, sub, mul of, e.g., polynomials mod $x^{761} - x - 1$.)

Typical “big-integer library”: a variable-length uint32 string $(f_0, f_1, \ldots, f_{\ell - 1})$ represents the nonnegative integer $f_0 + 2^{32} f_1 + \cdots + 2^{32(\ell - 1)} f_{\ell - 1}$.

Uniqueness: $\ell = 0$ or $f_{\ell - 1} \neq 0$. 
Library provides functions acting on this representation: (1) $f, g \mapsto f \cdot g$; (2) $f, g \mapsto f \mod g$; etc.
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But these functions take variable time to ensure uniqueness!

Need a different representation for constant-time arithmetic. Can also gain speed this way.
Constant-time bigint library: a constant-length uint32 string 
\((f_0, f_1, \ldots, f_{\ell-1})\) represents the nonnegative integer 
\[ f_0 + 2^{32} f_1 + \cdots + 2^{32}(\ell-1) f_{\ell-1}. \]

Adding two \(\ell\)-limb integers: always allocate \(\ell + 1\) limbs. Don’t remove top zero limb.
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Can also track bounds more refined than \(2^0, 2^{32}, 2^{64}, 2^{96}, \ldots\); but no limbs→bounds data flow.
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Adding two $\ell$-limb integers:
always allocate $\ell + 1$ limbs.
Don’t remove top zero limb.

Can also track bounds more
refined than $2^0, 2^{32}, 2^{64}, 2^{96}, \ldots$;
but no limbs $\rightarrow$ bounds data flow.

$f \mod p$ is as short as $p$. 
Usually faster representation:

\[ \text{uint32 string } (f_0, f_1, \ldots, f_9) \]

represents

\[ f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9. \]

Constant bound on each \( f_i \).

More limbs than before, but save time by avoiding overflows and delaying carries.

After multiplication, replace \( 2^{255} \) with 19.
Usually faster representation:

\texttt{uint32} string \((f_0, f_1, \ldots, f_9)\)
represents \(f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9.\)

Constant bound on each \(f_i\).

More limbs than before, but save time by avoiding overflows and delaying carries.

After multiplication, replace \(2^{255}\) with 19.

Slightly faster on some CPUs: \texttt{int32} string \((f_0, f_1, \ldots, f_9)\).
```c
int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
    f7_2 * (int64) g7_19;
...
int64 h4 = f0g4 + f1g3_2
    + f2g2 + f3g1_2
    + f4g0 + f5g9_38
    + f6g8_19 + f7g7_38
    + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```
Initial computation of $h_0, \ldots, h_9$ is polynomial multiplication modulo $x^{10} - 19$. 
Exercise: Which polynomials are being multiplied?
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**squeeze** the product into limited-size representation suitable for next multiplication.
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Exercise: Which polynomials are being multiplied?

Reduction modulo $x^{10} - 19$ and carries such as $h_4 \rightarrow h_5$ squeeze the product into limited-size representation suitable for next multiplication.

At end of computation: freeze representation into unique representation suitable for network transmission.
Much more about ECC speed:
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Progress in deploying proven fast software: see, e.g., 2015 Bernstein–Schwabe “gfverif”; 2017 HACL* X25519 in Firefox.
gfverif has verified ref10 implementation of X25519, plus occasional annotations, against the following specification:

\[ p = 2^{255} - 19 \]
\[ A = 486662 \]
\[ x_2, z_2, x_3, z_3 = 1, 0, x_1, 1 \]

for i in reversed(range(255)):
  ni = bit(n, i)
  x2, x3 = cswap(x2, x3, ni)
  z2, z3 = cswap(z2, z3, ni)
  x3, z3 = (4*(x2*x3-z2*z3)**2, 4*x1*(x2*z3-z2*x3)**2)
  x2, z2 = ((x2**2-z2**2)**2, 4*x2*z2*(x2**2+A*x2*z2+z2**2))
x3, z3 = (x3%p, z3%p)
x2, z2 = (x2%p, z2%p)
cut(x2)
cut(x3)
cut(z2)
cut(z3)
x2, x3 = cswap(x2, x3, ni)
z2, z3 = cswap(z2, z3, ni)
cut(x2)
cut(z2)
return x2*pow(z2, p-2, p)

What’s verified: output of ref10 is the same as spec mod p, and is between 0 and p – 1.
“What a difference a prime makes”

NIST P-256 prime $p$ is

$$2^{256} - 2^{224} + 2^{192} + 2^{96} - 1.$$ 

ECDSA standard specifies reduction procedure given an integer “$A$ less than $p^2$”:

Write $A$ as

$$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0),$$

meaning $\sum_i A_i 2^{32i}$.

Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as
Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo $p$ “by adding or subtracting a few copies” of $p$. 
What is “a few copies”?  
Variable-time loop is unsafe.
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Correct but quite slow: 
conditionally add $4p$, 
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Delay until end of computation?  
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Even worse: what about platforms where $2^{32}$ isn’t best radix?
There are many more ways that cryptographic design choices affect difficulty of building fast correct constant-time software.

e.g. ECDSA needs divisions of scalars. EdDSA doesn’t.

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What’s better use of time: implementing ECDSA, or upgrading protocol to EdDSA?