

# High-speed cryptography

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## Do we care about speed?

Almost all software is much slower than it could be.

Is software applied to much data?

Usually not. Usually the wasted CPU time is negligible.

But *crypto* software should be applied to all communication.

Crypto that's too slow  $\Rightarrow$   
fewer users  $\Rightarrow$  fewer cryptanalysts  
 $\Rightarrow$  less attractive for everybody.

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e.g. ECDSA verification computes  $(S^{-1}H(M))B + (S^{-1}R)A$ .

OpenSSL has complicated code for fast computation of  $S^{-1}$ .

Much simpler code would make verification considerably slower.

## Applications pursue speed

e.g. Latest “DNSSEC operational practices” recommendation

(2012) says “No one has broken a regular 1024-bit [RSA] key . . .

it is estimated that most zones can safely use 1024-bit keys for at least the next ten years . . .

Signing and verifying with a 2048-bit key takes longer than with a 1024-bit key . . . public operations (such as verification) are about four times slower.”

DNSSEC key sizes, 2016.11.28:

2048-bit DNSSEC master key  
controlled by U.S.

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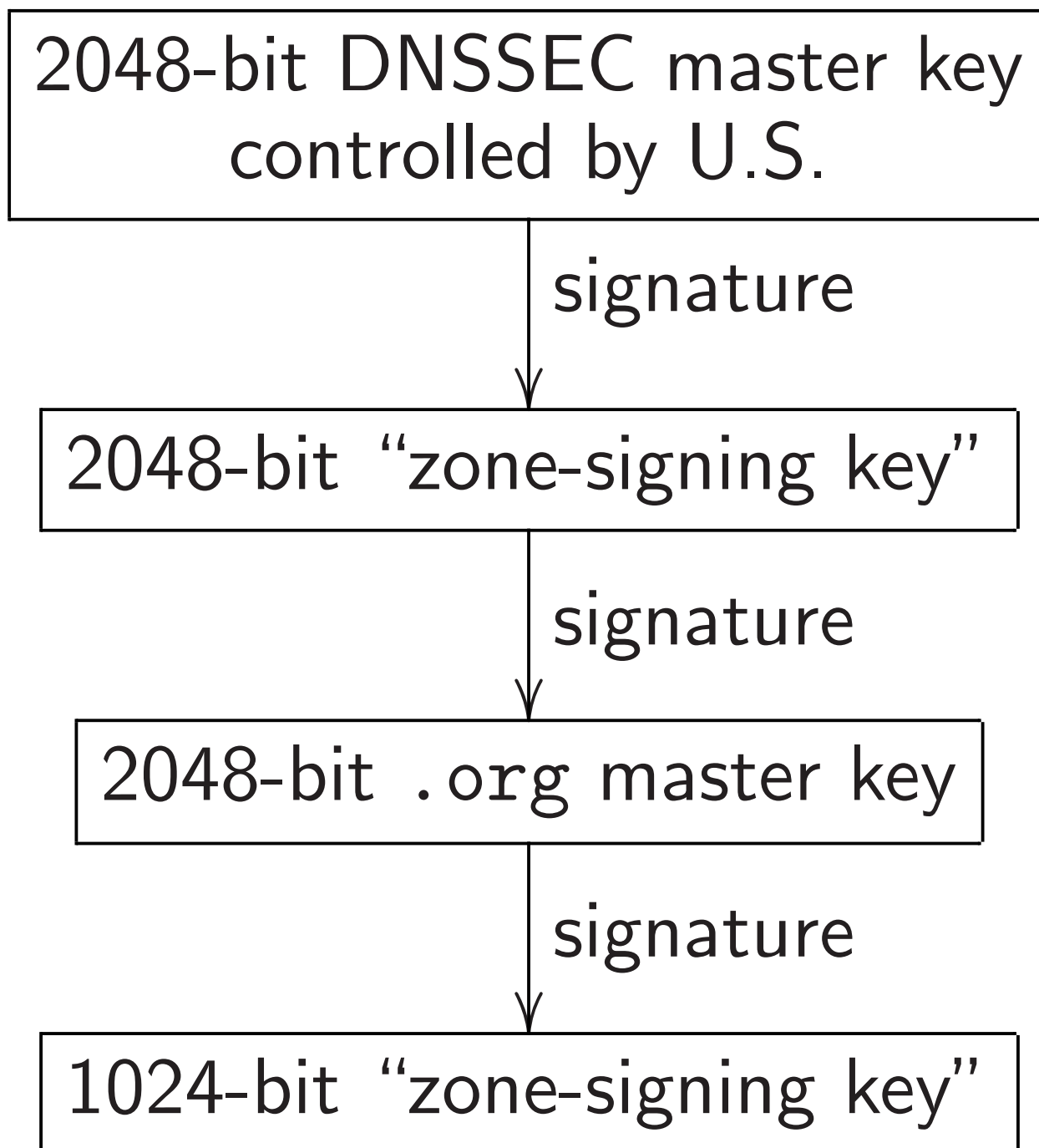
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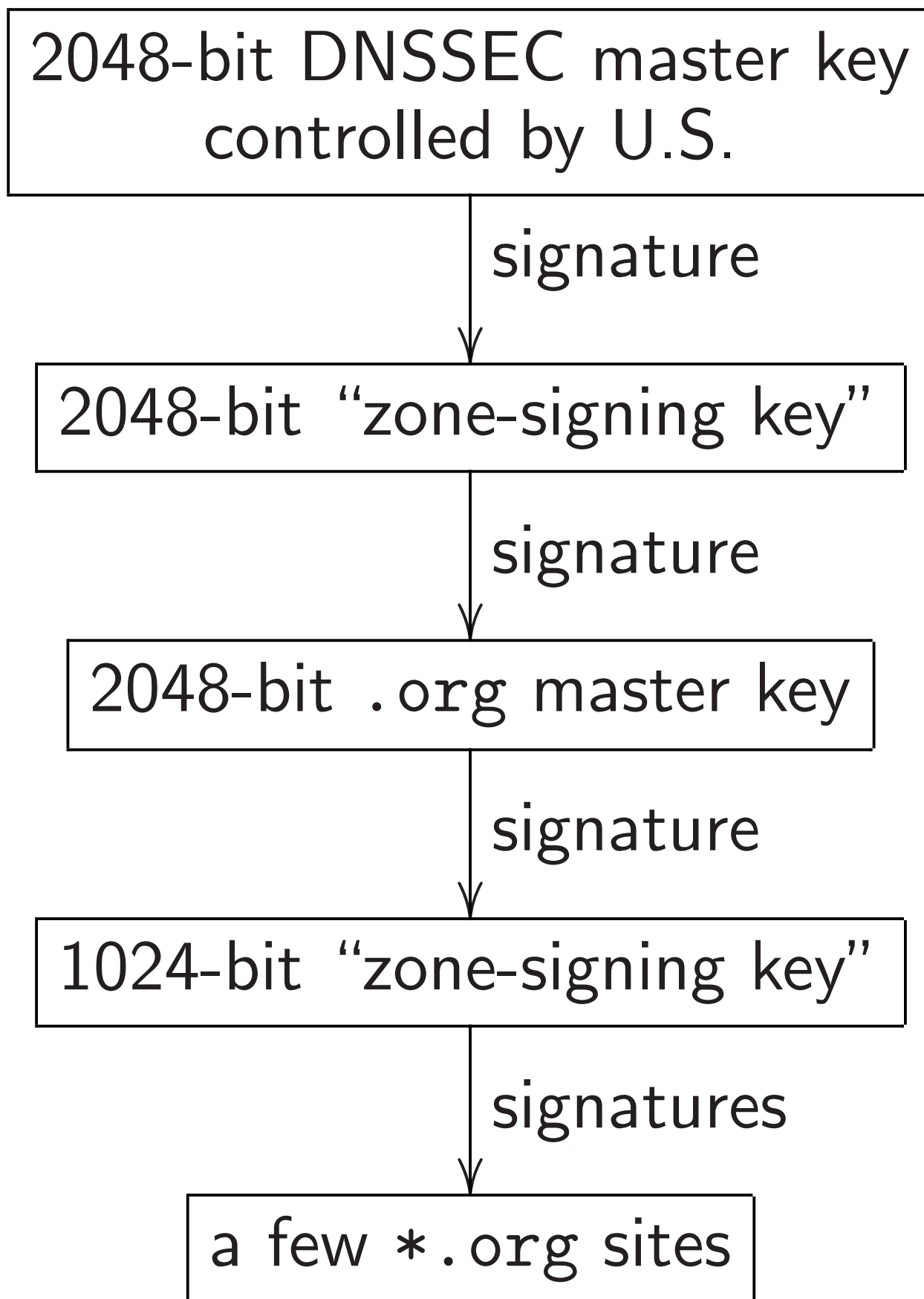
2048-bit .org master key



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2011 Weimerskirch survey of security for car communication:

“V2V safety applications will broadcast 10 messages per second, and a vehicle will receive 1,000 or more messages per second. There are two approaches available to process such a high amount of messages: (1) only messages that might impose an actual impact to a vehicle are processed, or (2) dedicated security hardware to process all messages is applied.”

2014 Ghoreishizadeh–Yalcin–  
Pullini–Micheli–Burleson–Carrara  
“A lightweight cryptographic  
system for implantable  
biosensors”: “This design uses  
the recently standardized SHA-3  
Keccak secure hash function  
implemented in an authenticated  
encryption mode . . . By selecting  
the newly standardized Keccak  
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large amount of analysis and  
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we have used the same number of rounds for all in order to **guarantee the security claim** of the Keccak proposal. However, instead of using the standard sizes for bitrate and capacity, we reduced the overall state size in order to achieve a compact implementation with a security level that would not have been possible at this cost with any other authenticated encryption scheme. The data block size and state size are selected as 4 and 100 bits, respectively.”

## Standards pursue speed

e.g. NIST's final AES report:

“Security was the most important factor in the evaluation . . .

Rijndael appears to offer an *adequate* security margin. . . .

Serpent appears to offer a *high* security margin.”

(Emphasis added.)

So why didn't Serpent win?

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So why didn't Serpent win?

Maybe side-channel security?



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Hardware speed: “Serpent is well suited to restricted-space environments . . . Fully pipelined implementations of Serpent offer the highest throughput of any of the finalists for non-feedback modes. . . . Efficiency is generally very good, and Serpent’s speed is independent of key size.”

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Great! Why didn’t Serpent win?

Aha: Software speed!

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Conclusion: “NIST judged Rijndael to be the best overall algorithm for the AES. Rijndael appears to be consistently a very good performer in both hardware and software [and offers good key agility, low memory, easy defense, fast defense, flexibility, parallelism].”

## Want fast *and* secure

Bad examples:

The pursuit of speed  
damages security.

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Good examples:

Obtain better speed  
*without* damaging security.

If security level was too low,  
scale up: better security  
for the same performance.



Success story: ECC.

Extensive work on speed of  
ECC at a high security level  
 $\Rightarrow$  modern ECC is fast enough  
for practically all applications.

Requires serious analysis  
and optimization of algorithms.

Not just “polynomial time”;  
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RSA and Rabin–Williams are even faster for signature verification, but slower for keygen, signing, sending keys, sending sigs.

## Some signature-system history

1985 ElGamal:  $\mathbf{F}_p^*$  signatures.

1990 Schnorr: ElGamal  
plus various improvements.

Patented until 2008.

1991 DSA, announced by NIST,  
later credited to NSA: ElGamal  
with one Schnorr improvement.

1999 ECDSA: replacing  $\mathbf{F}_p^*$  in  
DSA with an elliptic-curve group.

2011 EdDSA (e.g., Ed25519):  
Schnorr plus more improvements.

ElGamal verification:

$(R, S)$  is signature of  $M$

if  $B^{H(M)} \equiv A^R R^S \pmod{p}$

and  $R, S \in \{0, 1, \dots, p - 2\}$ .

Here  $p$  is standard prime,

$B$  is standard base,

$A$  is signer's public key,

$H(M)$  is hash of message.

Secret key: random  $a$ .

Public key:  $A = B^a \pmod{p}$ .

To sign  $M$ : generate random  $r$ ,

compute  $R = B^r \pmod{p}$ ,

$S = r^{-1}(H(M) - aR) \pmod{p - 1}$ .

## Hash the exponent

Tweak:  $(R, S)$  is signature of  $M$   
if  $B^{H(M)} \equiv A^{H(R)} R^S \pmod{p}$   
and  $R, S \in \{0, 1, \dots, p - 2\}$ .

Signer: as before except  $S =$   
 $r^{-1}(H(M) - aH(R)) \pmod{p - 1}$ .

Speed impact: negligible.

Hashing  $R$  is very fast.

Security impact: seems to be  
serious obstacle to any attack  
strategy that relies on choosing  
a particular  $A$  exponent.

## Prime-order subgroup

Choose  $B$  to have order  $q$  for standard prime divisor  $q$  of  $p - 1$ .  
e.g. take 3000-bit  $p$ , 256-bit  $q$ .

Again verify  $B^{H(M)} \equiv A^{H(R)} R^S$ .

ECC:  $H(M)B = H(R)A + SR$ .

Signer: same except now

$$S = r^{-1}(H(M) - aH(R)) \bmod q.$$

Simpler security analysis.

Speed advantage: Smaller  $S$

(when  $q$  is smaller than  $p - 1$ ).

Less time to transmit signature.

## Two scalars

Verify  $B^{H(R)^{-1}H(M)} = AR^{H(R)^{-1}S}$ .

ECC:  $(H(R)^{-1}H(M))B = A + (H(R)^{-1}S)R$ .

Safe to assume that nobody will ever find  $H(R)$  divisible by  $q$ .

No security loss:

if  $B^{H(R)^{-1}H(M)} = AR^{H(R)^{-1}S}$

then  $B^{H(M)} = A^{H(R)}R^S$ .

Speed advantage: fewer scalars, outweighing cost of  $H(R)^{-1}$ .

## Precomputing quotient

Notation:  $\underline{S} = H(R)^{-1}S$ .

Send  $(R, \underline{S})$  instead of  $(R, S)$  as signature: i.e.,  $\underline{S}$  computed by signer instead of verifier.

Verify  $B^{H(R)^{-1}H(M)} = AR\underline{S}$ .

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From now on: Rename  $\underline{S}$  as  $S$ .

## Merge hashes: collision resilience

$$B^{H(R,M)} = AR^S.$$

$$\text{ECC: } H(R, M)B = A + SR.$$

Speed advantage:  $H(R, M)$   
is faster than  $H(R)^{-1}H(M)$ .

Security advantage: Imagine  
attacker somehow finding  
innocent  $M$  and dangerous  $M'$   
with  $H(M) = H(M')$ .

Using  $H(R)^{-1}H(M)$ : if signer  
signs  $M$  then attacker reuses  
same signature for  $M'$ .

Using  $H(R, M)$ : no problem.

## Eliminate divisions

$$B^S = RA^{H(R,M)}.$$

$$\text{ECC: } SB = R + H(R, M)A.$$

Signer in previous system:

$$S = r^{-1}(H(R, M) - a) \bmod q.$$

Signer in this system:

$$S = r + H(R, M)a \bmod q.$$

Speed advantage:

Skip all inversions.

Security analysis is similar, slightly simpler. See, e.g., 2000 Pointcheval–Stern.

## Signature compression

Schnorr signature is

$(H(R, M), S)$  instead of  $(R, S)$ .

Given  $(h, S)$ : verifier  
recovers  $R = B^S / A^h$ ,  
checks  $h = H(R, M)$ .

ECC:  $R = SB - hA$ .

Speed advantage sending sigs  
when  $H(R, M)$  is shorter than  $R$ .

No security impact:

anyone can compress sigs.

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Schnorr chooses half-size  $H$ :

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More serious objection:

multi-target preimage attacks.



## DSA and ECDSA

DSA is ElGamal plus

- prime-order subgroups;
- $A^{-1}$  instead of  $A$ ;
- two scalars.

Much worse than Schnorr: DSA

- does not hash  $R$ ;
- does not merge hashes;
- is not collision-resilient;
- requires inversion for signer;
- requires inversion for verifier (or three exponents).

## EdDSA

EdDSA is Schnorr with

- complete twisted Edwards curve;
- no signature compression;
- double-size  $H$  output;
- $A$  as extra  $H$  input;
- deterministic  $R$ .

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Extra  $H$  input:  $H(R, A, M)$ .

Speed impact: negligible.

Alleviates concerns that several public keys could be attacked simultaneously.

Why no signature compression:

1. ECC signatures are short even without compression.

64 bytes for signature using high-security curve.

2. Security of shorter  $H$  needs thorough analysis.

3. Double-size  $H$  alleviates concerns regarding  $H$  security.

4. Avoiding compression allows another speedup: batch signature verification.