Simplicity

D. J. Bernstein
University of Illinois at Chicago &
Technische Universiteit Eindhoven

Joint work with:
Tanja Lange
Technische Universiteit Eindhoven

NIST's ECC standards
= NSA's prime choices
+ NSA's curve choices
+ NSA's coordinate choices
+ NSA's computation choices
+ NSA's protocol choices.

NIST's ECC standards create
unnecessary complexity
in ECC implementations.

This unnecessary complexity

- scares away implementors,
- reduces ECC adoption,
- interferes with optimization,
- keeps ECC out of small devices,
- scares away auditors,
- interferes with verification, and
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Replace c with 19
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Replace c with $19q + r$ whe
$r = c \bmod 2^{255}$, $q = \lfloor c/2^{25}$
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Allocate 350 bits for $19q +$

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Repeat same compression:
350 bits $\rightarrow$ 256 bits.
Small enough for next mult.

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To **completely** reduce 256 bits
mod p, do two iterations of
constant-time conditional sub.

One conditional sub:
replace c with $c - (1 - s)p$
where s is sign bit in $c - p$.

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bits allocated for b:
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NIST P-
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NIST P-256 prime $2^{256} - 2^{224} + 2^{192}$

ECDSA standard s reduction procedu an integer "A less

Write A as $(A_{15}, A_{14}, A_{13}, A_{12}$ A_8, A_7, A_6, A_5, A meaning $\sum_i A_i 2^{32}$

Define $T; S_1; S_2; S_3; S_4; D$ as

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Constant-time NIST P-256

NIST P-256 prime p is
$2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$

ECDSA standard specifies
reduction procedure given
an integer "A less than p^2":

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$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10},$
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$(A_{15}, A_1$

$(0, A_{15},$

$(A_{15}, A_1$

$(A_8, A_{13}$

$(A_{10}, A_8$

$(A_{11}, A_9$

$(A_{12}, 0,$

$(A_{13}, 0,$

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$(0, A_{15}, A_{14}, A_{13}, A$

$(A_{15}, A_{14}, 0, 0, 0, A$

$(A_8, A_{13}, A_{15}, A_{14},$

$(A_{10}, A_8, 0, 0, 0, A_1$

$(A_{11}, A_9, 0, 0, A_{15},$

$(A_{12}, 0, A_{10}, A_9, A_8$

$(A_{13}, 0, A_{11}, A_{10}, A$

Compute $T + 2S_1$

$S_4 - D_1 - D_2 - D$

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$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A$
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0$
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0$
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8$
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11},$
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_1$
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13},$
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}$
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15},$

Compute $T + 2S_1 + 2S_2 +$
$S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by addin
subtracting a few copies" of

Constant-time NIST P-256

NIST P-256 prime p is $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

ECDSA standard specifies reduction procedure given an integer "A less than p^2":

Write A as $(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$, meaning $\sum_i A_i 2^{32i}$.

Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

t-time NIST P-256

-256 prime p is
$^{224} + 2^{192} + 2^{96} - 1$.

standard specifies
n procedure given
er "A less than p^2":

as
$_4, A_{13}, A_{12}, A_{11}, A_{10}, A_9,$
$A_6, A_5, A_4, A_3, A_2, A_1, A_0)$,
$\sum_i A_i 2^{32i}$.

$_2; S_3; S_4; D_1; D_2; D_3; D_4$

7

8

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

What is
A loop?
presuma

ST P-256

p is
$^2 + 2^{96} - 1$.

specifies
re given
than p^2":

$_2, A_{11}, A_{10}, A_9,$
$_4, A_3, A_2, A_1, A_0)$,
^{2i}.

$D_1; D_2; D_3; D_4$

7

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

8

What is "a few co
A loop? **Variable**
presumably a secu

1.

, A_9,
$A_1, A_0)$,

$; D_4$

7

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

8

What is "a few copies"?
A loop? **Variable time**,
presumably a security proble

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

What is "a few copies"?

A loop? **Variable time**, presumably a security problem.

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

What is "a few copies"?
A loop? **Variable time**,
presumably a security problem.

Correct but quite slow:
conditionally add $4p$,
conditionally add $2p$,
conditionally add p,
conditionally sub $4p$,
conditionally sub $2p$,
conditionally sub p.

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

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conditionally add $2p$,
conditionally add p,
conditionally sub $4p$,
conditionally sub $2p$,
conditionally sub p.

Delay until end of computation?
Trouble: "A less than p^2".

$(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$;
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$;
$(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$;
$(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$;
$(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$;
$(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$;
$(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$;
$(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$;
$(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo p "by adding or subtracting a few copies" of p.

What is "a few copies"?
A loop? **Variable time**,
presumably a security problem.

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conditionally add $4p$,
conditionally add $2p$,
conditionally add p,
conditionally sub $4p$,
conditionally sub $2p$,
conditionally sub p.

Delay until end of computation?
Trouble: "A less than p^2".

Even worse: what about platforms where 2^{32} isn't best radix?

$A_5, A_4, A_3, A_2, A_1, A_0$);

$_4, A_{13}, A_{12}, A_{11}, 0, 0, 0$);

$A_{14}, A_{13}, A_{12}, 0, 0, 0$);

$_4, 0, 0, 0, A_{10}, A_9, A_8$);

$, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9$);

$, 0, 0, 0, A_{13}, A_{12}, A_{11}$);

$, 0, 0, A_{15}, A_{14}, A_{13}, A_{12}$);

$A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13}$);

$A_{11}, A_{10}, A_9, 0, A_{15}, A_{14}$).

e $T + 2S_1 + 2S_2 + S_3 +$

$- D_2 - D_3 - D_4$.

modulo p "by adding or

ing a few copies" of p.

What is "a few copies"?

A loop? **Variable time**,

presumably a security problem.

Correct but quite slow:

conditionally add $4p$,

conditionally add $2p$,

conditionally add p,

conditionally sub $4p$,

conditionally sub $2p$,

conditionally sub p.

Delay until end of computation?

Trouble: "A less than p^2".

Even worse: what about platforms where 2^{32} isn't best radix?

The Mo

```
x2,z2,x
for i i
  bit =
  x2,x3
  z2,z3
  x3,z3

  x2,z2
    4*x
  x2,x3
  z2,z3
return
```

$A_3, A_2, A_1, A_0)$;
$A_2, A_{11}, 0, 0, 0)$;
$A_{12}, 0, 0, 0)$;
$A_{10}, A_9, A_8)$;
$A_{13}, A_{11}, A_{10}, A_9)$;
$A_{13}, A_{12}, A_{11})$;
$A_{14}, A_{13}, A_{12})$;
$A_8, A_{15}, A_{14}, A_{13})$;
$A_9, 0, A_{15}, A_{14})$.

$+ 2S_2 + S_3 +$
$D_3 - D_4$.

"by adding or
copies" of p.

What is "a few copies"?
A loop? **Variable time**,
presumably a security problem.

Correct but quite slow:
conditionally add $4p$,
conditionally add $2p$,
conditionally add p,
conditionally sub $4p$,
conditionally sub $2p$,
conditionally sub p.

Delay until end of computation?
Trouble: "A less than p^2".

Even worse: what about platforms
where 2^{32} isn't best radix?

The Montgomery

```
x2,z2,x3,z3 = 1,
for i in reverse
  bit = 1 & (n >
  x2,x3 = cswap(
  z2,z3 = cswap(
  x3,z3 = ((x2*x
          x1*(x2*z
  x2,z2 = ((x2^2
    4*x2*z2*(x2^
  x2,x3 = cswap(
  z2,z3 = cswap(
return x2*z2^(p-
```

A_0);
, 0);
);
);
A_{10}, A_9);
$_1$);
A_{12});
$_{\cdot}, A_{13}$);
A_{14}).

$S_3 +$

g or
p.

What is "a few copies"?
A loop? **Variable time**,
presumably a security problem.

Correct but quite slow:
conditionally add $4p$,
conditionally add $2p$,
conditionally add p,
conditionally sub $4p$,
conditionally sub $2p$,
conditionally sub p.

Delay until end of computation?
Trouble: "A less than p^2".

Even worse: what about platforms
where 2^{32} isn't best radix?

The Montgomery ladder

```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(2
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit
  z2,z3 = cswap(z2,z3,bit
  x3,z3 = ((x2*x3-z2*z3)^
          x1*(x2*z3-z2*x3)^
  x2,z2 = ((x2^2-z2^2)^2,
    4*x2*z2*(x2^2+A*x2*z2
  x2,x3 = cswap(x2,x3,bit
  z2,z3 = cswap(z2,z3,bit
return x2*z2^(p-2)
```

What is "a few copies"?

A loop? **Variable time**,
presumably a security problem.

Correct but quite slow:
conditionally add $4p$,
conditionally add $2p$,
conditionally add p,
conditionally sub $4p$,
conditionally sub $2p$,
conditionally sub p.

Delay until end of computation?
Trouble: "A less than p^2".

Even worse: what about platforms
where 2^{32} isn't best radix?

The Montgomery ladder

```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
  x3,z3 = ((x2*x3-z2*z3)^2,
           x1*(x2*z3-z2*x3)^2)
  x2,z2 = ((x2^2-z2^2)^2,
    4*x2*z2*(x2^2+A*x2*z2+z2^2))
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
return x2*z2^(p-2)
```

"a few copies"?

Variable time,

bly a security problem.

but quite slow:

nally add $4p$,

nally add $2p$,

nally add p,

nally sub $4p$,

nally sub $2p$,

nally sub p.

ntil end of computation?

"A less than p^2".

rse: what about platforms

^{32} isn't best radix?

The Montgomery ladder

```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
  x3,z3 = ((x2*x3-z2*z3)^2,
           x1*(x2*z3-z2*x3)^2)
  x2,z2 = ((x2^2-z2^2)^2,
    4*x2*z2*(x2^2+A*x2*z2+z2^2))
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
return x2*z2^(p-2)
```

Simple;

compute

on $y^2 =$

when A^2

pies"?

time,

rity problem.

slow:

$4p$,

$2p$,

p,

$4p$,

$2p$,

p.

computation?

han p^2".

about platforms

st radix?

The Montgomery ladder

```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
  x3,z3 = ((x2*x3-z2*z3)^2,
           x1*(x2*z3-z2*x3)^2)
  x2,z2 = ((x2^2-z2^2)^2,
    4*x2*z2*(x2^2+A*x2*z2+z2^2))
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
return x2*z2^(p-2)
```

Simple; fast; **alwa**

computes scalar m

on $y^2 = x^3 + Ax^2$

when $A^2 - 4$ is no

em.

ion?

tforms

The Montgomery ladder

```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
  x3,z3 = ((x2*x3-z2*z3)^2,
           x1*(x2*z3-z2*x3)^2)
  x2,z2 = ((x2^2-z2^2)^2,
    4*x2*z2*(x2^2+A*x2*z2+z2^2))
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
return x2*z2^(p-2)
```


Simple; fast; **always**

computes scalar multiplicati

on $y^2 = x^3 + Ax^2 + x$

when $A^2 - 4$ is non-square.

The Montgomery ladder

```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
  x3,z3 = ((x2*x3-z2*z3)^2,
          x1*(x2*z3-z2*x3)^2)
  x2,z2 = ((x2^2-z2^2)^2,
    4*x2*z2*(x2^2+A*x2*z2+z2^2))
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
return x2*z2^(p-2)
```


Simple; fast; **always**

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on $y^2 = x^3 + Ax^2 + x$

when $A^2 - 4$ is non-square.

The Montgomery ladder

```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
  x3,z3 = ((x2*x3-z2*z3)^2,
          x1*(x2*z3-z2*x3)^2)
  x2,z2 = ((x2^2-z2^2)^2,
    4*x2*z2*(x2^2+A*x2*z2+z2^2))
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
return x2*z2^(p-2)
```


Simple; fast; **always**
computes scalar multiplication
on $y^2 = x^3 + Ax^2 + x$
when $A^2 - 4$ is non-square.

With some extra lines
can compute (x, y) output
given (x, y) input.
But simpler to use just x,
as proposed by 1985 Miller.

The Montgomery ladder

```
x2,z2,x3,z3 = 1,0,x1,1
for i in reversed(range(255)):
  bit = 1 & (n >> i)
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
  x3,z3 = ((x2*x3-z2*z3)^2,
           x1*(x2*z3-z2*x3)^2)
  x2,z2 = ((x2^2-z2^2)^2,
    4*x2*z2*(x2^2+A*x2*z2+z2^2))
  x2,x3 = cswap(x2,x3,bit)
  z2,z3 = cswap(z2,z3,bit)
return x2*z2^(p-2)
```


Simple; fast; **always**
computes scalar multiplication
on $y^2 = x^3 + Ax^2 + x$
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can compute (x, y) output
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as proposed by 1985 Miller.

Adaptations to NIST curves
are much slower; not as simple;
not proven to always work.
Other scalar-mult methods:
proven but much more complex.

ntgomery ladder

```
3,z3 = 1,0,x1,1
n reversed(range(255)):
 1 & (n >> i)
 = cswap(x2,x3,bit)
 = cswap(z2,z3,bit)
 = ((x2*x3-z2*z3)^2,
 x1*(x2*z3-z2*x3)^2)
 = ((x2^2-z2^2)^2,
2*z2*(x2^2+A*x2*z2+z2^2))
 = cswap(x2,x3,bit)
 = cswap(z2,z3,bit)
x2*z2^(p-2)
```


Simple; fast; **always**
computes scalar multiplication
on $y^2 = x^3 + Ax^2 + x$
when $A^2 - 4$ is non-square.

With some extra lines
can compute (x, y) output
given (x, y) input.
But simpler to use just x,
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are much slower; not as simple;
not proven to always work.
Other scalar-mult methods:
proven but much more complex.

"Hey, yo
that x_1 i

ladder

```
0,x1,1
d(range(255)):
> i)
x2,x3,bit)
z2,z3,bit)
3-z2*z3)^2,
3-z2*x3)^2)
-z2^2)^2,
2+A*x2*z2+z2^2))
x2,x3,bit)
z2,z3,bit)
2)
```


Simple; fast; **always**
computes scalar multiplication
on $y^2 = x^3 + Ax^2 + x$
when $A^2 - 4$ is non-square.

With some extra lines
can compute (x, y) output
given (x, y) input.
But simpler to use just x,
as proposed by 1985 Miller.

Adaptations to NIST curves
are much slower; not as simple;
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Other scalar-mult methods:
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