NTRU Prime

Daniel J. Bernstein

University of Illinois at Chicago &
Technische Universiteit Eindhoven

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cr.yp.to/papers.html
#ntruprime is joint work with:

Chitchanok Chuengsatiansup
Tanja Lange
Christine van Vredendaal

Technische Universiteit Eindhoven

Focus of this talk: motivation.

Can we predict future attacks?

1996 Dobbertin–Bosselaers–Preneel "RIPEMD-160: a strengthened version of RIPEMD": "It is anticipated that these techniques can be used to produce collisions for MD5 and perhaps also for RIPEMD. This will probably require an additional effort, but it no longer seems as far away as it was a year ago."

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Define $C(x, y) = 4^x 9^y \bmod p$ for suitable ranges of x and y.

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UCLA press release: "According to Sahai, previously developed techniques for obfuscation presented only a 'speed bump,' forcing an attacker to spend some effort, perhaps a few days, trying to reverse-engineer the software. The new system, he said, puts up an 'iron wall' . . . a game-change in the field of cryptography."

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Use smallness: $fm + 3gr$.

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One way to exploit this: $c(1), h(1)$ are visible; $r(1)$ is guessable, sometimes standard. Attacker scans many ciphertexts to find some with large $m(1)$. Uses this to speed up m search.

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Why include $m(1)$ here?
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NTRU complicates m selection so that $m(1)$ is never large.
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Better: replace NTRU's $\mathbf{Z}[x]/(x^p - 1)$ with $\mathbf{Z}[x]/\Phi_p$.
Recall $\Phi_p = (x^p - 1)/(x - 1)$.

Can view poly m mod $x^p - 1$ as two parts: $m(1)$; m mod Φ_p.
Compatible with add, mult.
Why include $m(1)$ here?
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Or use other irreds. Ring-LWE typically uses $\Phi_{2048} = x^{1024} + 1$.

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Attackers can also use automorphisms in more ways

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$x \mapsto x^3$, $x \mapsto x^5$, $x \mapsto x^7$, etc. are automorphisms of $\mathcal{R} = \mathbf{Z}[x]/\Phi_{2^k}$.

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"Cyclotomic units" are defined as $\mathcal{R}^* \cap \{\pm x^{e_0} \prod_i (1-x^i)^{e_i}\}$. Weber's conjecture: all elements of $\mathcal{R}^*$ are cyclotomic units.

Experiments confirm that SV is quickly broken by LLL using, e.g., 1997 Washington textbook basis for cyclotomic units. Shortness of basis is critical; missing from bogus CGS analysis.

Attackers can also use automorphisms in more ways.

2016 Albrecht–Bai–Ducas "A subfield lattice attack on overstretched NTRU assumptions: Cryptanalysis of some FHE and Graded Encoding Schemes" use norms $g\sigma(g)$, and independently 2016 Cheon–Jeong–Lee ("The main technique of our algorithm is the reduction of a problem on a field to one in a subfield") use traces $g + \sigma(g)$, where σ is an order-2 automorphism.

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