Introduction to
quantum algorithms

Daniel J. Bernstein
University of Illinois at Chicago &
Technische Universiteit Eindhoven

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an element of $\{0,1\}^n$,
often viewed as representing
an element of $\{0,1,\ldots,2^n-1\}$.

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Similarly: swapping qubits i, j.

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$(q_0, q_1,$

Example
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$a_{16}, a_{17},$
$a_{24}, a_{25},$
$\mapsto (a_0, a$
a_8, a_9, a
$a_{16}, a_{17},$
$a_{24}, a_{25},$

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$a_5, a_6, a_7) \mapsto$

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$a_5, a_6, a_7)$

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$_q|^2 / \sum_r |a_r|^2.$

$a_4, a_7, a_6)$

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$a_8, a_9, a_{10}, a_{11}, a_{12}$
$a_{16}, a_{17}, a_{18}, a_{19}, a$
$a_{24}, a_{25}, a_{26}, a_{27}, a$
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$q_2)$,

$a_r|^2$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_4, a_5, a_6, a_7, a_0, a_1, a_2, a_3)$
is "complementing qubit 2":
$(q_0, q_1, q_2) \mapsto (q_0, q_1, q_2 \oplus 1)$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_4, a_2, a_6, a_1, a_5, a_3, a_7)$
is "swapping qubits 0 and 2":
$(q_0, q_1, q_2) \mapsto (q_2, q_1, q_0)$.

Complementing qubit 2
= swapping qubits 0 and 2
$\circ$ complementing qubit 0
$\circ$ swapping qubits 0 and 2.

Similarly: swapping qubits i, j.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7)$
$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$
is a "reversible XOR gate"
"controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1, q_1,$

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{2}$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{3}$
$\mapsto (a_0, a_1, a_3, a_2, a_4, a_5, a_7, a$
$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15},$
$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{2}$
$a_{24}, a_{25}, a_{27}, a_{26}, a_{28}, a_{29}, a_{3}$

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_4, a_5, a_6, a_7, a_0, a_1, a_2, a_3)$
is "complementing qubit 2":
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$(a_0, a_4, a_2, a_6, a_1, a_5, a_3, a_7)$
is "swapping qubits 0 and 2":
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Complementing qubit 2
= swapping qubits 0 and 2
∘ complementing qubit 0
∘ swapping qubits 0 and 2.

Similarly: swapping qubits i, j.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$
is a "reversible XOR gate" =
"controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1, q_1, q_2)$.

Example with more qubits:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$a_6, a_7, a_0, a_1, a_2, a_3)$
plementing qubit 2":
$q_2) \mapsto (q_0, q_1, q_2 \oplus 1)$.

$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$a_2, a_6, a_1, a_5, a_3, a_7)$
pping qubits 0 and 2":
$q_2) \mapsto (q_2, q_1, q_0)$.

menting qubit 2
ping qubits 0 and 2
nplementing qubit 0
pping qubits 0 and 2.

y: swapping qubits i, j.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$
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Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1,$
$(a_0, a_1,$
is a "To
"control
$(q_0, q_1,$

Example
$(a_0, a_1,$
$a_8, a_9,$
$a_{16}, a_{17},$
$a_{24}, a_{25},$
$\mapsto (a_0,$
$a_8, a_9,$
$a_{16}, a_{17},$
$a_{24}, a_{25},$

$a_5, a_6, a_7) \mapsto$

$a_1, a_2, a_3)$

g qubit 2":

$, q_1, q_2 \oplus 1)$.

$a_5, a_6, a_7) \mapsto$

$a_5, a_3, a_7)$

ts 0 and 2":

$, q_1, q_0)$.

ubit 2

s 0 and 2

ng qubit 0

its 0 and 2.

g qubits i, j.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$
is a "reversible XOR gate" = "controlled NOT gate":
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Example with more qubits:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4,$
$(a_0, a_1, a_2, a_3, a_4,$
is a "Toffoli gate"
"controlled contro
$(q_0, q_1, q_2) \mapsto (q_0$

Example with mor

$(a_0, a_1, a_2, a_3, a_4,$
$a_8, a_9, a_{10}, a_{11}, a_{12}$
$a_{16}, a_{17}, a_{18}, a_{19},$
$a_{24}, a_{25}, a_{26}, a_{27},$
$\mapsto (a_0, a_1, a_2, a_3,$
$a_8, a_9, a_{10}, a_{11}, a_{12}$
$a_{16}, a_{17}, a_{18}, a_{19},$
$a_{24}, a_{25}, a_{26}, a_{27},$

$\mapsto$

:

1).

$\mapsto$

":

2.

, j.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$
is a "reversible XOR gate" $=$
"controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7)$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" $=$
"controlled controlled NOT
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q$

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{2}$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{3}$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{2}$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{3}$

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$
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Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
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$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
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Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$a_3, a_2, a_4, a_5, a_7, a_6)$
ersible XOR gate" $=$
led NOT gate":
$q_2) \mapsto (q_0 \oplus q_1, q_1, q_2)$.

with more qubits:
$a_2, a_3, a_4, a_5, a_6, a_7,$
$_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$_1, a_3, a_2, a_4, a_5, a_7, a_6,$
$_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
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Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

<u>Reversib</u>

Say p is
of $\{0, 1,$

General
these fas
to obtai
$(a_{p(0)}, a$
$\mapsto (a_0, a$

$a_5, a_6, a_7) \mapsto$
$a_5, a_7, a_6)$
OR gate" $=$
gate":
$\oplus q_1, q_1, q_2)$.

e qubits:
$a_5, a_6, a_7,$
$, a_{13}, a_{14}, a_{15},$
$_{20}, a_{21}, a_{22}, a_{23},$
$_{28}, a_{29}, a_{30}, a_{31})$
$_4, a_5, a_7, a_6,$
$, a_{13}, a_{15}, a_{14},$
$_{20}, a_{21}, a_{23}, a_{22},$
$_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" $=$
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible comput

Say p is a permuta
of $\{0, 1, \ldots, 2^n -$

General strategy to
these fast quantur
to obtain index pe
$(a_{p(0)}, a_{p(1)}, \ldots, a$
$\mapsto (a_0, a_1, \ldots, a_{2^n}$

$\mapsto$

$=$

$q_2)$.

a_{15},

$2, a_{23}$,

$0, a_{31})$

6,

a_{14},

$3, a_{22}$,

$1, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" $=$
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation
of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose
these fast quantum operatio
to obtain index permutation
$(a_{p(0)}, a_{p(1)}, \ldots, a_{p(2^n-1)})$
$\mapsto (a_0, a_1, \ldots, a_{2^n-1})$:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" $=$
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose these fast quantum operations to obtain index permutation
$(a_{p(0)}, a_{p(1)}, \ldots, a_{p(2^n-1)})$
$\mapsto (a_0, a_1, \ldots, a_{2^n-1})$:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" $=$
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

<u>Reversible computation</u>

Say p is a permutation
of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose
these fast quantum operations
to obtain index permutation
$(a_{p(0)}, a_{p(1)}, \ldots, a_{p(2^n-1)})$
$\mapsto (a_0, a_1, \ldots, a_{2^n-1})$:

1. Build a traditional circuit
to compute $j \mapsto p(j)$
using NOT/XOR/AND gates.

2. Convert into reversible gates:
e.g., convert AND into Toffoli.

$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$a_2, a_3, a_4, a_5, a_7, a_6)$

ffoli gate" =

lled controlled NOT gate":

$q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

e with more qubits:

$a_2, a_3, a_4, a_5, a_6, a_7,$

$_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$

$a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$

$a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$

$_1, a_2, a_3, a_4, a_5, a_7, a_6,$

$_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$

$a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$

$a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose these fast quantum operations to obtain index permutation $(a_{p(0)}, a_{p(1)}, \ldots, a_{p(2^n-1)}) \mapsto (a_0, a_1, \ldots, a_{2^n-1})$:

1. Build a traditional circuit to compute $j \mapsto p(j)$ using NOT/XOR/AND gates.

2. Convert into reversible gates: e.g., convert AND into Toffoli.

Example

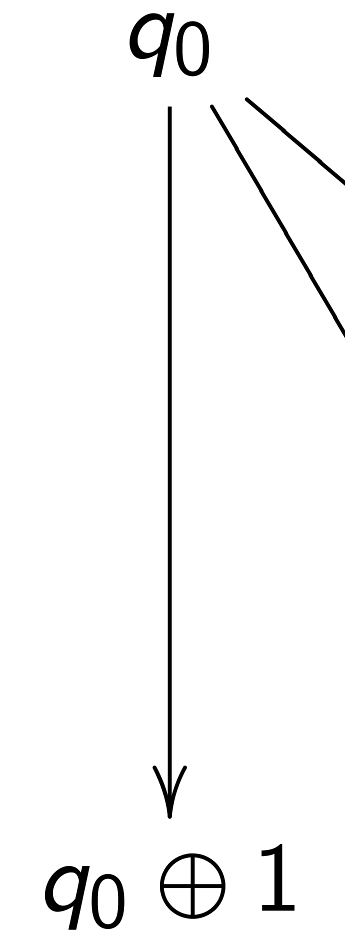
$(a_0, a_1,$

$(a_7, a_0,$

permuta

1. Build

to comp



$a_5, a_6, a_7) \mapsto$

$a_5, a_7, a_6)$

$=$

lled NOT gate":

$\oplus\, q_1 q_2, q_1, q_2)$.

e qubits:

$a_5, a_6, a_7,$

$, a_{13}, a_{14}, a_{15},$

$20, a_{21}, a_{22}, a_{23},$

$28, a_{29}, a_{30}, a_{31})$

$4, a_5, a_7, a_6,$

$, a_{13}, a_{15}, a_{14},$

$20, a_{21}, a_{23}, a_{22},$

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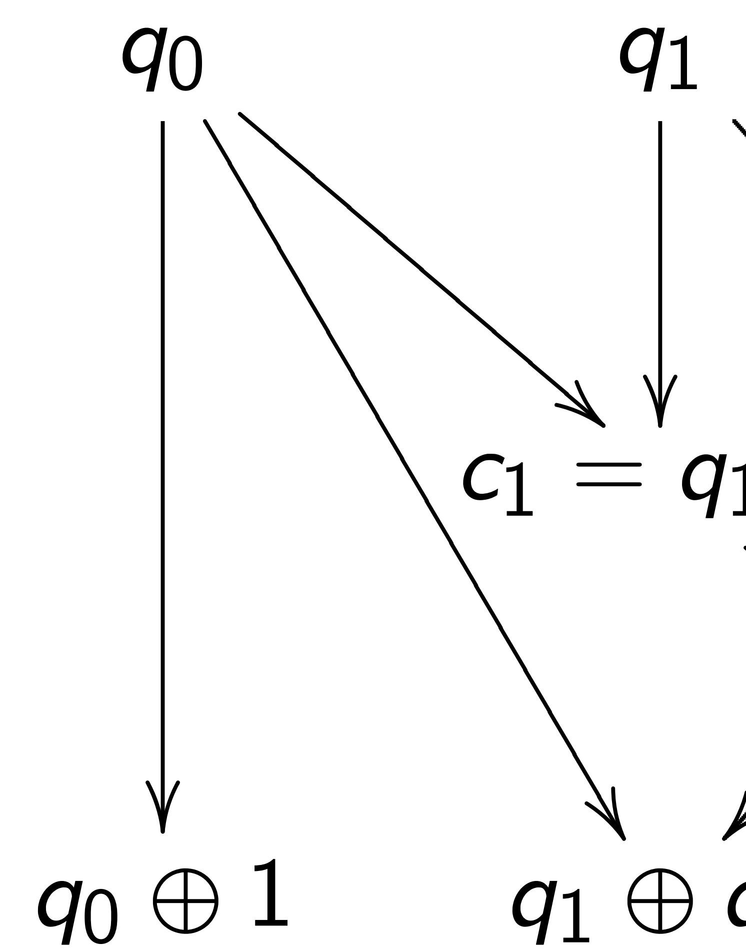
$(a_0, a_1, a_2, a_3, a_4, a$

$(a_7, a_0, a_1, a_2, a_3, a$

permutation $q \mapsto$

1. Build a traditio

to compute $q \mapsto q$



$\mapsto$

gate”:

$_1, q_2)$.

a_{15},

$_2, a_{23}$,

$_0, a_{31})$

$_6$,

a_{14},

$_3, a_{22}$,

$_1, a_{30})$.

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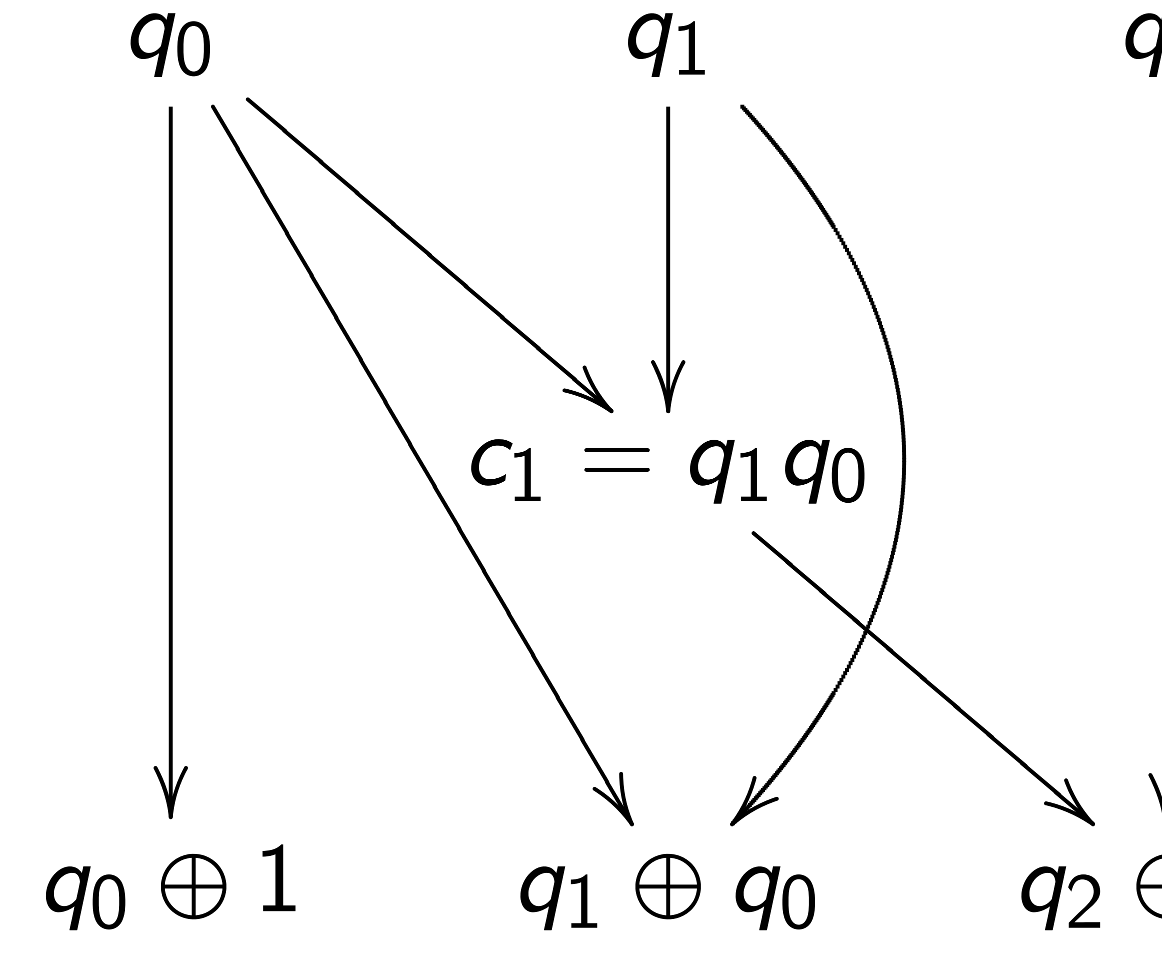
1. Build a traditional circuit to compute $q \mapsto q + 1$ mod



Reversible computation

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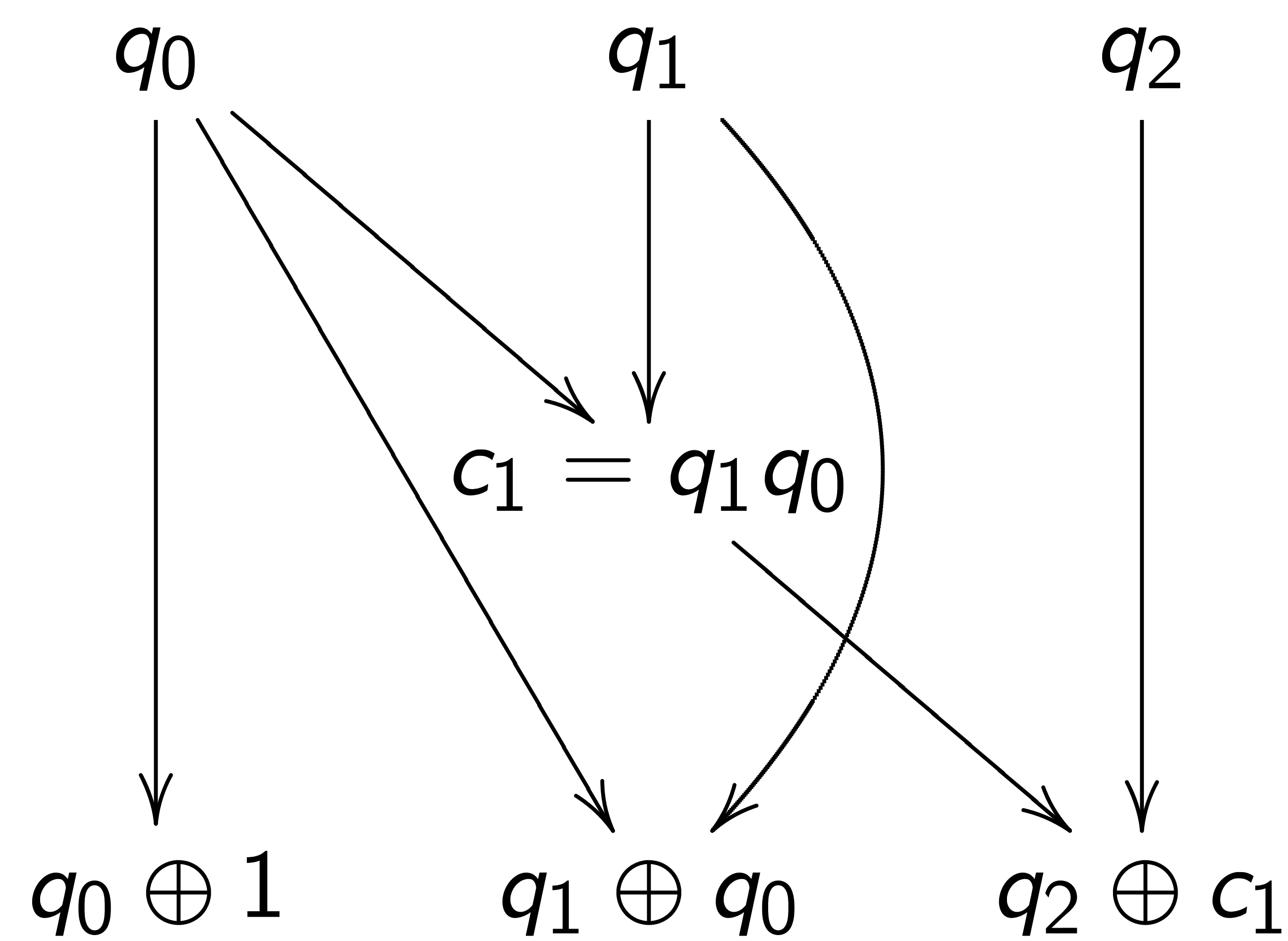
1. Build a traditional circuit to compute $q \mapsto q + 1 \bmod 8$.



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$_1, \ldots, a_{2^n-1})$:

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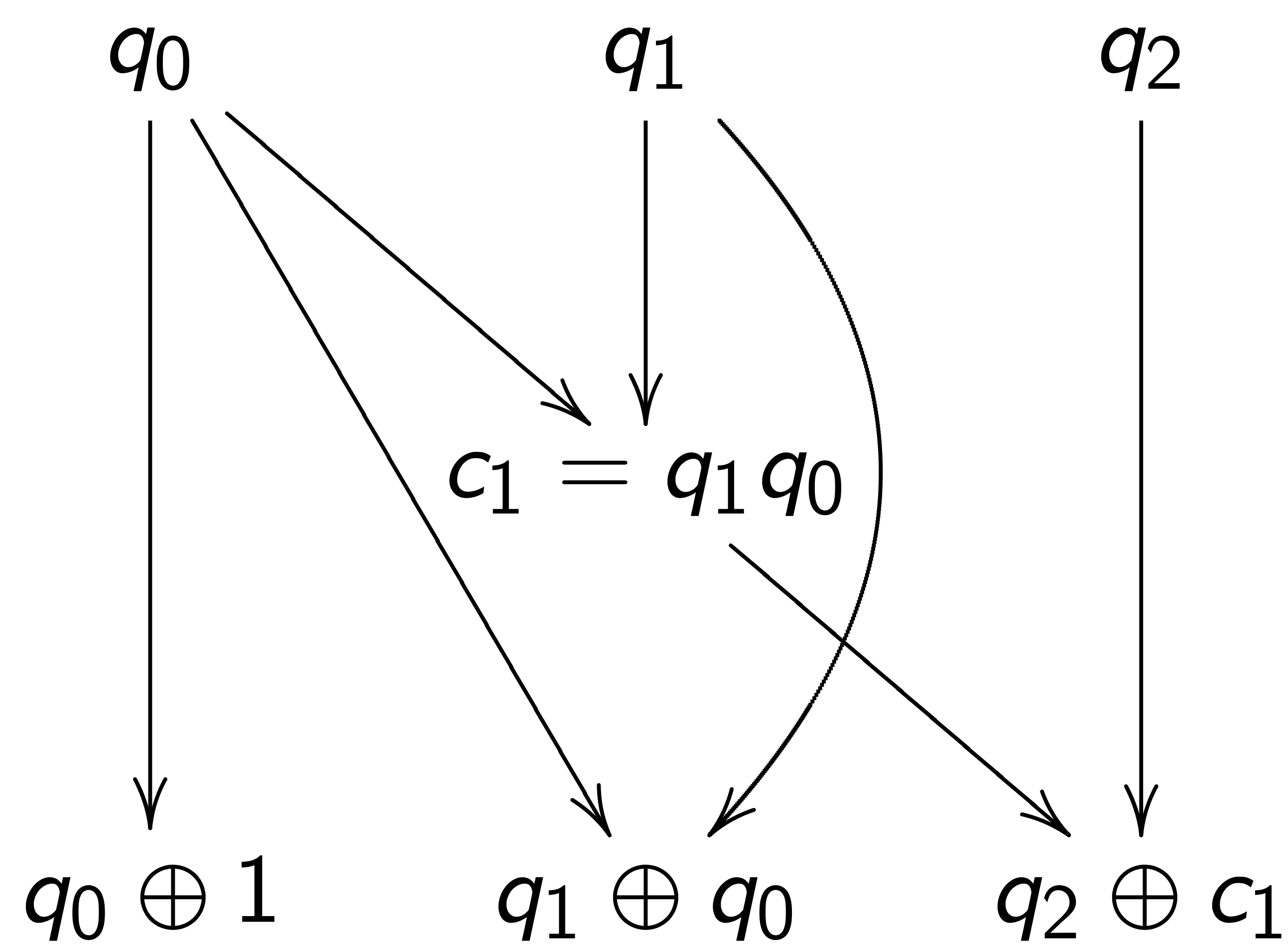

2. Conv

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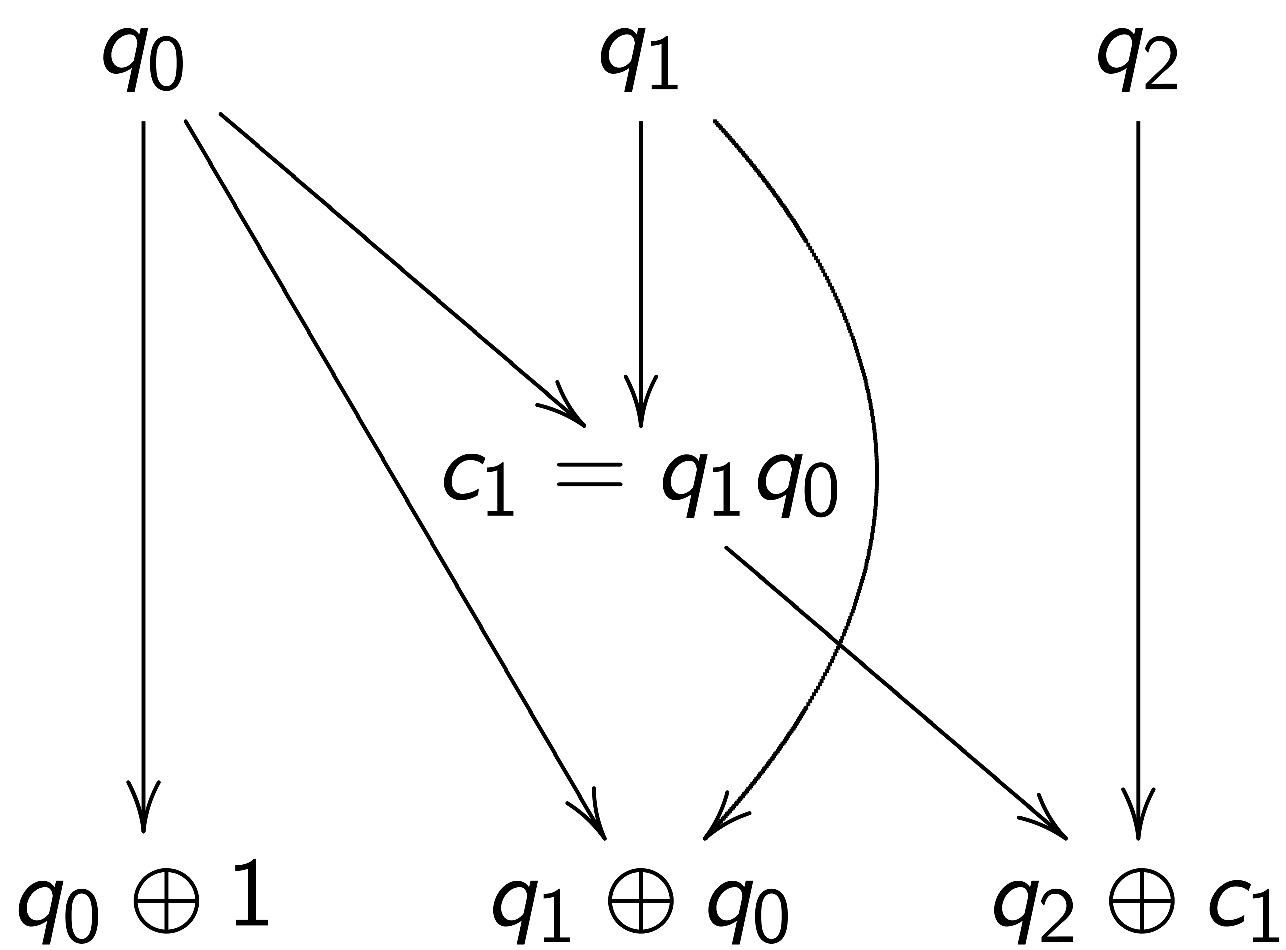

2. Convert into re

Toffoli for $q_2 \leftarrow q$

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$(a_0, a_1, a_2, a_7, a_4, a$

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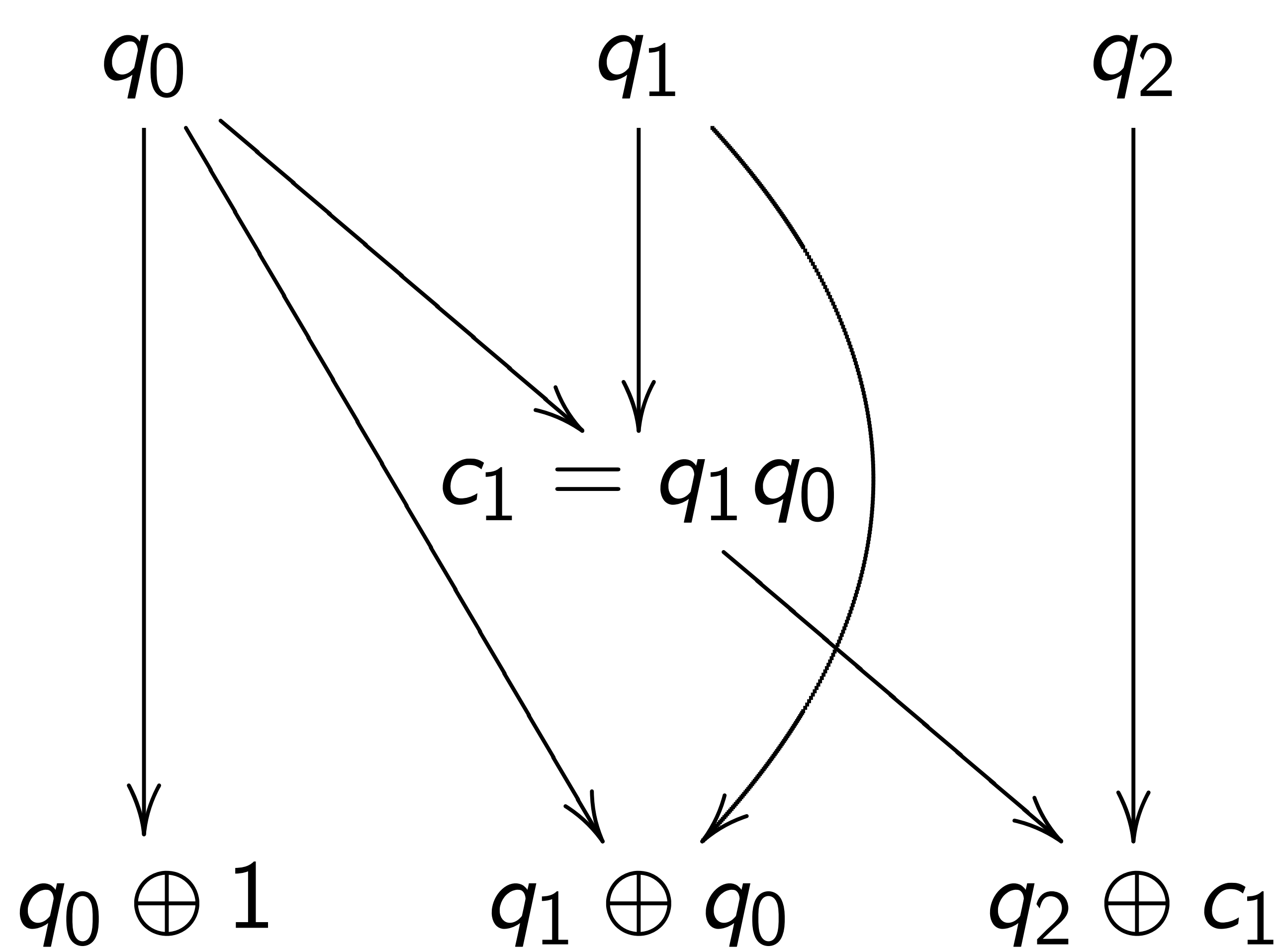

2. Convert into reversible ga

Toffoli for $q_2 \leftarrow q_2 \oplus q_1 q_0$:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7)$
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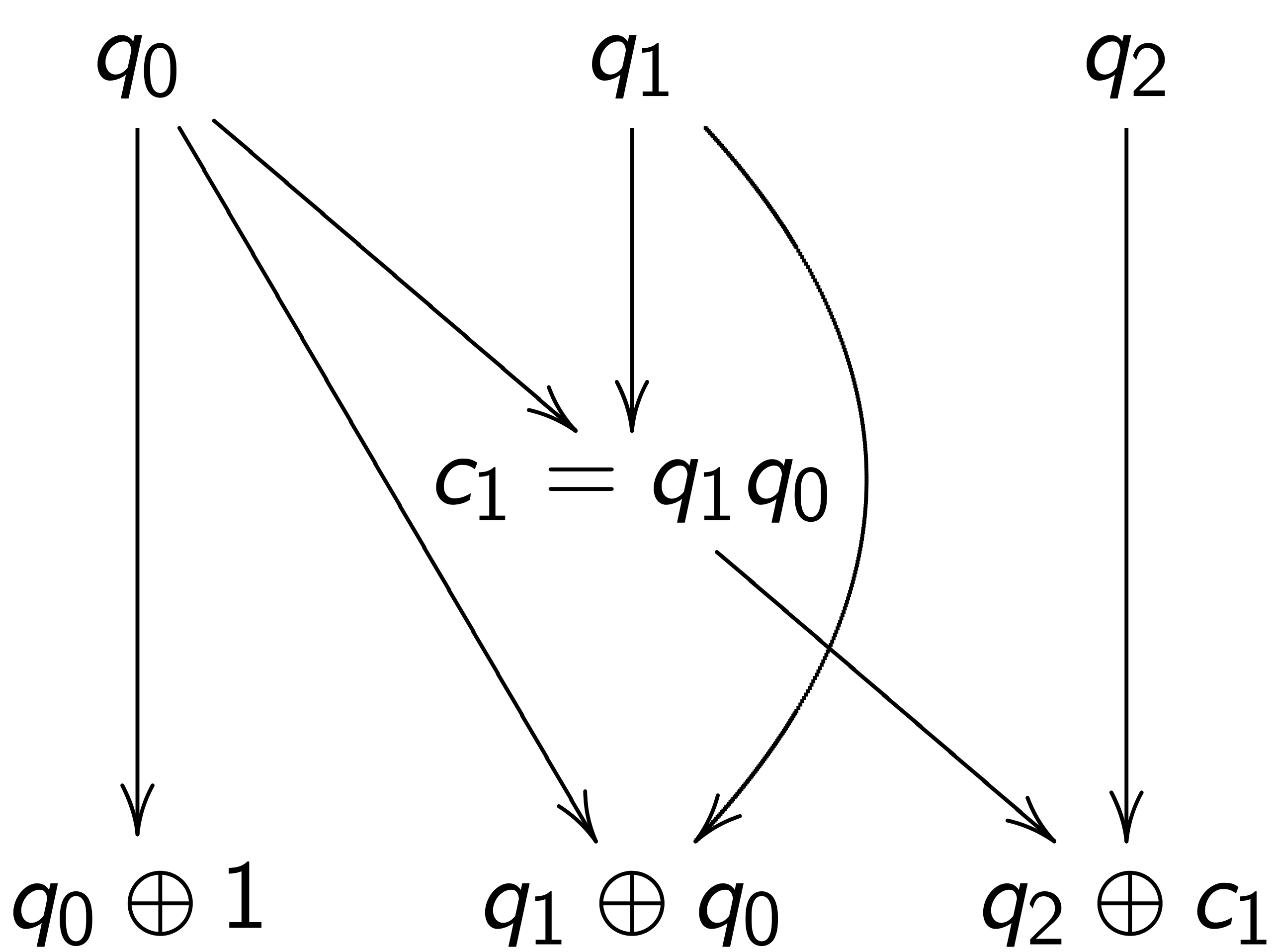

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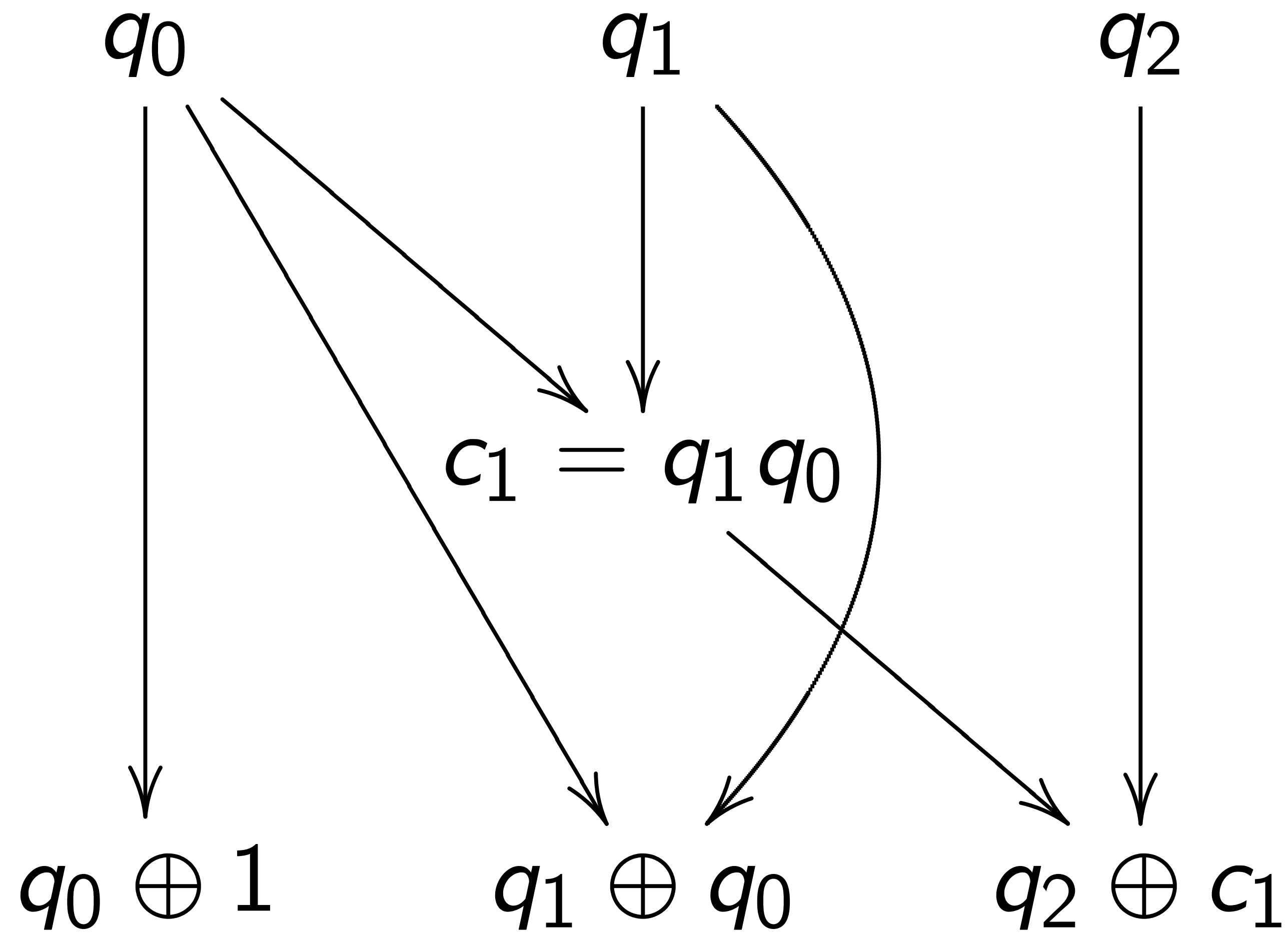

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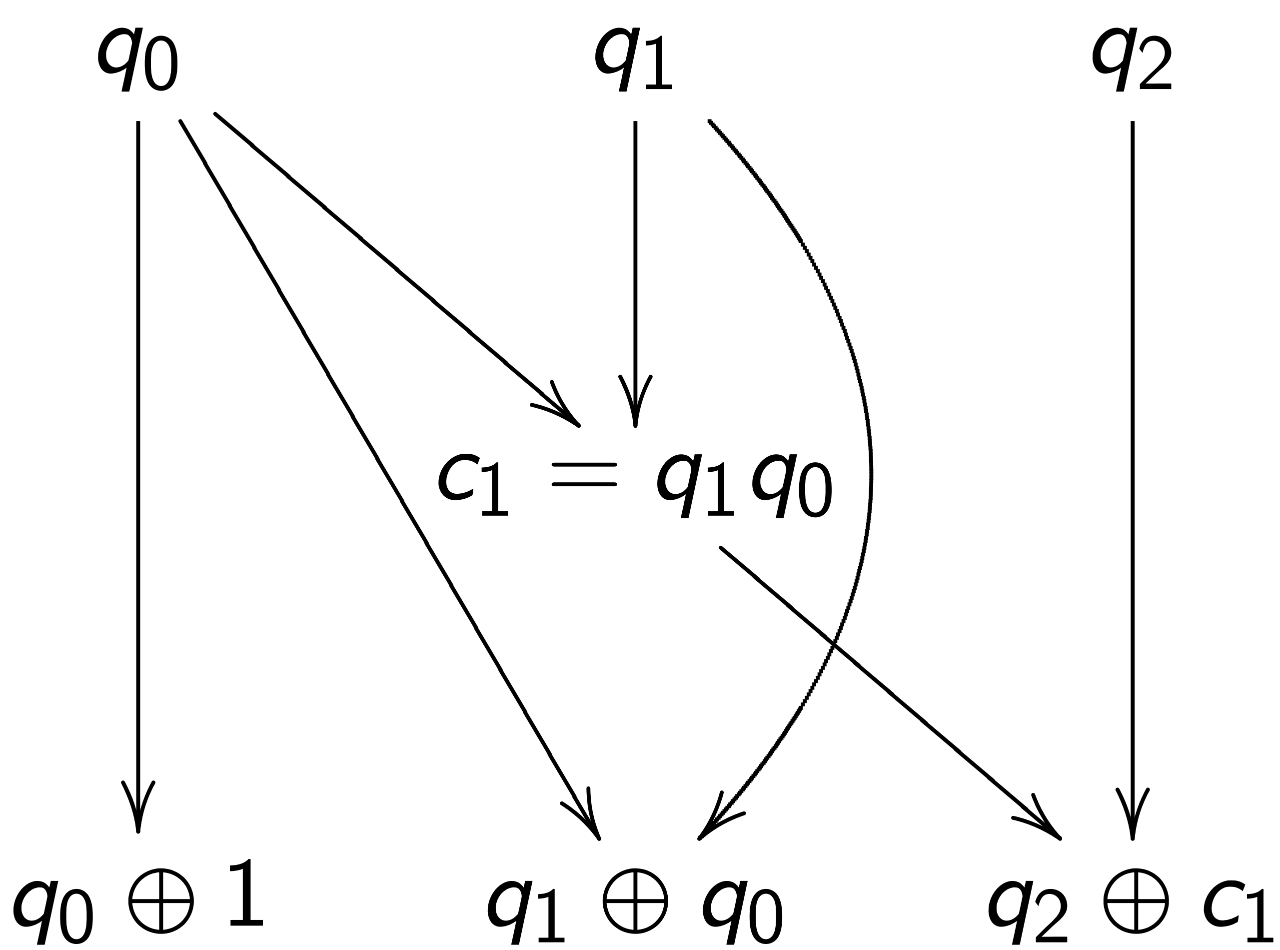

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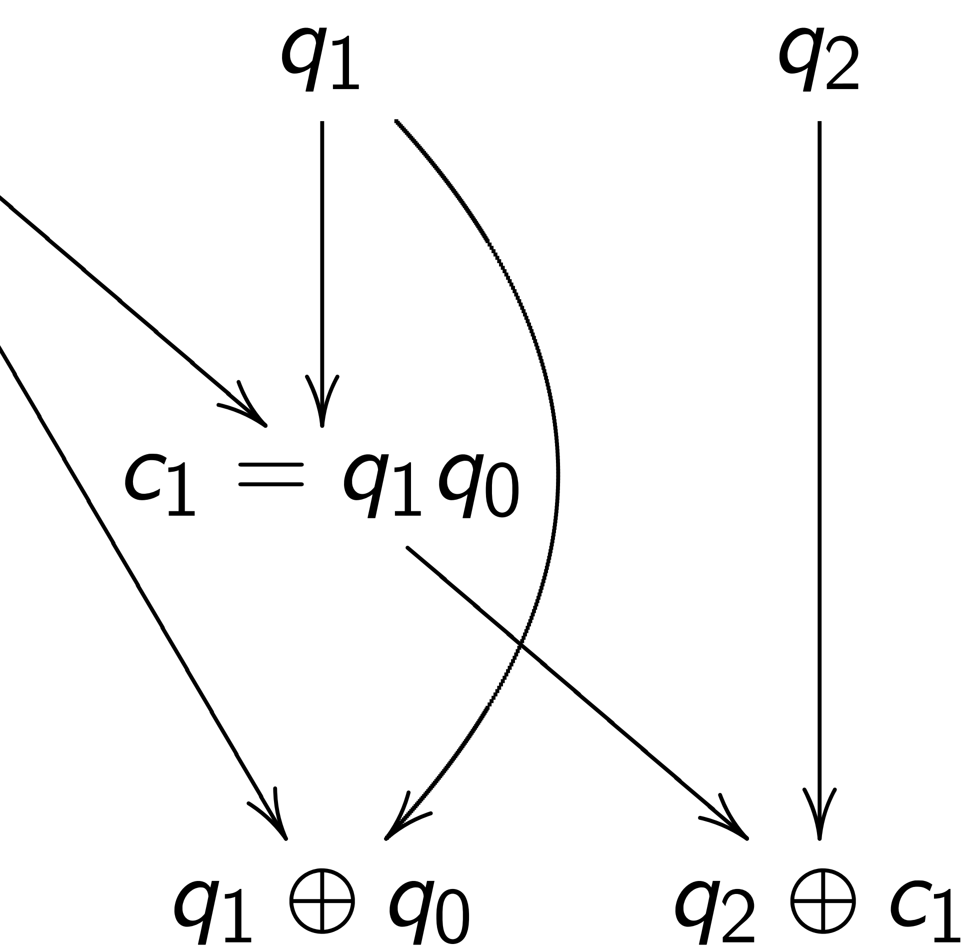

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Also, it didn't need extra storage: circuit operated "in place" after computation $c_1 \leftarrow q_1 q_0$ was merged into $q_2 \leftarrow q_2 \oplus c_1$.

Typical circuits aren't in-place.

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Start fro

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$b_{i+1} = 1$

$b_{i+2} = 1$

...

$b_T = 1 \oplus$

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${}_2 \oplus q_1 q_0$:

$a_5, a_6, a_7) \mapsto$

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$a_5, a_6, a_3) \mapsto$

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$b_T = 1 \oplus b_{f(T)} b_{g(T)}$;
specified outputs.

Reversible but dirty:
inputs $b_1, b_2, \ldots, b_T$;
$b_{i+1} \leftarrow 1 \oplus b_{i+1} \oplus b_{f(i+1)} b_{g(i+1)}$;
$b_{i+2} \leftarrow 1 \oplus b_{i+2} \oplus b_{f(i+2)} b_{g(i+2)}$;
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$b_T \leftarrow 1 \oplus b_T \oplus b_{f(T)} b_{g(T)}$.
Same outputs if all of $b_{i+1}, \ldots, b_T$ started as 0.

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$b_T = 1 \oplus b_{f(T)} b_{g(T)}$;
specified outputs.

Reversible but dirty:
inputs $b_1, b_2, \ldots, b_T$;
$b_{i+1} \leftarrow 1 \oplus b_{i+1} \oplus b_{f(i+1)} b_{g(i+1)}$;
$b_{i+2} \leftarrow 1 \oplus b_{i+2} \oplus b_{f(i+2)} b_{g(i+2)}$;
...
$b_T \leftarrow 1 \oplus b_T \oplus b_{f(T)} b_{g(T)}$.
Same outputs if all of
$b_{i+1}, \ldots, b_T$ started as 0.

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Start from any circuit:

inputs $b_1, b_2, \ldots, b_i$;

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(inputs, dirt, outputs).

Dirty reversible computation

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Original computation:
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$(\text{inputs}, \text{dirt}, \text{outputs})$.

Dirty reversible computation:
$(\text{inputs}, \text{zeros}, \text{zeros}) \mapsto$
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Given fast circuit f
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Reversible and clean:
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Original computation:
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Dirty reversible computation:
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(inputs, dirt, outputs).

Clean reversible computation:
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(inputs, zeros, outputs).

Given fast circuit for p
and fast circuit for p^{-1},
build fast reversible circuit f
$(x, \text{zeros}) \mapsto (p(x), \text{zeros})$.

Reversible and clean:
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build fast reversible circuit for
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Given fast circuit for p
and fast circuit for p^{-1},
build fast reversible circuit for
$(x, \text{zeros}) \mapsto (p(x), \text{zeros})$.

Replace reversible bit operations
with Toffoli gates etc.
permuting $\mathbf{C}^{2^{n+z}} \to \mathbf{C}^{2^{n+z}}$.

Permutation on first 2^n entries is
$(a_{p(0)}, a_{p(1)}, \ldots, a_{p(2^n-1)})$
$\mapsto (a_0, a_1, \ldots, a_{2^n-1})$.

Typically prepare vectors
supported on first 2^n entries
so don't care how permutation
acts on last $2^{n+z} - 2^n$ entries.

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Warning: Number of **qubits**
$\approx$ number of **bit operations**
in original p, p^{-1} circuits.

This can be much larger
than number of **bits stored**
in the original circuits.

Many useful techniques
to compress into fewer qubit
but often these lose time.
Many subtle tradeoffs.

Crude "poly-time" analyses
don't care about this,
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Given fast circuit for p
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$(x, \text{zeros}) \mapsto (p(x), \text{zeros})$.

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Fast quantum operations, pa

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Fast quantum operations, part 2

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Same for qubit 1:
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Qubit 0 and then qubit 1:
$(a_0, a_1, a_2, a_3) \mapsto$
$(a_0 + a_1, a_0 - a_1, a_2 + a_3, a_2 - a_3) \mapsto$
$(a_0 + a_1 + a_2 + a_3, a_0 - a_1 + a_2 - a_3,$
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$a_0 + a_1 - a_2 - a_3, a_0 - a_1 - a_2 + a_3)$.

Repeat n times: e
$(1, 0, 0, \ldots, 0) \mapsto$ (

Measuring $(1, 0, 0,$
always produces 0

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can produce any o
$\Pr[\text{output} = q] =$

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Fast quantum operations, part 2

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$(a_0 + a_1 + a_2 + a_3, a_0 - a_1 + a_2 - a_3,$
$a_0 + a_1 - a_2 - a_3, a_0 - a_1 - a_2 + a_3)$.

Repeat n times: e.g.,
$(1, 0, 0, \ldots, 0) \mapsto (1, 1, 1, \ldots$

Measuring $(1, 0, 0, \ldots, 0)$
always produces 0.

Measuring $(1, 1, 1, \ldots, 1)$
can produce any output:
$\Pr[\text{output} = q] = 1/2^n$.

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$(a_0 + a_1, a_0 - a_1, a_2 + a_3, a_2 - a_3) \mapsto$
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$(a_0 + a_1 + a_2 + a_3, a_0 - a_1 + a_2 - a_3,$
$a_0 + a_1 - a_2 - a_3, a_0 - a_1 - a_2 + a_3)$.

Repeat n times: e.g.,
$(1, 0, 0, \ldots, 0) \mapsto (1, 1, 1, \ldots, 1)$.

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always produces 0.

Measuring $(1, 1, 1, \ldots, 1)$
can produce any output:
$\Pr[\text{output} = q] = 1/2^n$.

Aside from "normalization"
(irrelevant to measurement),
have Hadamard = Hadamard^{-1},
so easily work backwards
from "uniform superposition"
$(1, 1, 1, \ldots, 1)$ to "pure state"
$(1, 0, 0, \ldots, 0)$.

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$\mapsto (a_0 + a_1, a_0 - a_1)$.

$a_2, a_3) \mapsto$

$, a_0 - a_1, a_2 + a_3, a_2 - a_3)$.

r qubit 1:

$a_2, a_3) \mapsto$

$, a_1 + a_3, a_0 - a_2, a_1 - a_3)$.

and then qubit 1:

$a_2, a_3) \mapsto$

$a_0 - a_1, a_2 + a_3, a_2 - a_3) \mapsto$

$+ a_2 + a_3, a_0 - a_1 + a_2 - a_3,$

$- a_2 - a_3, a_0 - a_1 - a_2 + a_3)$.

Repeat n times: e.g., $(1, 0, 0, \ldots, 0) \mapsto (1, 1, 1, \ldots, 1)$.

Measuring $(1, 0, 0, \ldots, 0)$ always produces 0.

Measuring $(1, 1, 1, \ldots, 1)$ can produce any output: $\Pr[\text{output} = q] = 1/2^n$.

Aside from "normalization" (irrelevant to measurement), have Hadamard = Hadamard^{-1}, so easily work backwards from "uniform superposition" $(1, 1, 1, \ldots, 1)$ to "pure state" $(1, 0, 0, \ldots, 0)$.

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Assume: nonzero $s \in \{0, 1\}$ satisfies $f(x) = f(x \oplus s)$ for every $x \in \{0, 1\}^n$. Can we find this period s, given a fast circuit for f?

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Compute f for many inputs,
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Success probability is very low
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for reversibility.

Prepare $n + m + z$
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Simon’s algorithm
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Say f maps n bits to m bits
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Prepare $n + m + z$ qubits
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vector $(1, 0, 0, \ldots)$.

Use n-fold Hadamard
to move first n qubits
into uniform superposition:
$(1, 1, 1, \ldots, 1, 0, 0, \ldots)$
with 2^n entries 1, others 0.

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$(1, 1, 1, \ldots, 1, 0, 0, \ldots)$
with 2^n entries 1, others 0.

Apply fast vector permutatio
for reversible f computation
1 in position $(q, 0, 0)$
moves to position $(q, f(q), 0$

Note symmetry between
1 at $(q, f(q), 0)$ and
1 at $(q \oplus s, f(q), 0)$.

Apply n-fold Hadamard.

Measure. By symmetry,
output is orthogonal to s.

Repeat $n + 10$ times.
Use Gaussian elimination
to (probably) find s.

Simon's algorithm
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Measure. By symmetry, output is orthogonal to s.

Repeat $n + 10$ times. Use Gaussian elimination to (probably) find s.

Grover's algorithm

Assume: unique s
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<u>Grover's algorithm</u>

Assume: unique $s \in \{0, 1\}^n$
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Traditional algorithm to find
compute f for many inputs,
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Success probability is very lo
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Assume: unique $s \in \{0, 1\}^n$ has $f(s) = 0$.

Traditional algorithm to find s: compute f for many inputs, hope to find output 0. Success probability is very low until #inputs approaches 2^n.

Grover's algorithm takes only $2^{n/2}$ reversible computations of f. Typically: reversibility overhead is small enough that this easily beats traditional algorithm.

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Start from uniform
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Step 1: Set $a \leftarrow b$
$b_q = -a_q$ if $f(q) =$
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Repeat steps 1 an
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Start from uniform superpos

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Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average
This is also fast.

Repeat steps 1 and 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this fi

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Measure the n qubits. With high probability this finds s.

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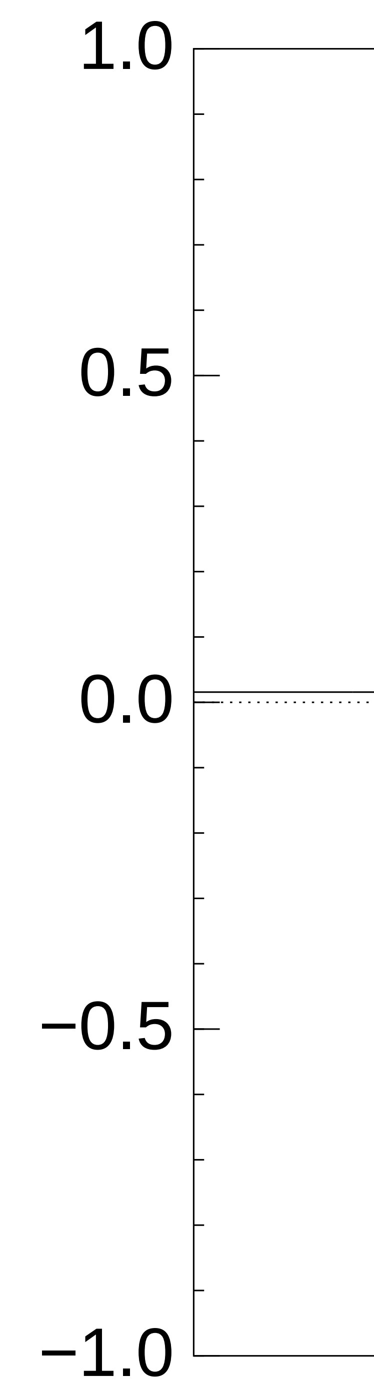

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Graph of $q \mapsto a_q$
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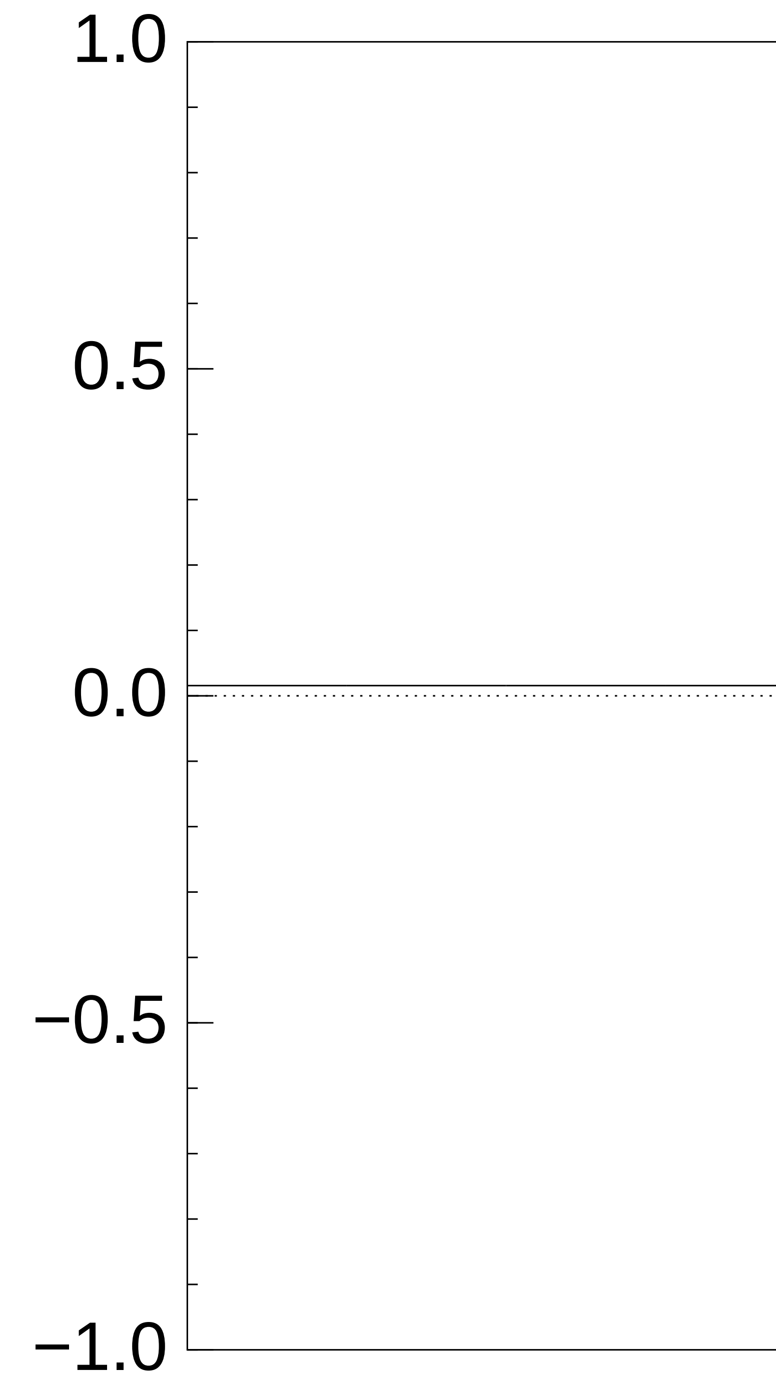

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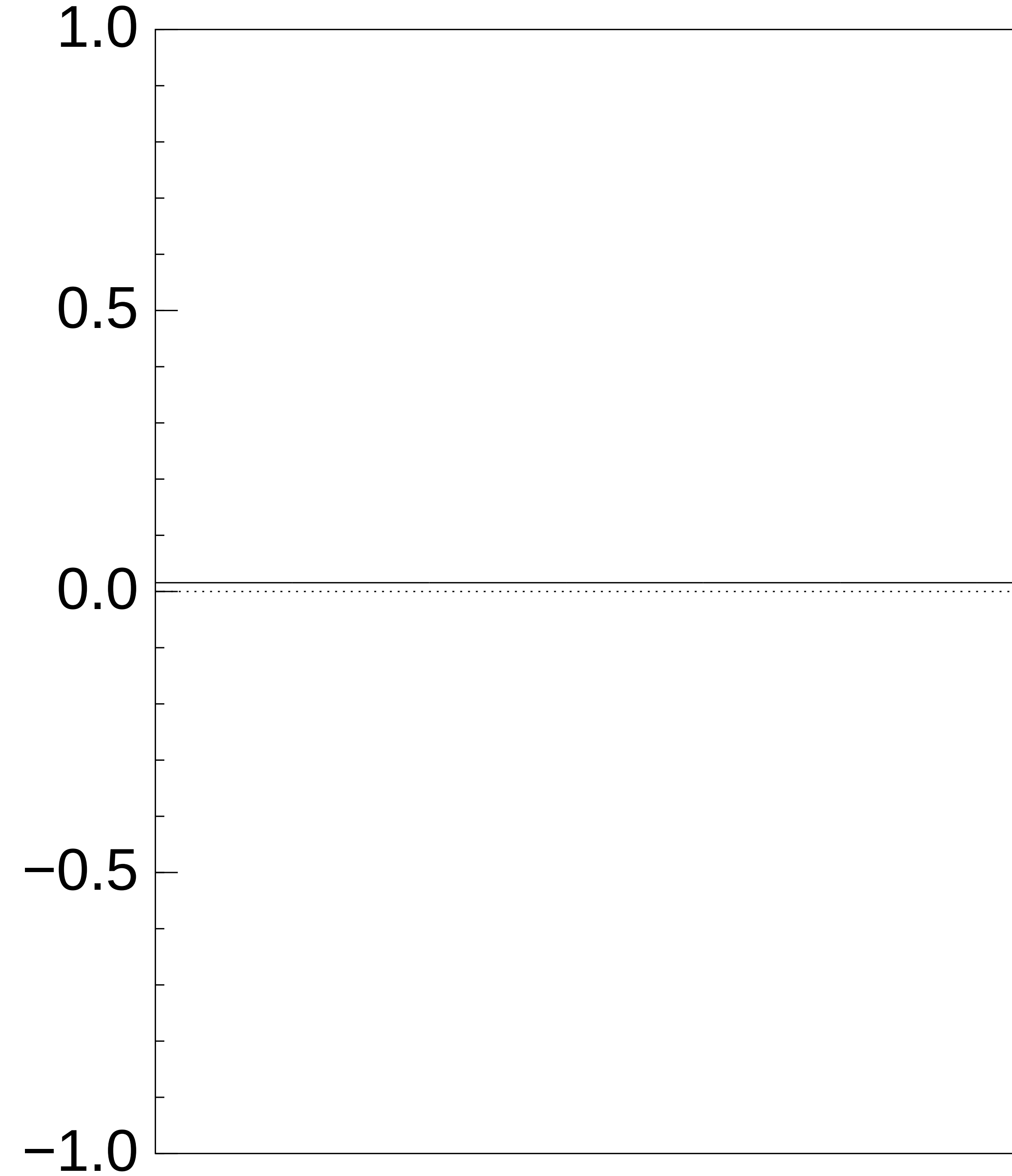
Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after 0 steps:



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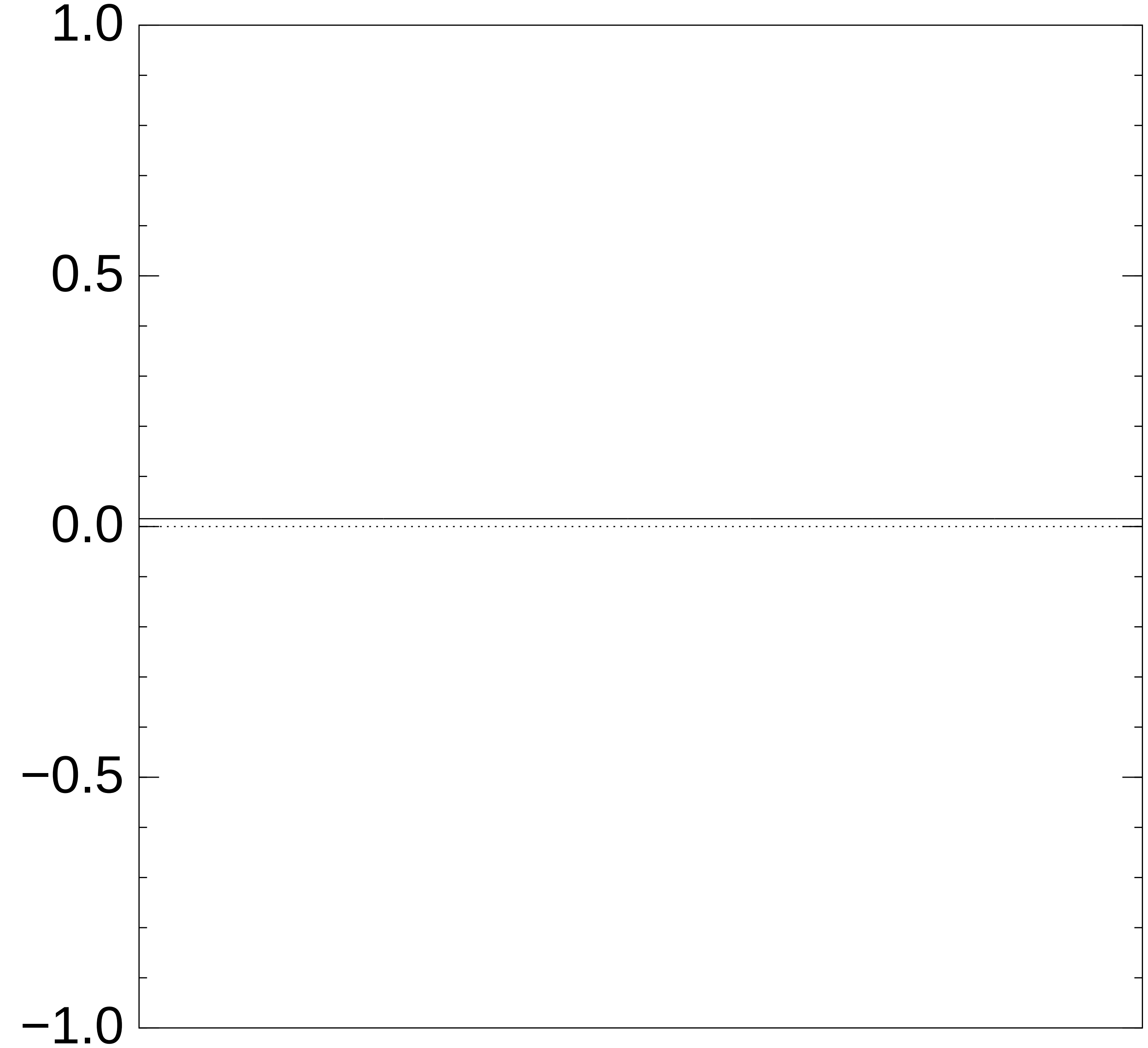
Graph of $q \mapsto a_q$ for an example with $n = 12$ after 0 steps:



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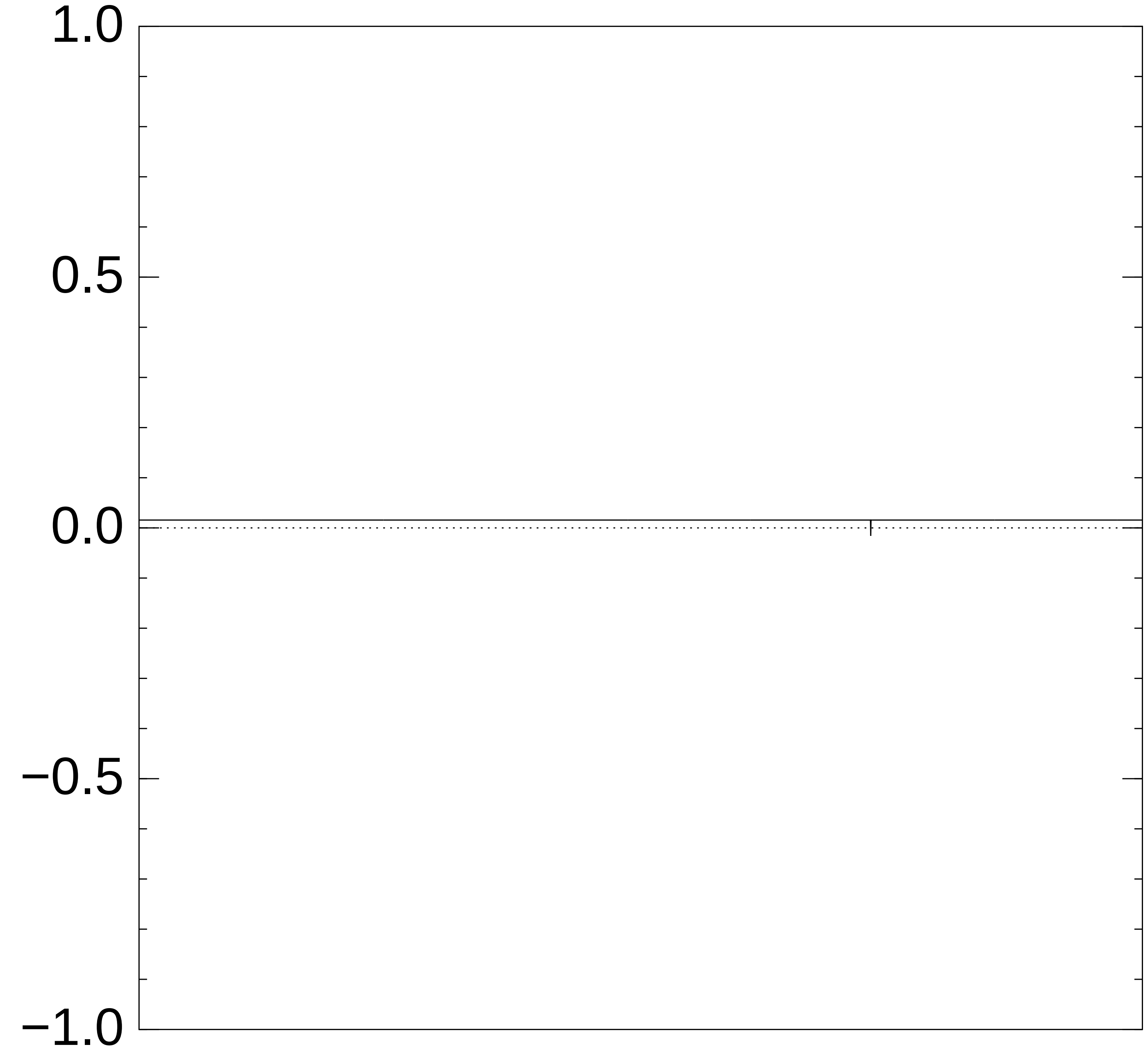
Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1:



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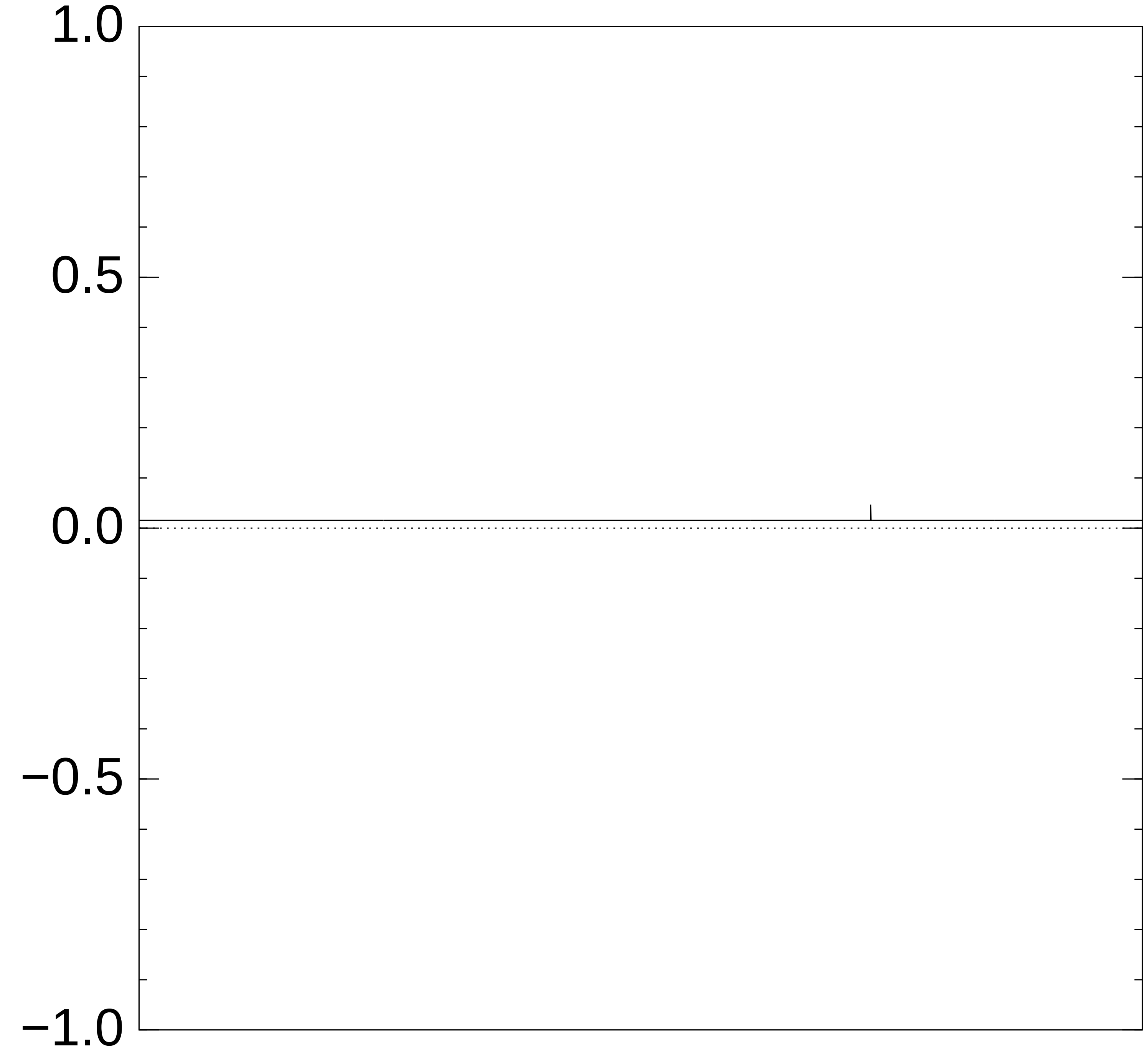
Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1 + Step 2:



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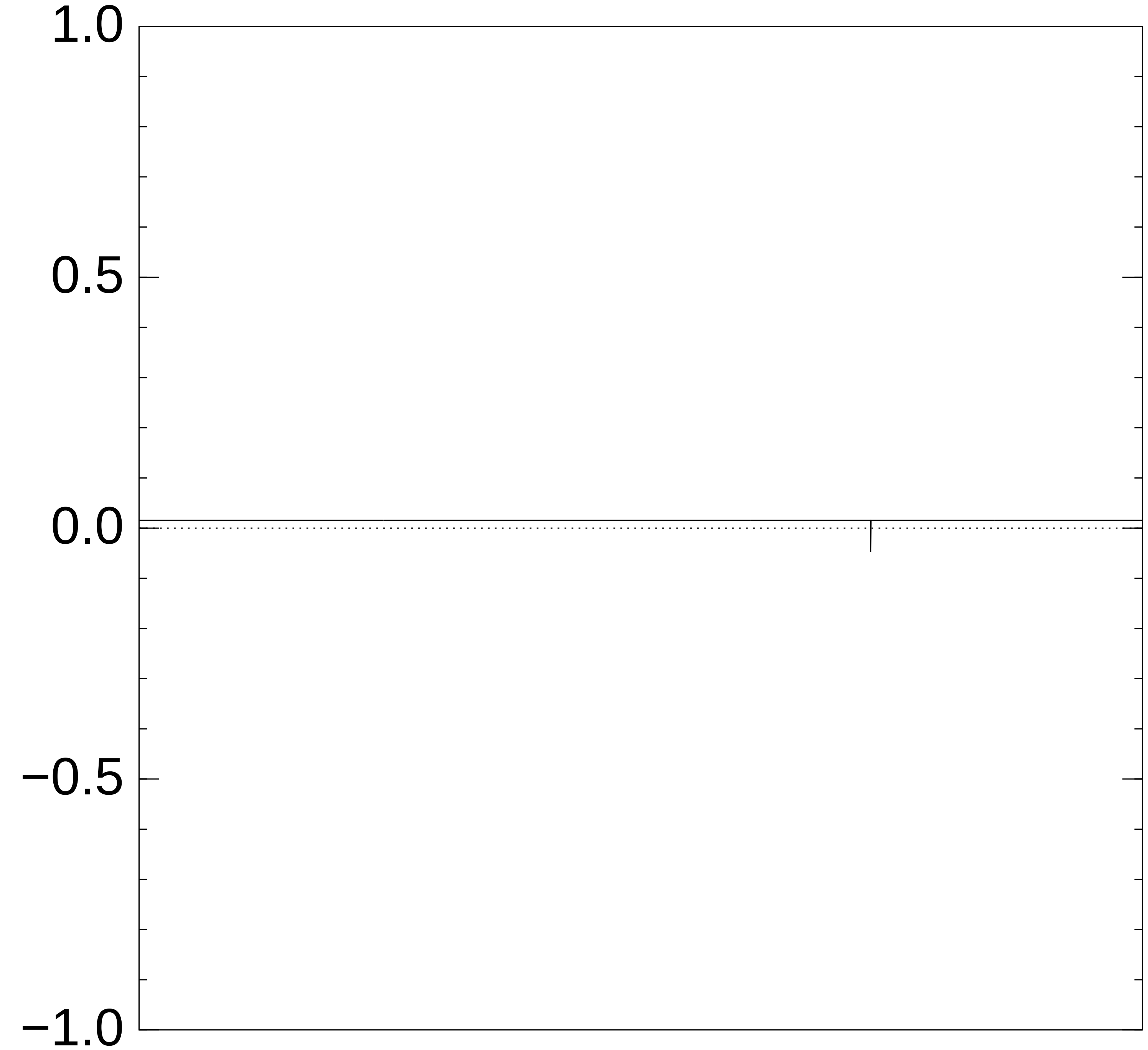
Graph of $q \mapsto a_q$
for an example with $n = 12$
after Step $1 +$ Step $2 +$ Step 1:



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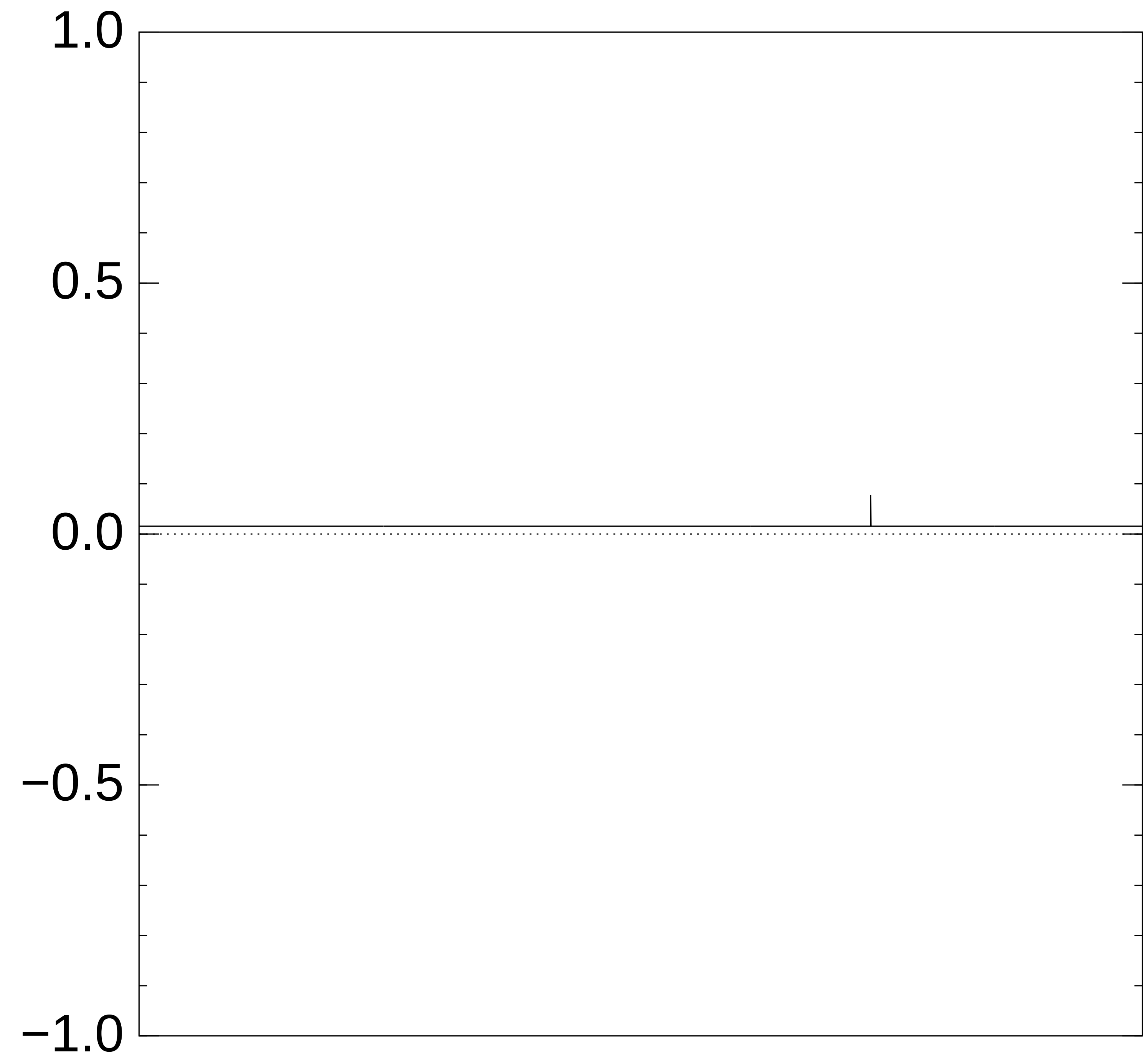
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$
for an example with $n = 12$
after $2 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

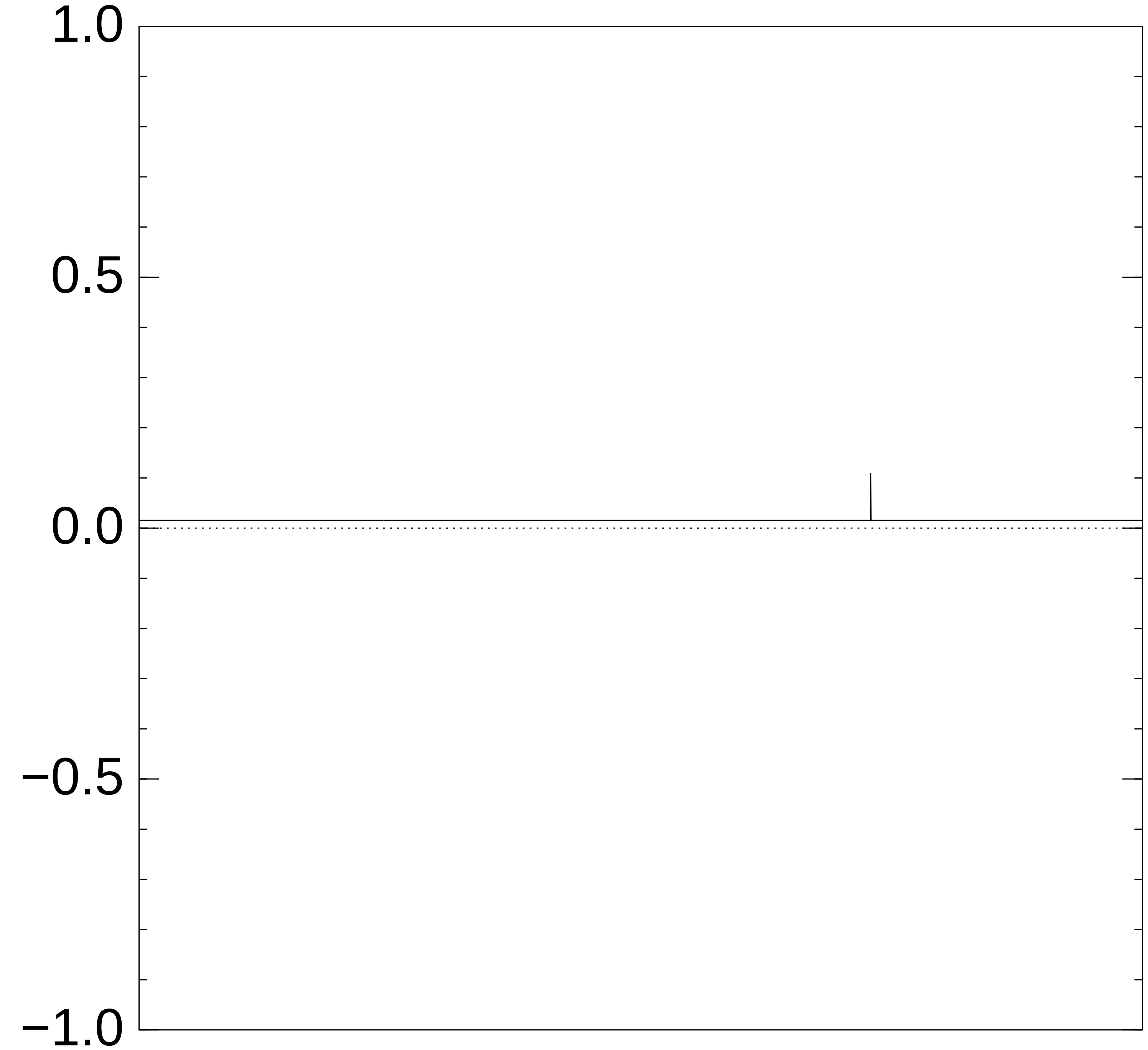
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$
for an example with $n = 12$
after $3 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

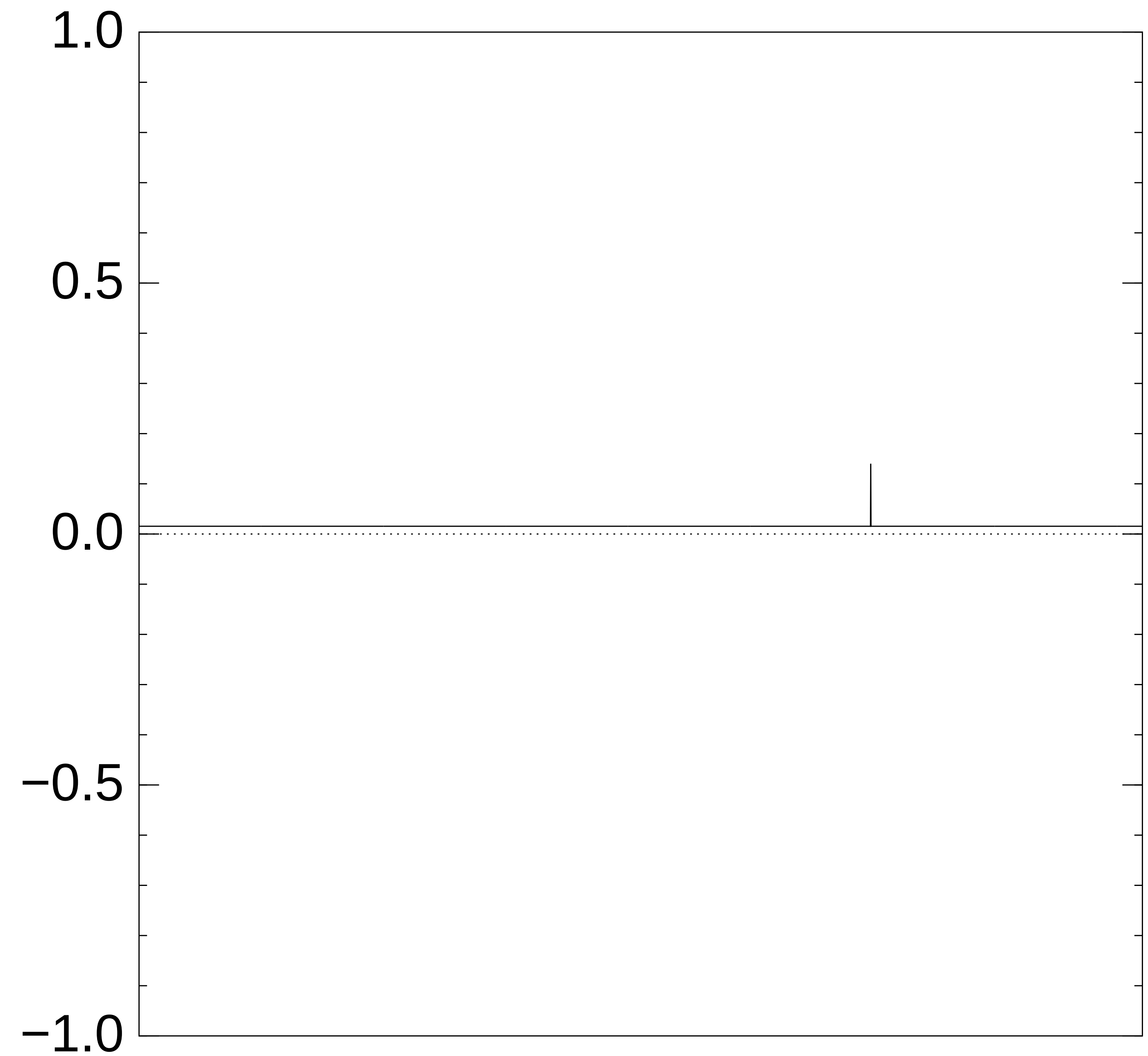
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$
for an example with $n = 12$
after $4 \times$ (Step $1 +$ Step 2):



Start from uniform superposition over all n-bit strings q.

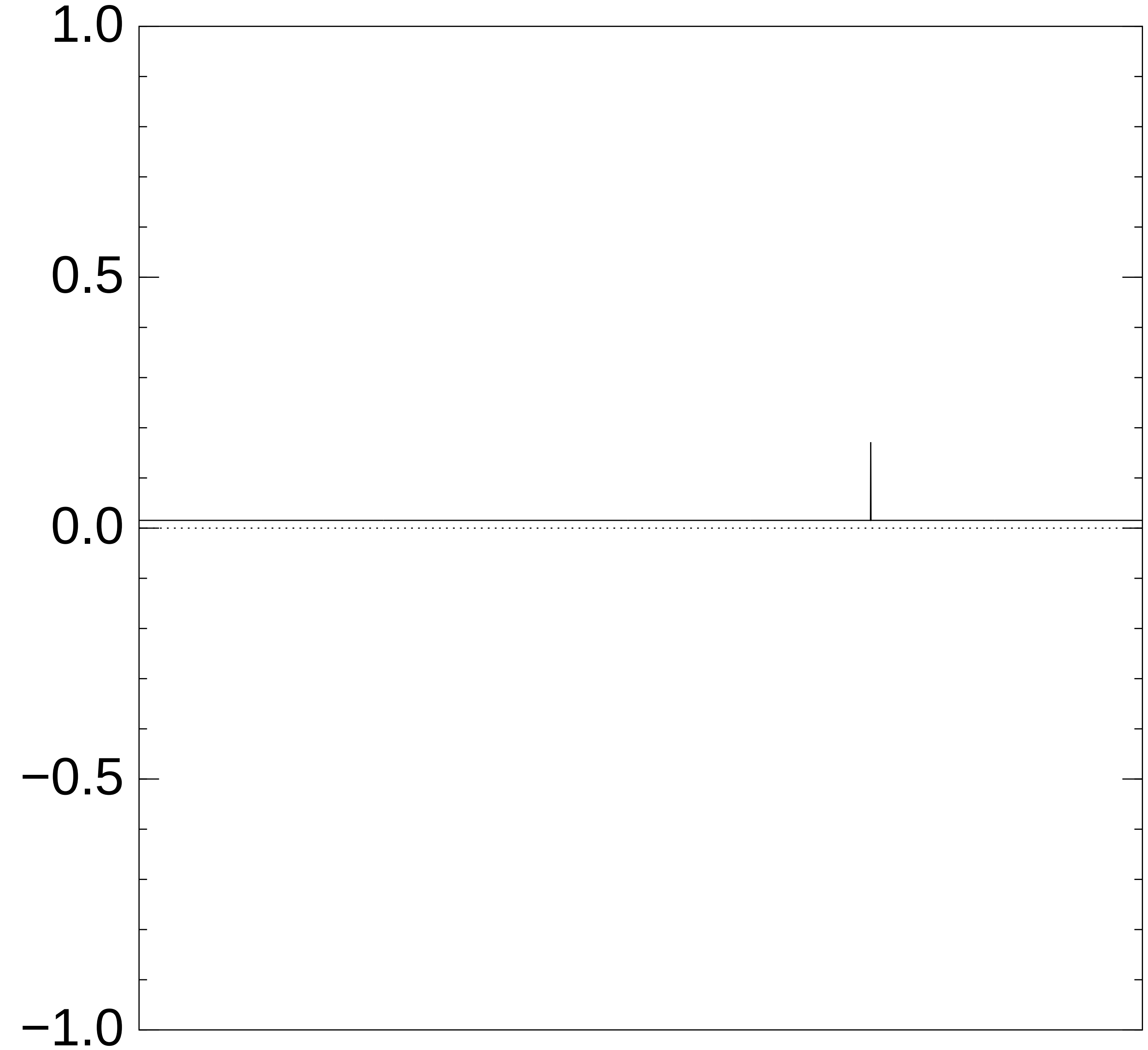
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $5 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

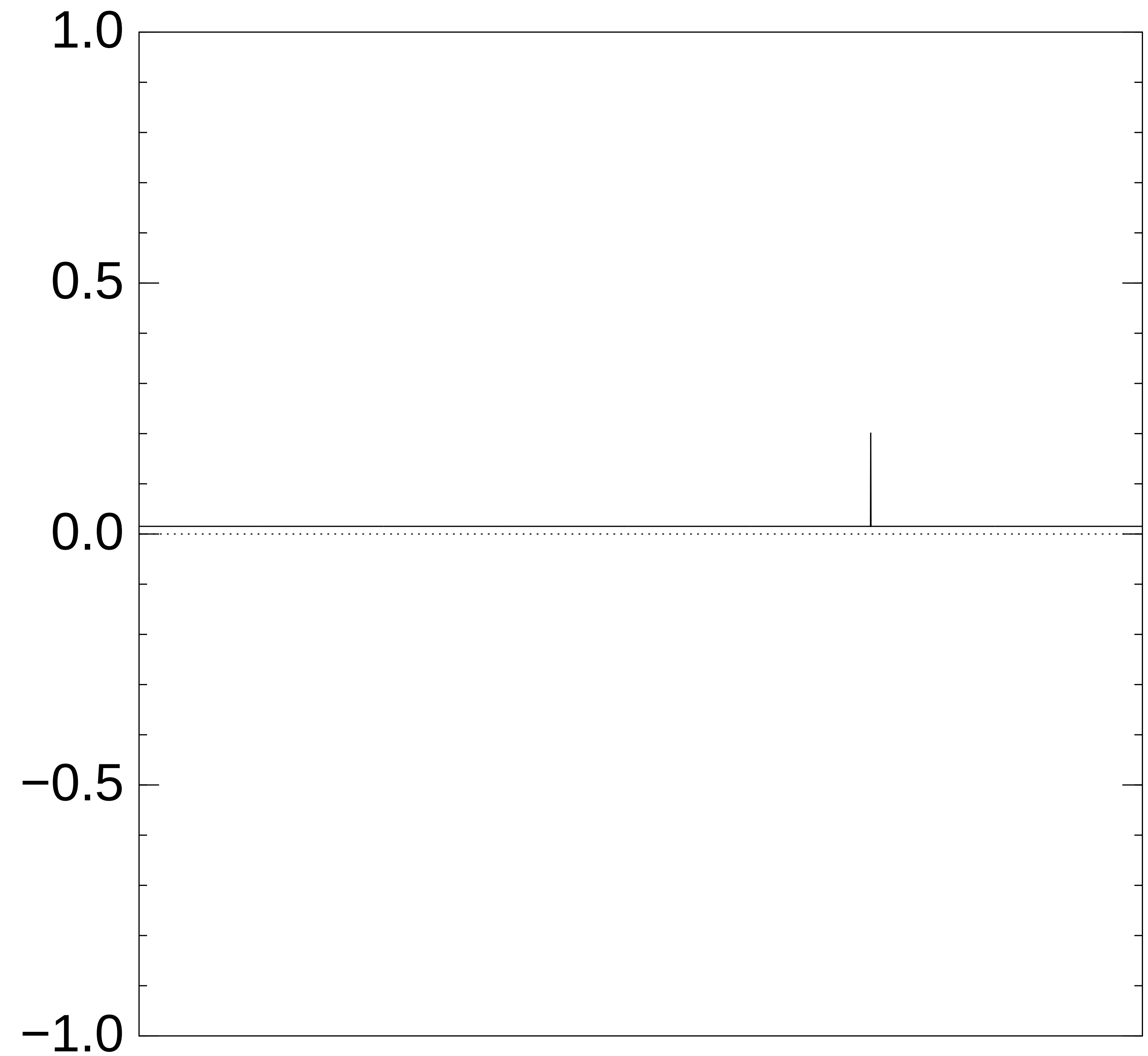
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$
for an example with $n = 12$
after $6 \times$ (Step $1 +$ Step 2):



Start from uniform superposition over all n-bit strings q.

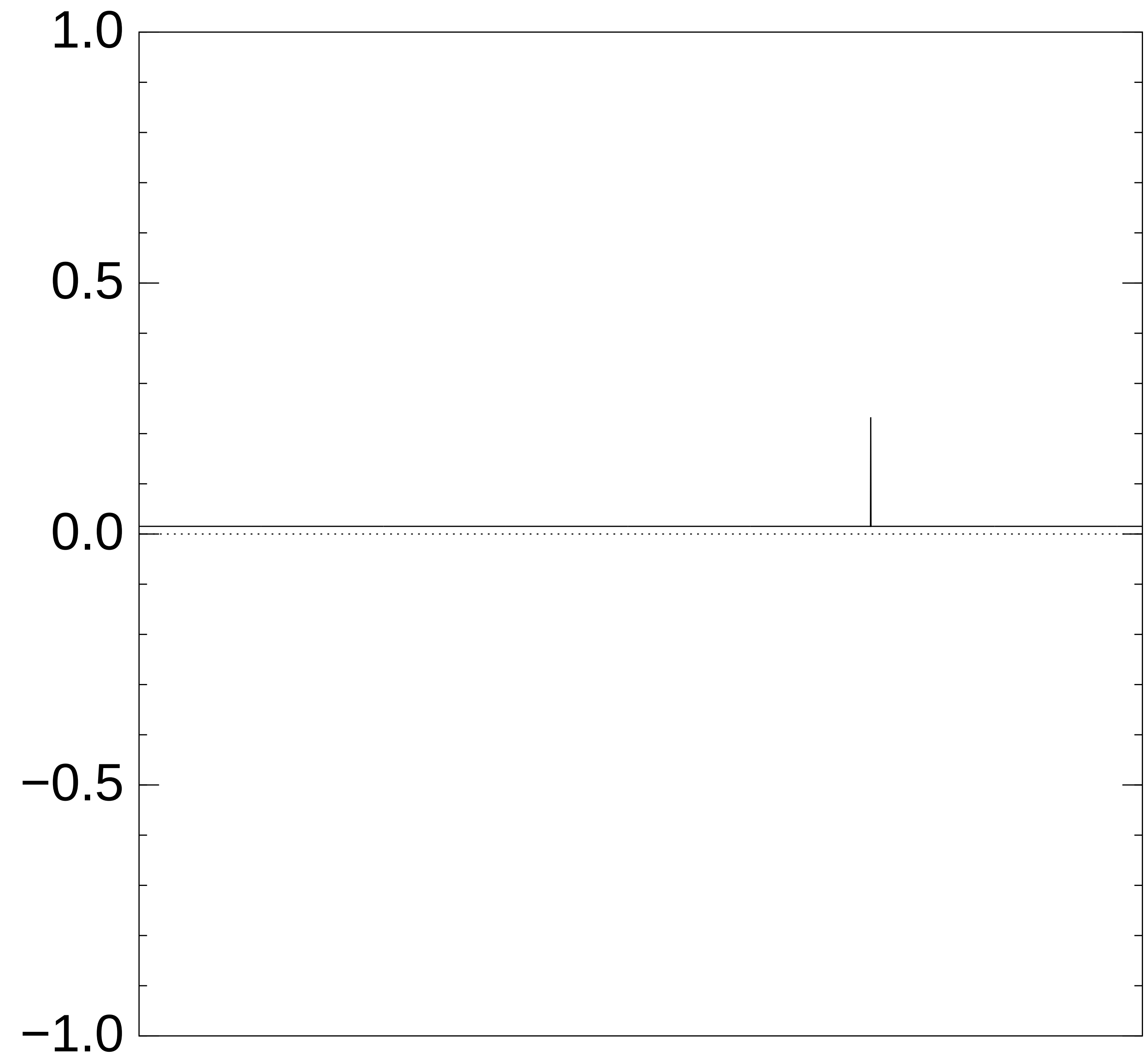
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $7 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over all n-bit strings q.

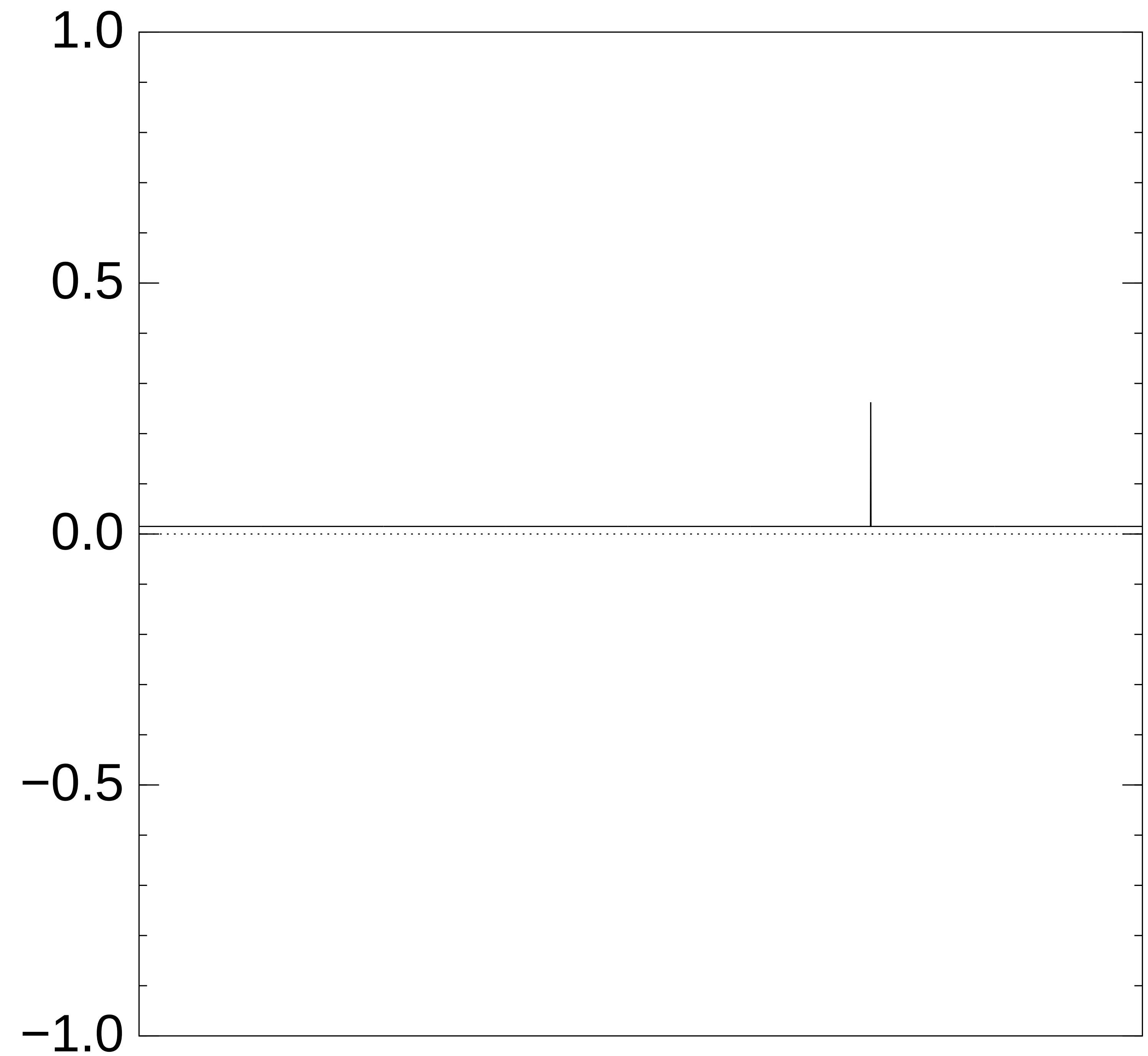
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $8 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

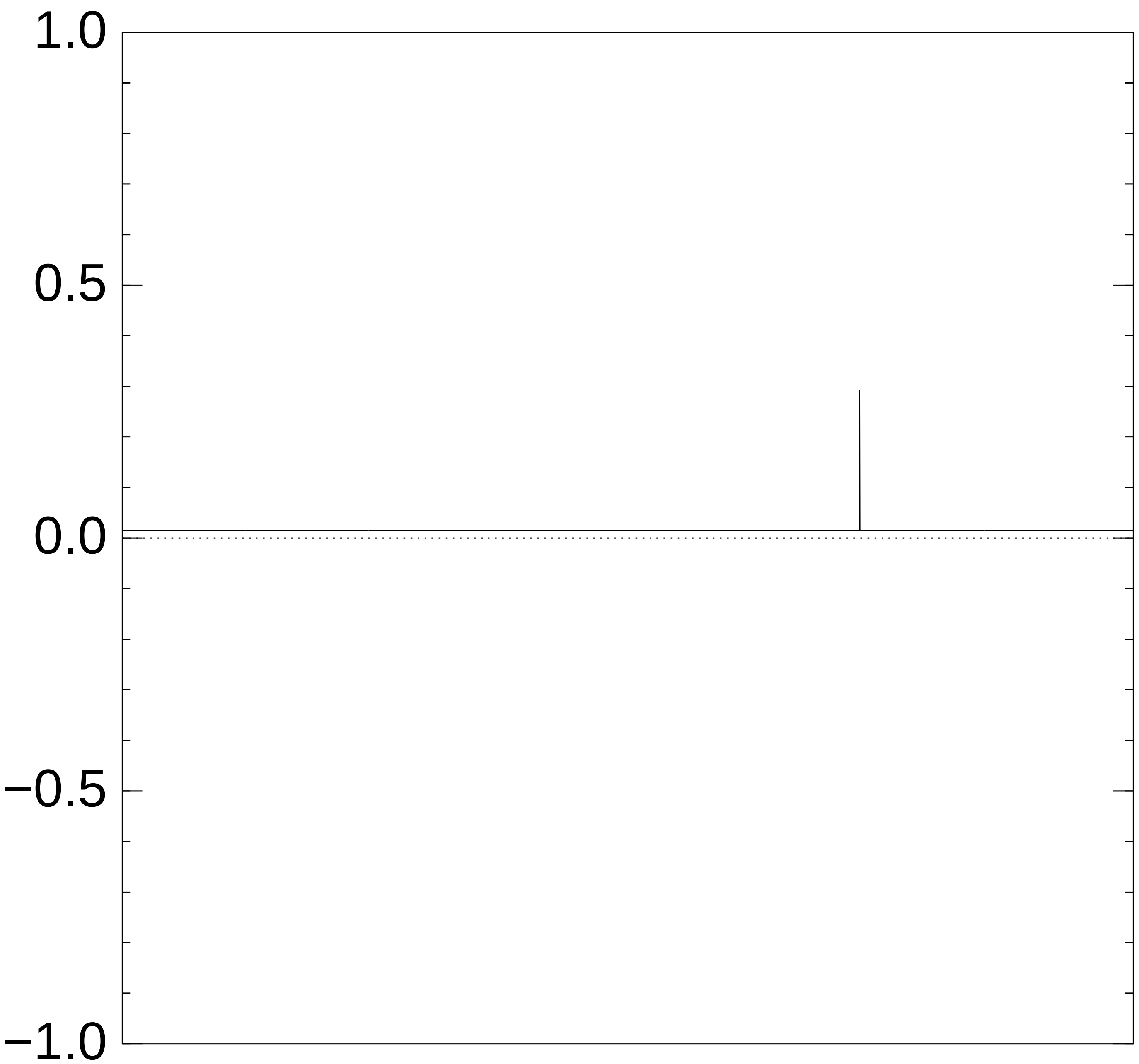
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2
about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$
for an example with $n = 12$
after $9 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over all n-bit strings q.

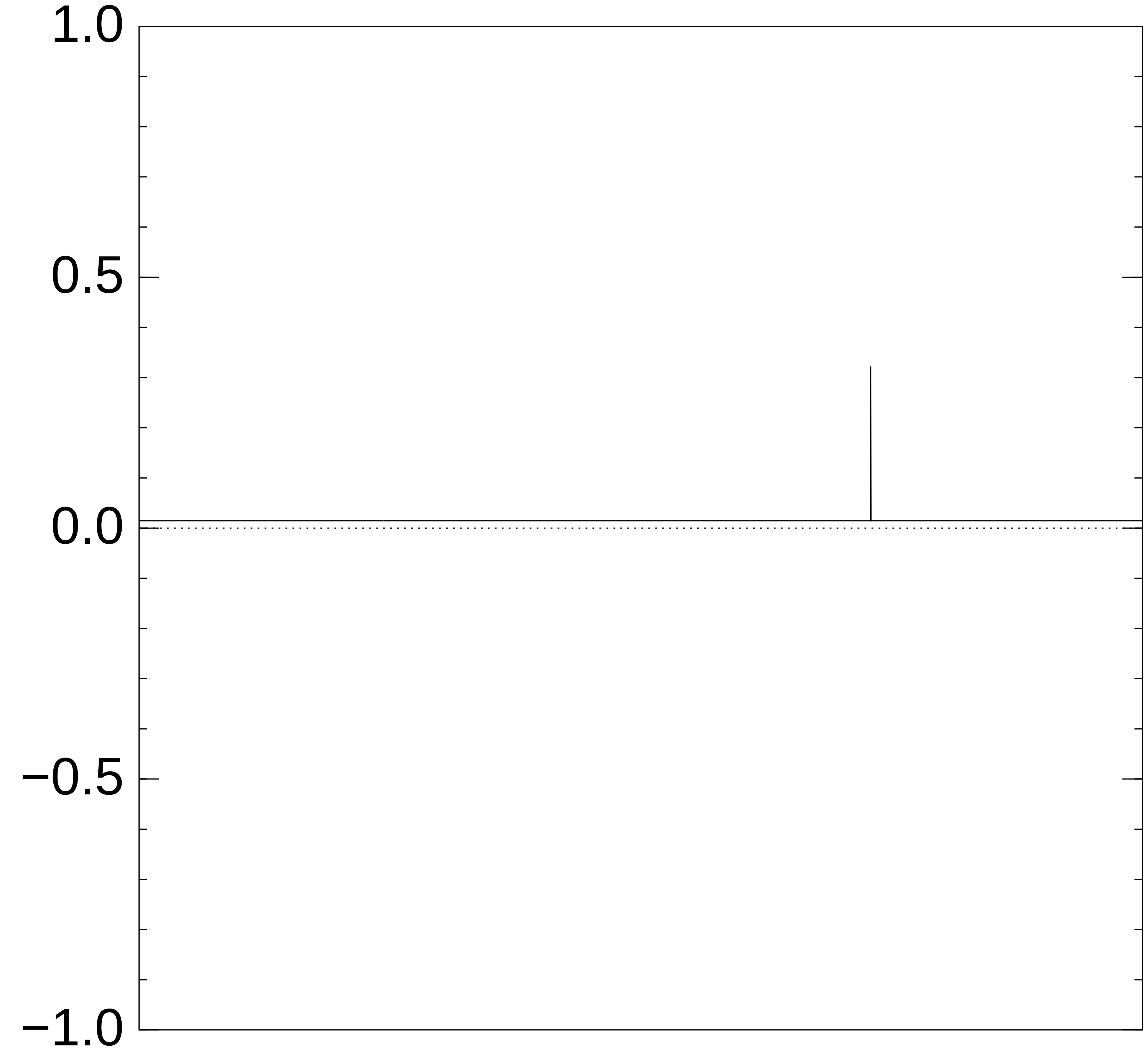
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $10 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

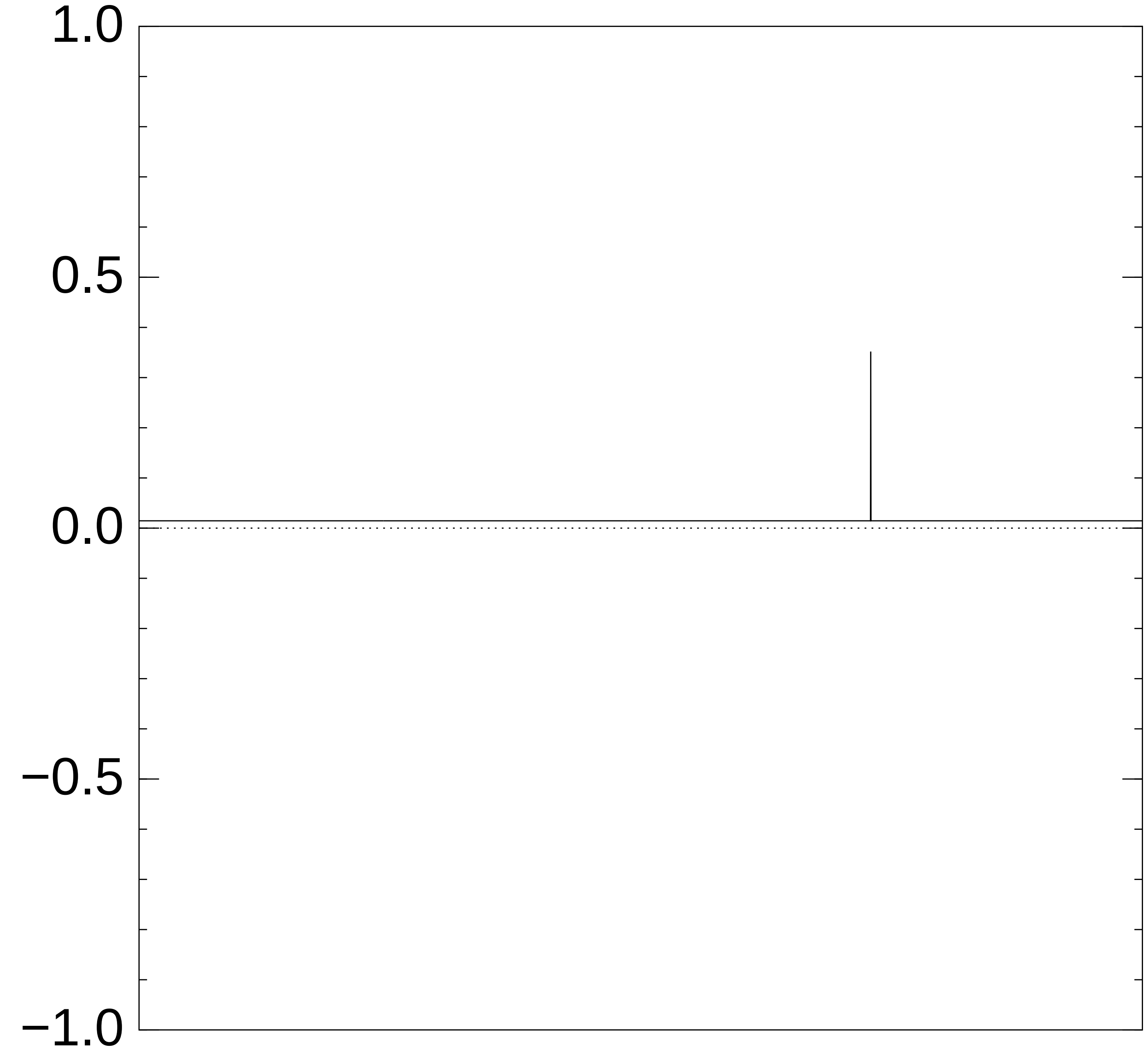
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $11 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

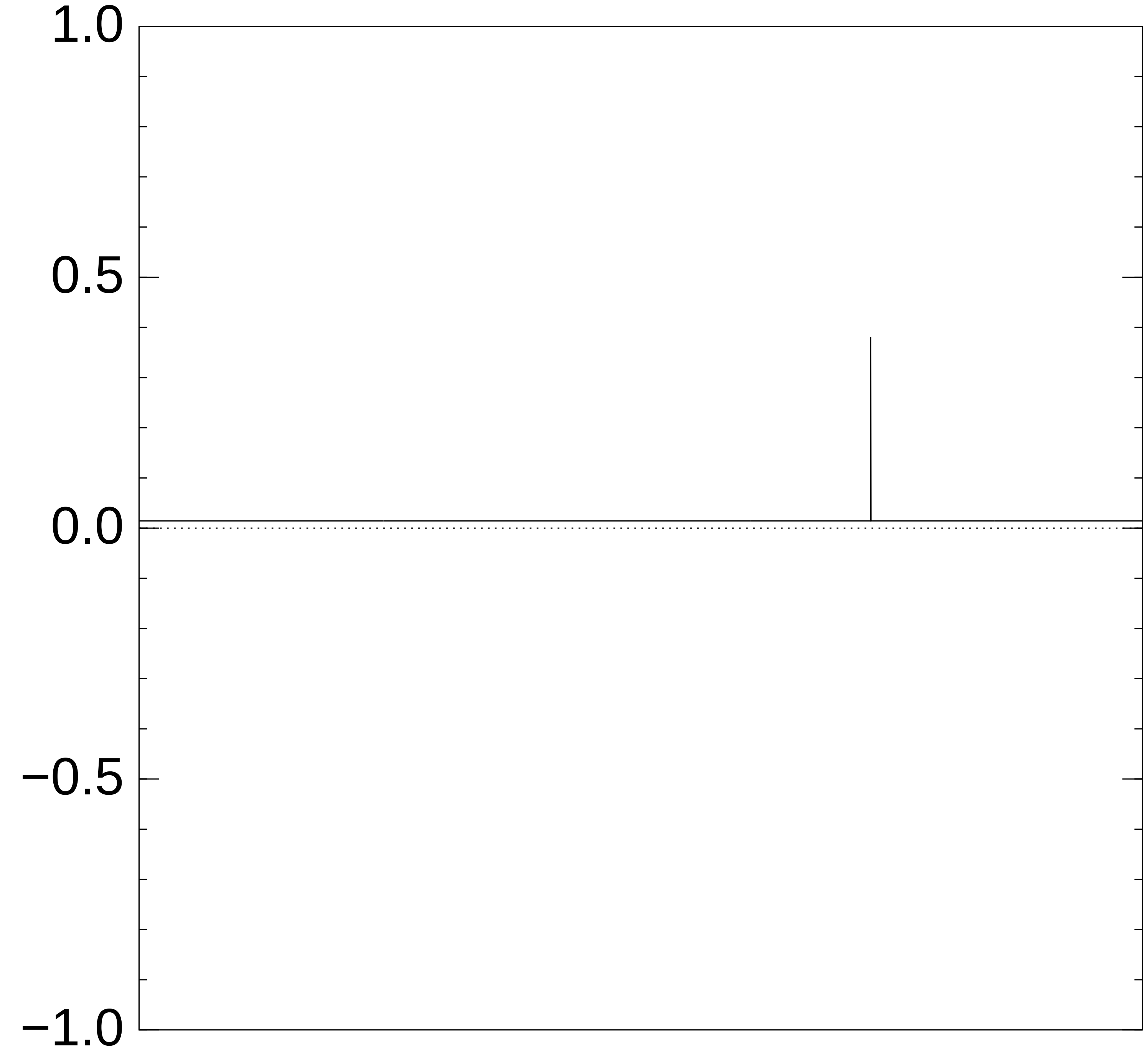
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$
for an example with $n = 12$
after $12 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over all n-bit strings q.

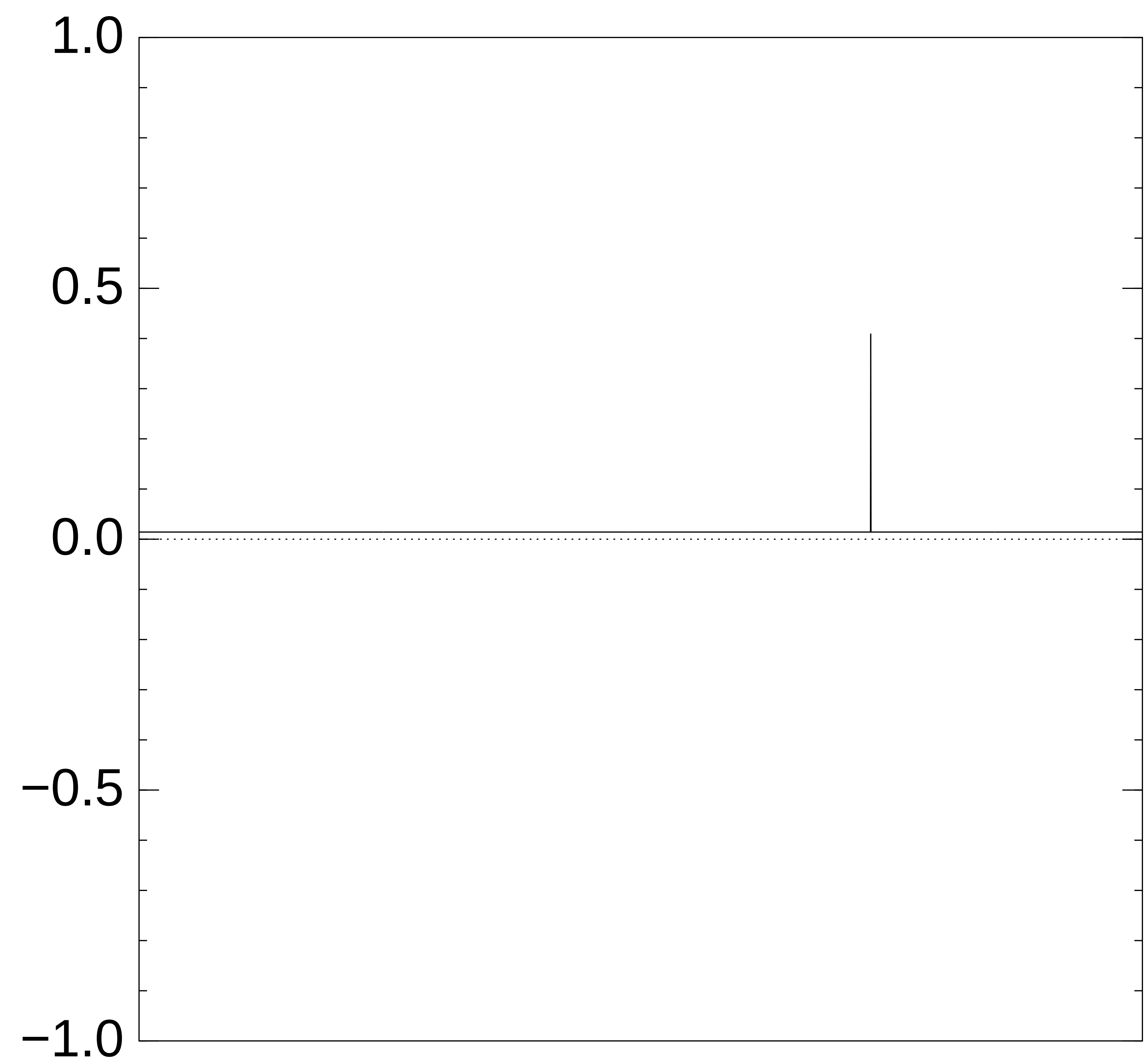
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$
for an example with $n = 12$
after $13 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

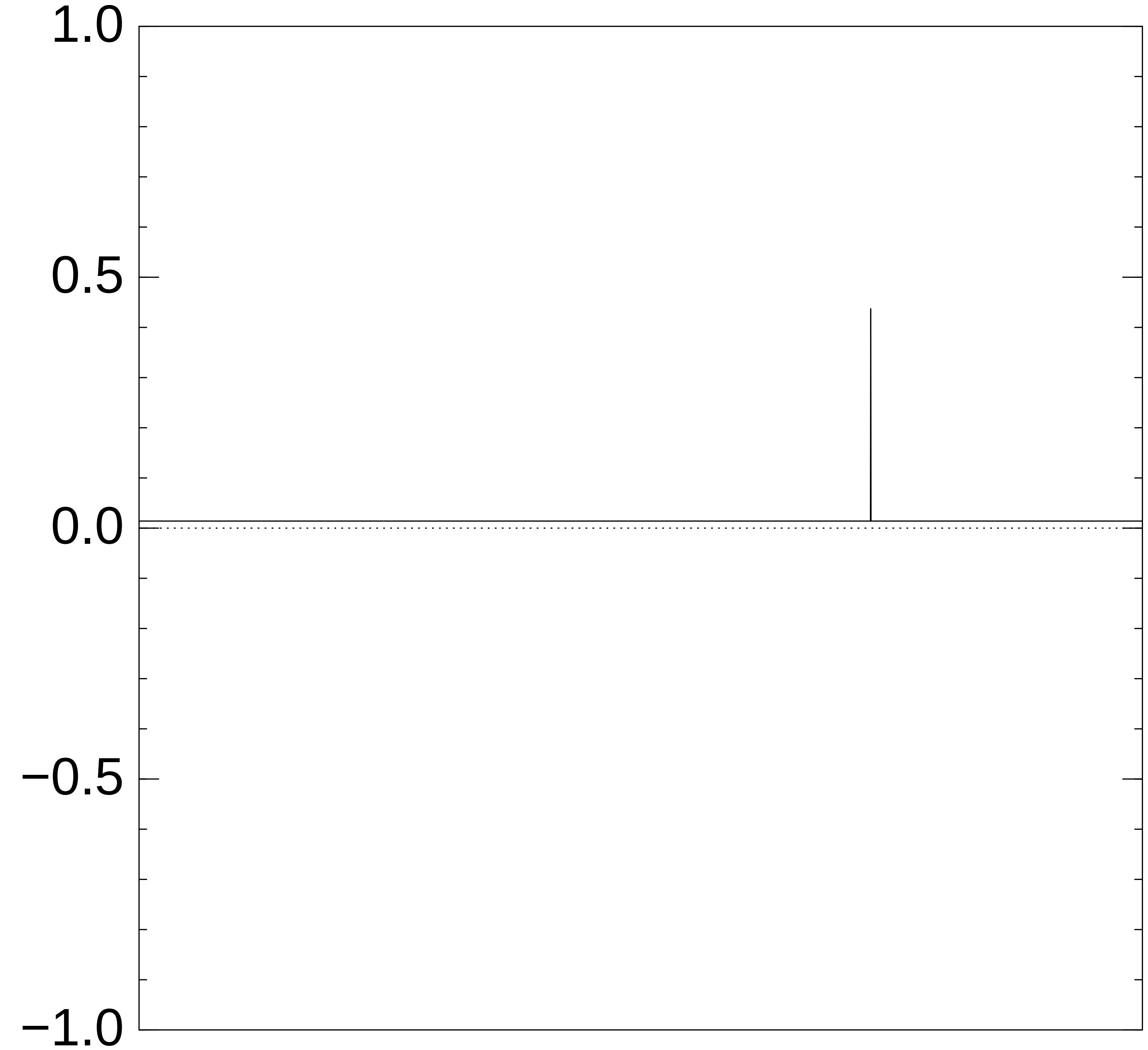
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $14 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

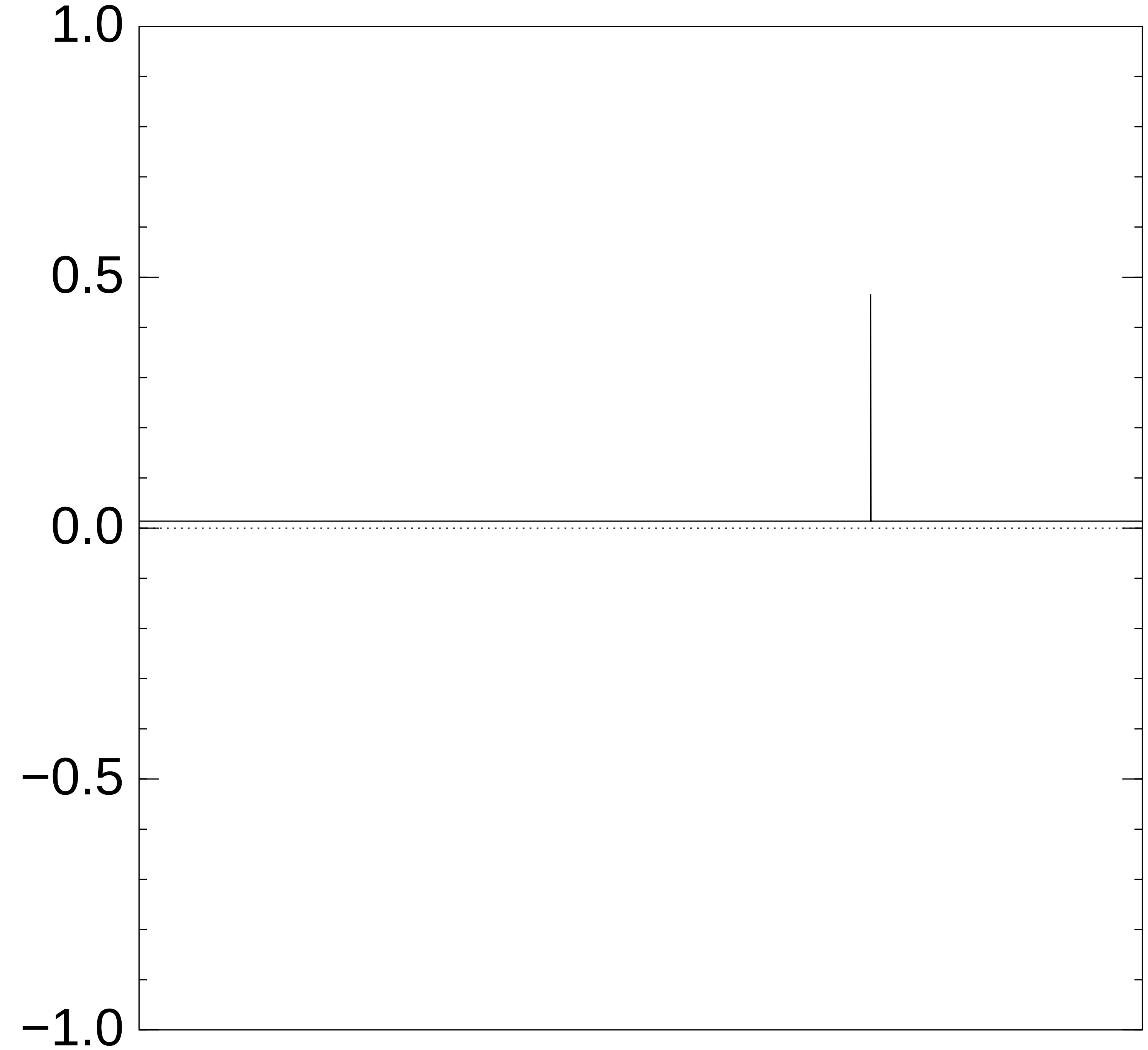
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $15 \times (\text{Step 1} + \text{Step 2})$:



Start from uniform superposition over all n-bit strings q.

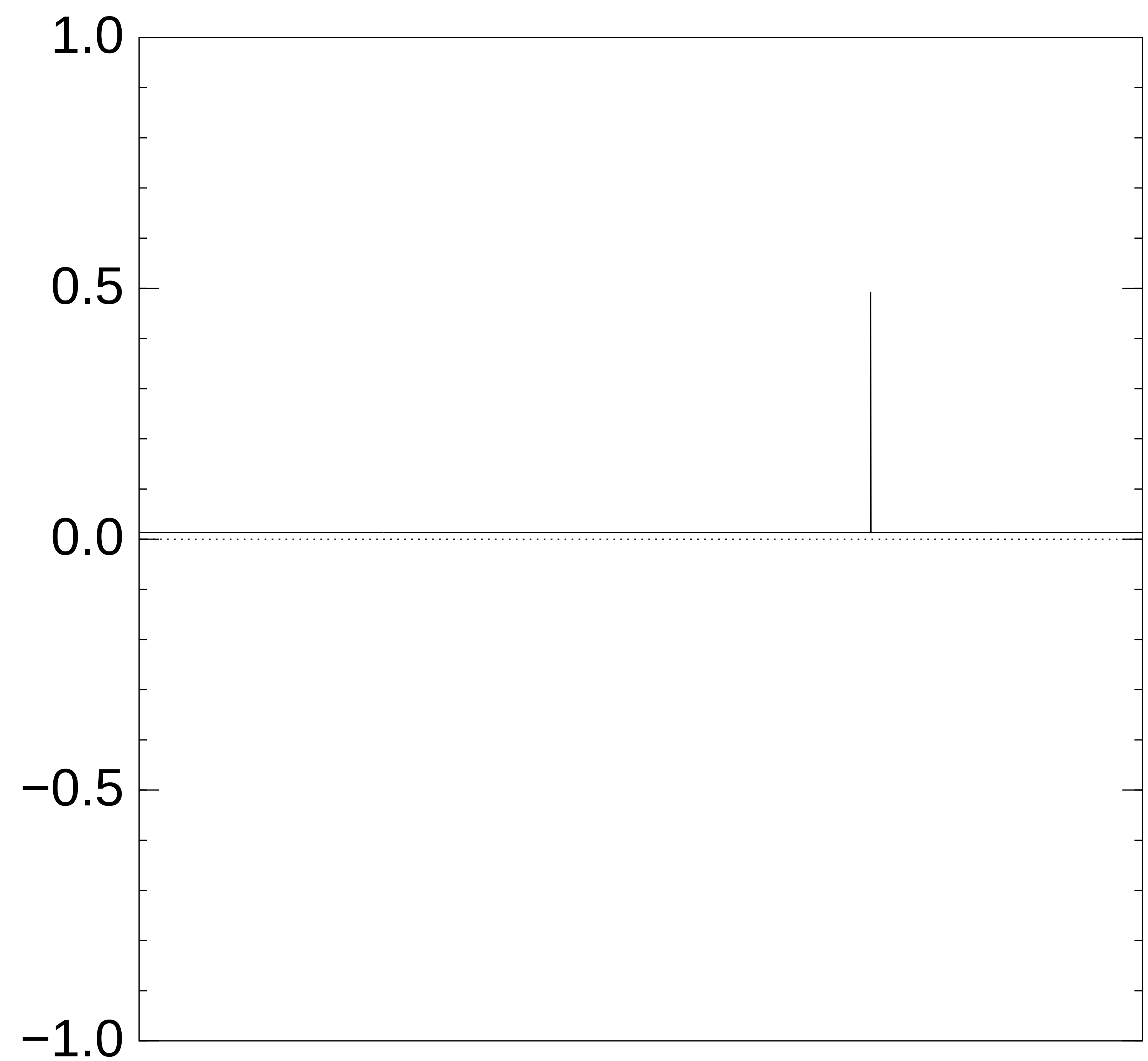
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $16 \times$ (Step $1 +$ Step 2):



Start from uniform superposition over all n-bit strings q.

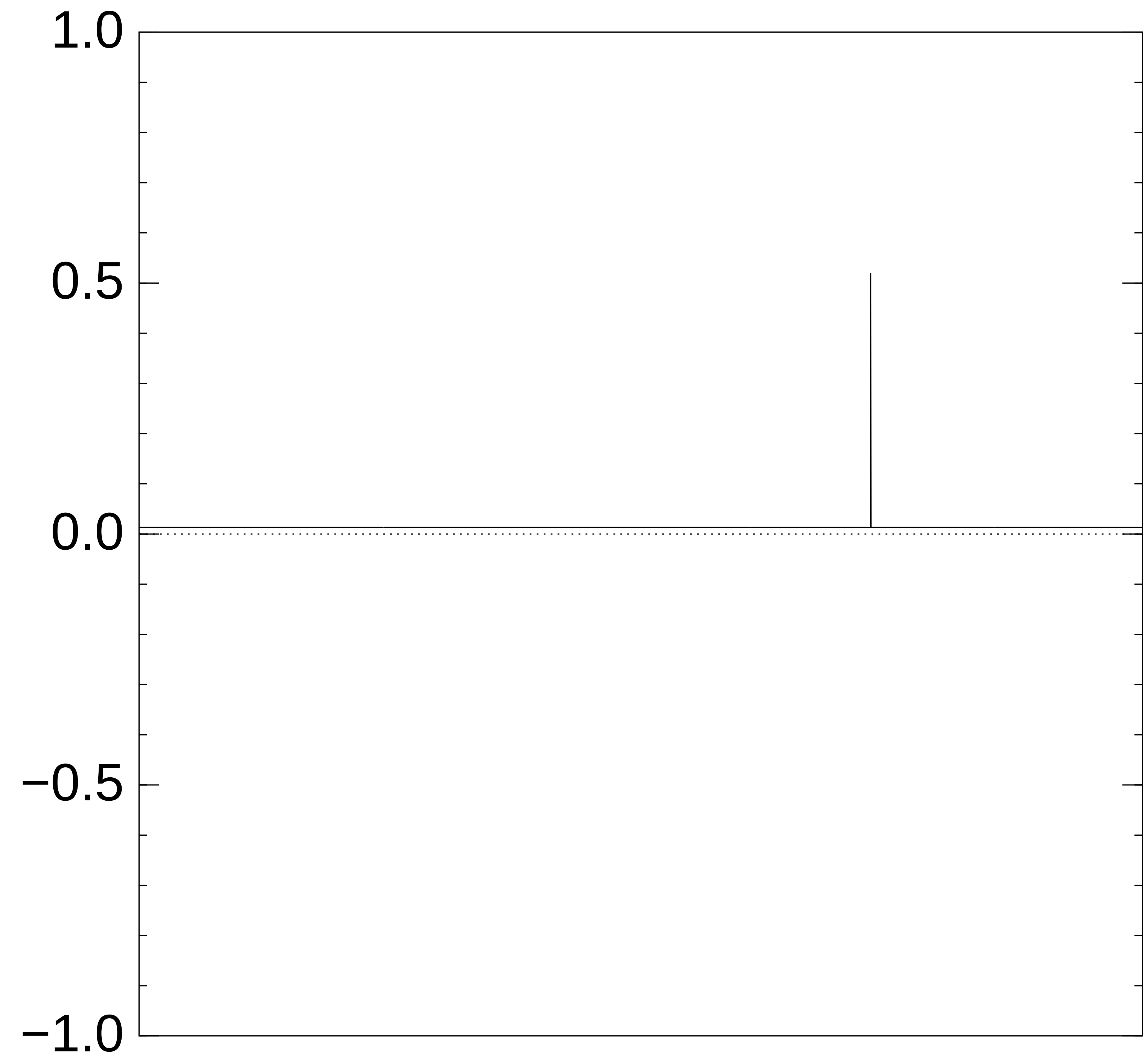
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$
for an example with $n = 12$
after $17 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

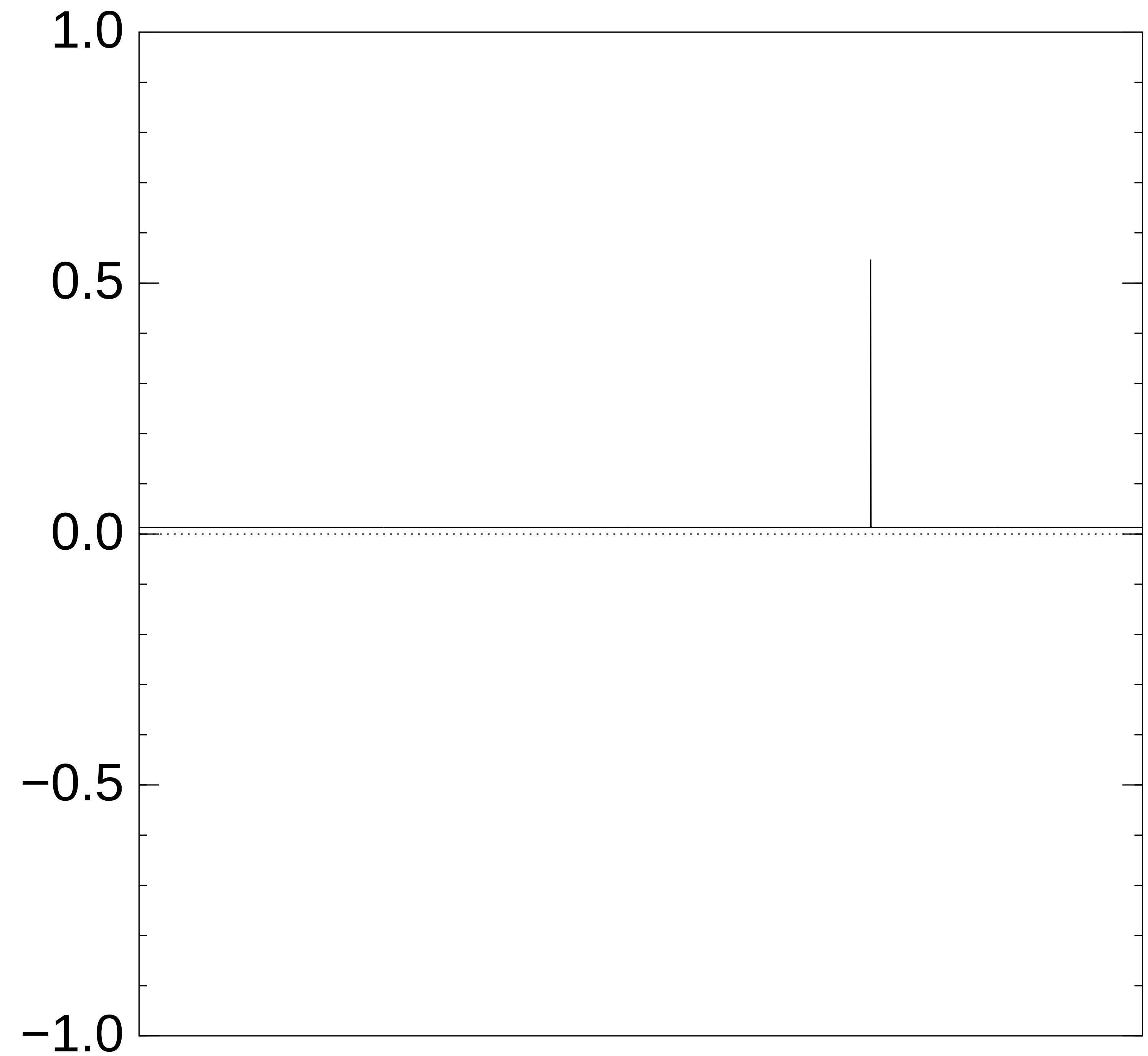
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $18 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

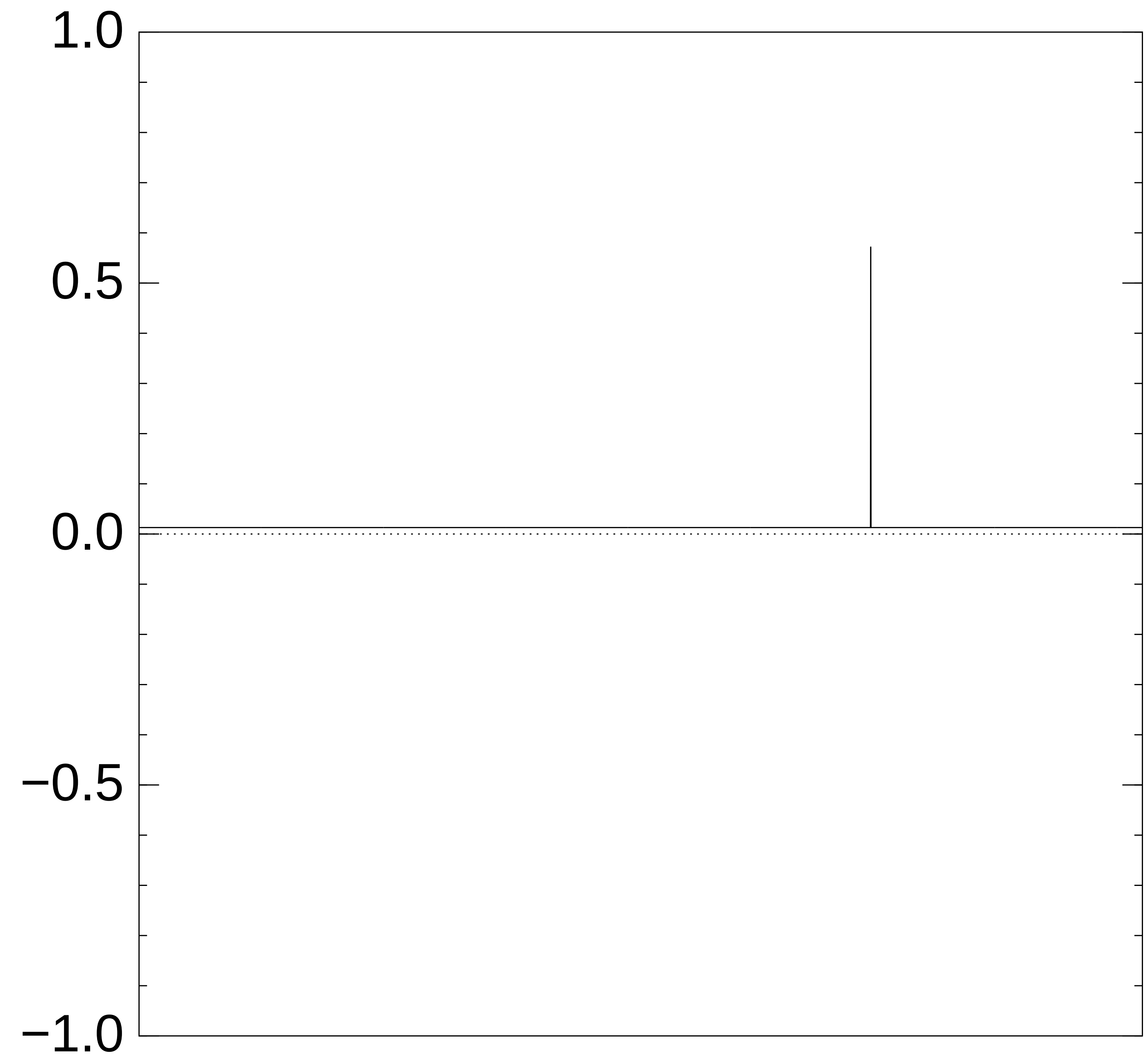
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $19 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

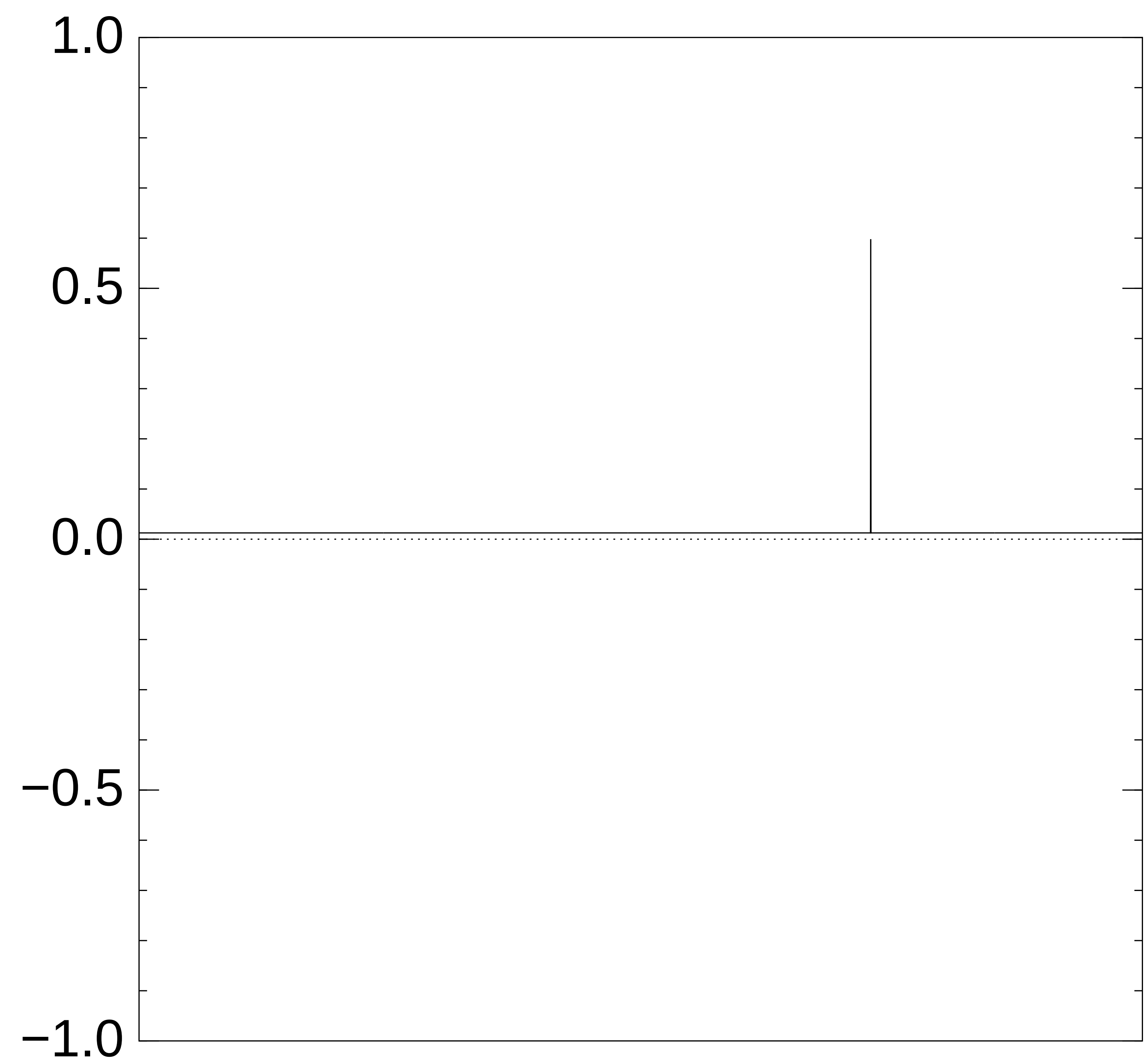
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $20 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

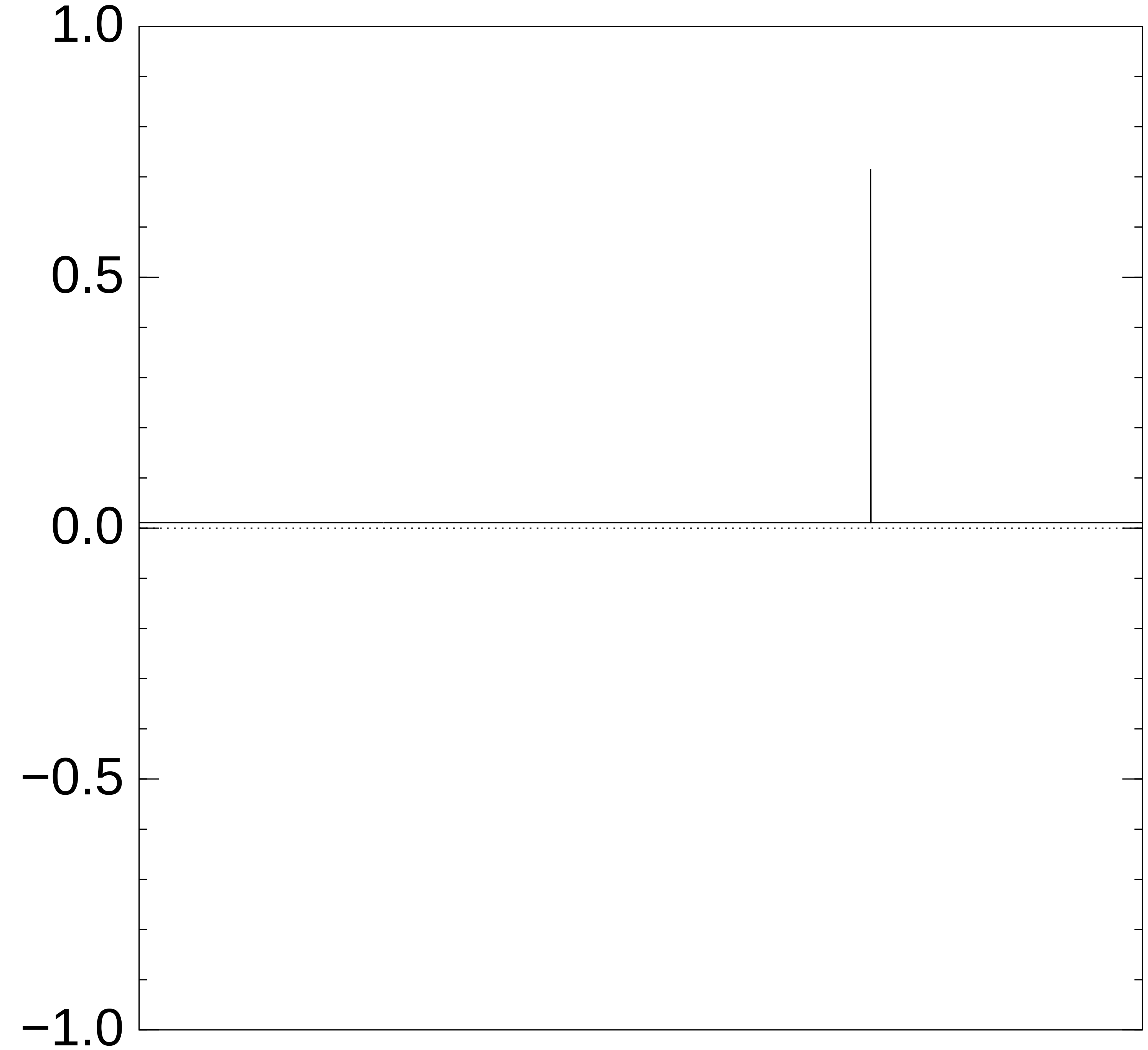
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $25 \times$ (Step $1 +$ Step 2):



Start from uniform superposition over all n-bit strings q.

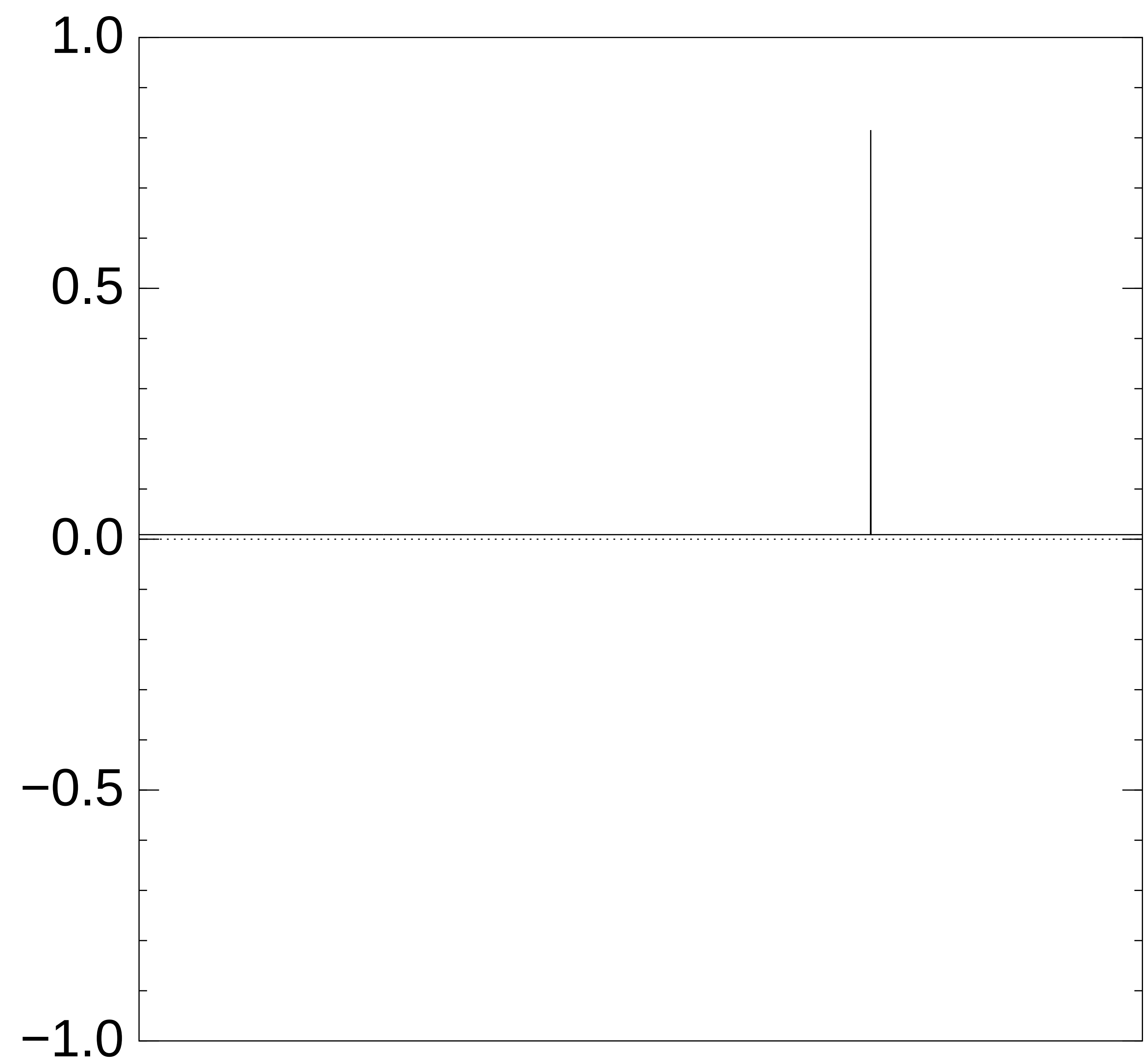
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $30 \times$ (Step $1 +$ Step 2):



Start from uniform superposition over all n-bit strings q.

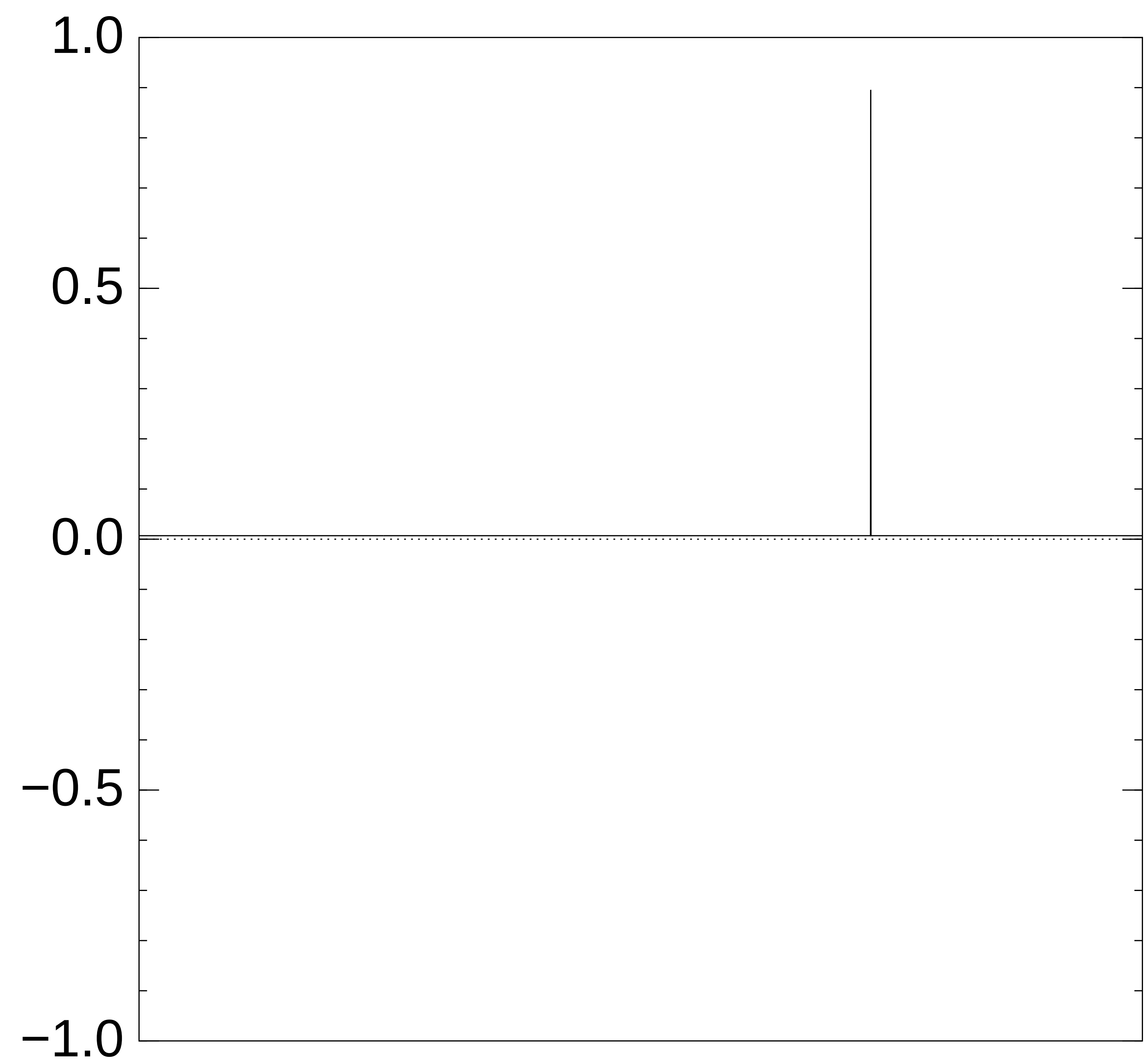
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $35 \times$ (Step $1 +$ Step 2):



Good moment to stop, measure.

Start from uniform superposition over all n-bit strings q.

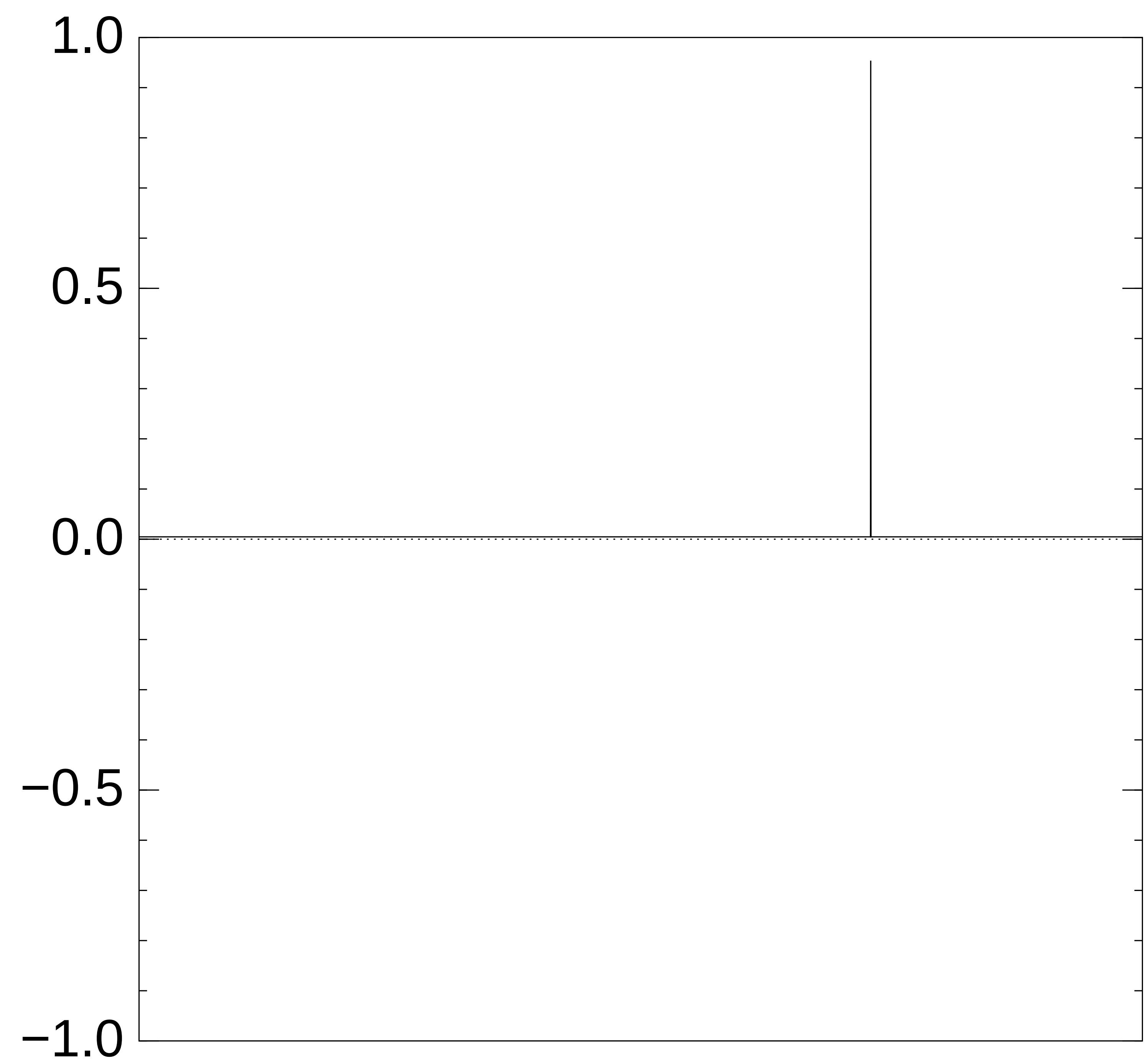
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $40 \times$ (Step $1 +$ Step 2):



Start from uniform superposition over all n-bit strings q.

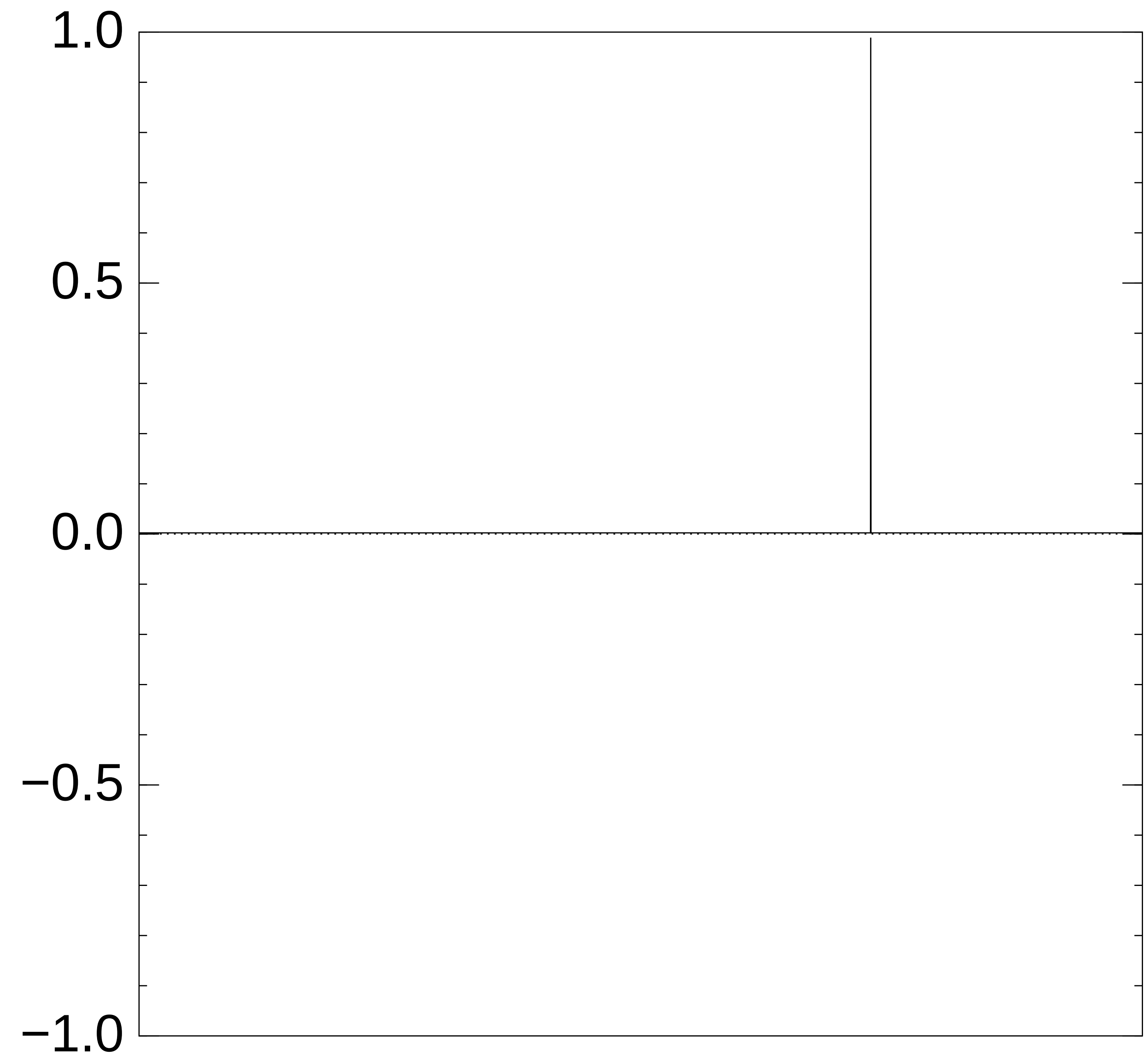
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $45 \times$ (Step $1 +$ Step 2):



Start from uniform superposition over all n-bit strings q.

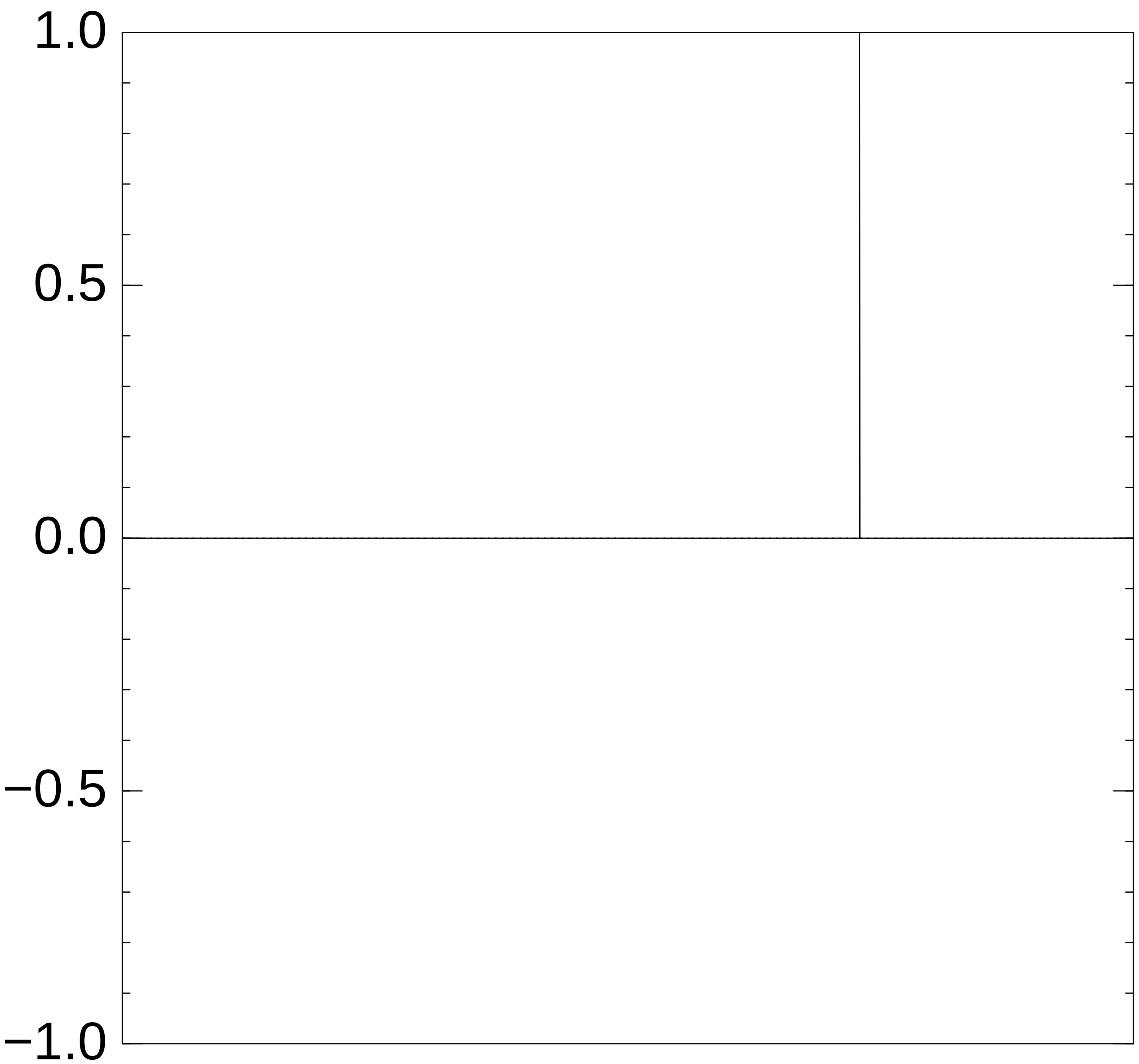
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $50 \times$ (Step $1 +$ Step 2):



Traditional stopping point.

Start from uniform superposition over all n-bit strings q.

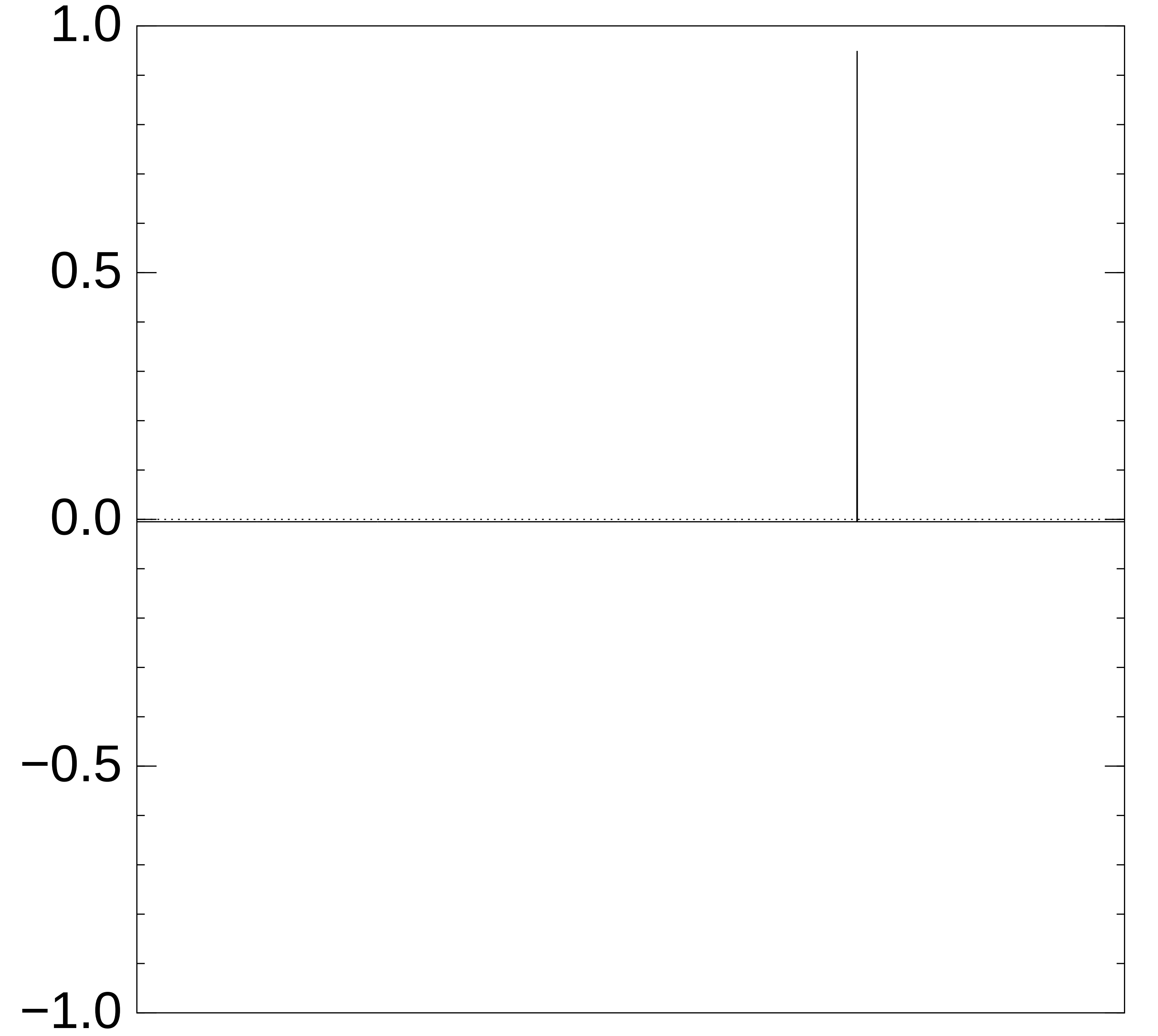
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $60 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over all n-bit strings q.

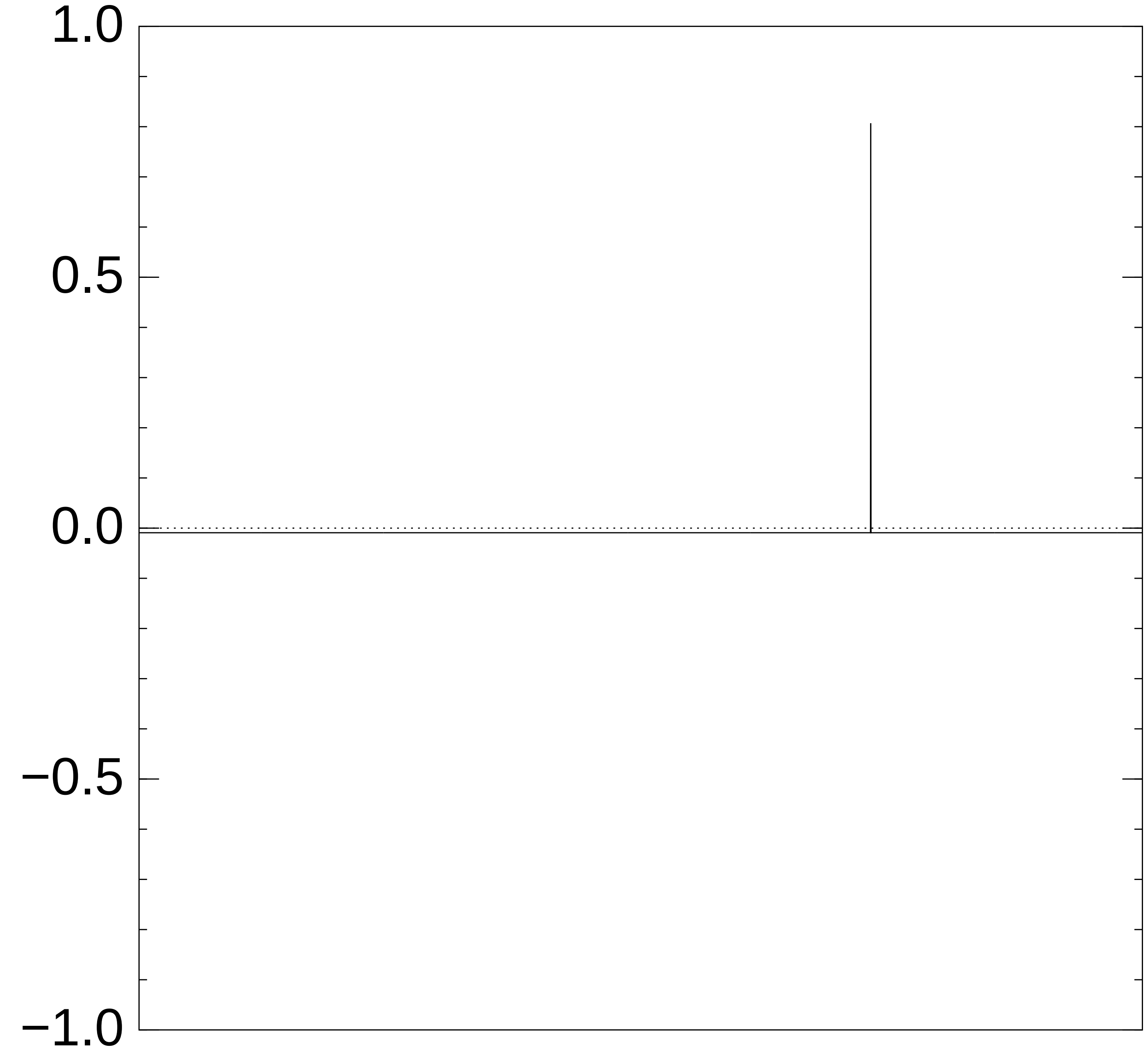
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$
for an example with $n = 12$
after $70 \times$ (Step $1 +$ Step 2):



Start from uniform superposition over all n-bit strings q.

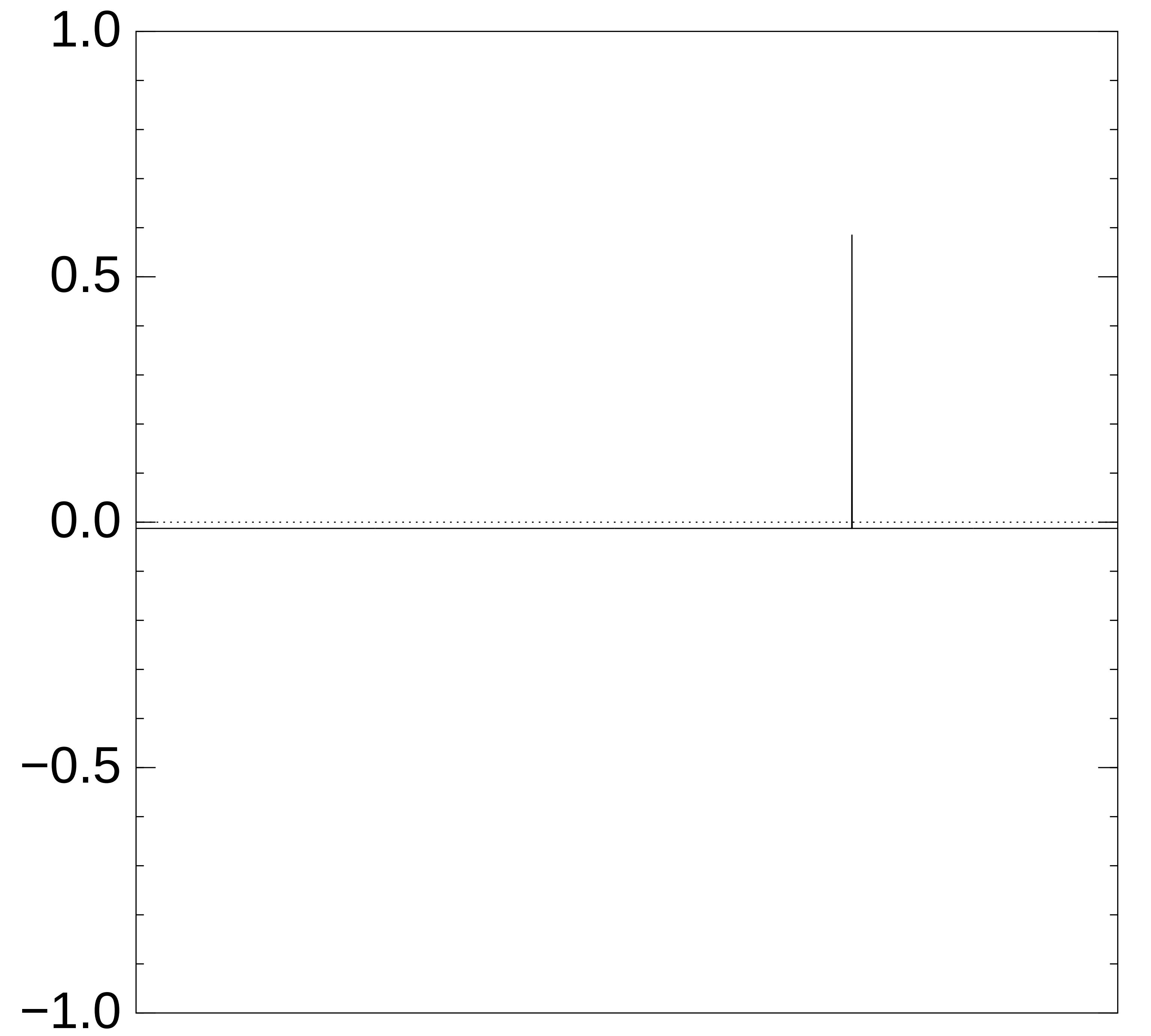
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $80 \times (\text{Step } 1 + \text{Step } 2)$:



Start from uniform superposition over all n-bit strings q.

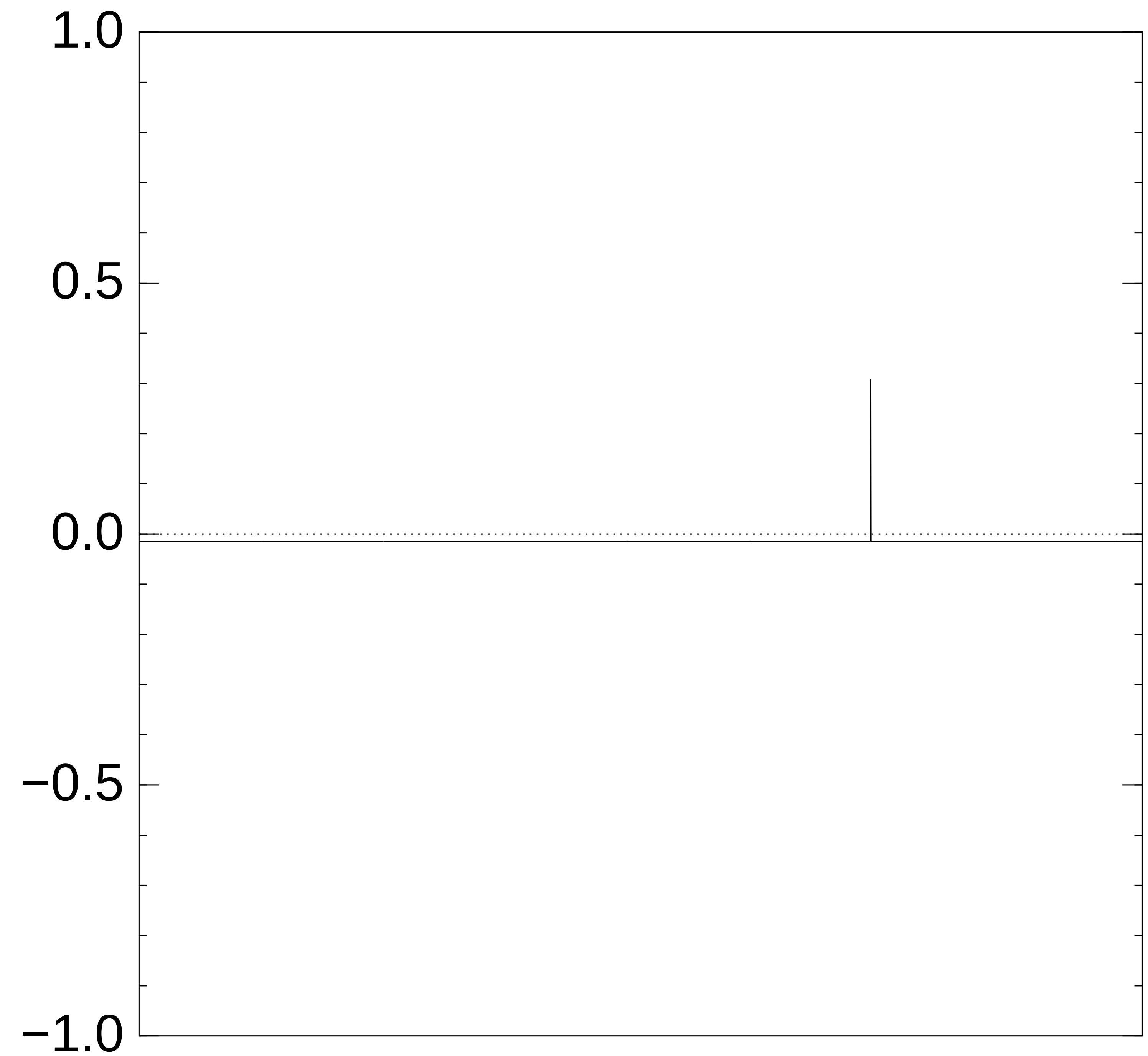
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $90 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

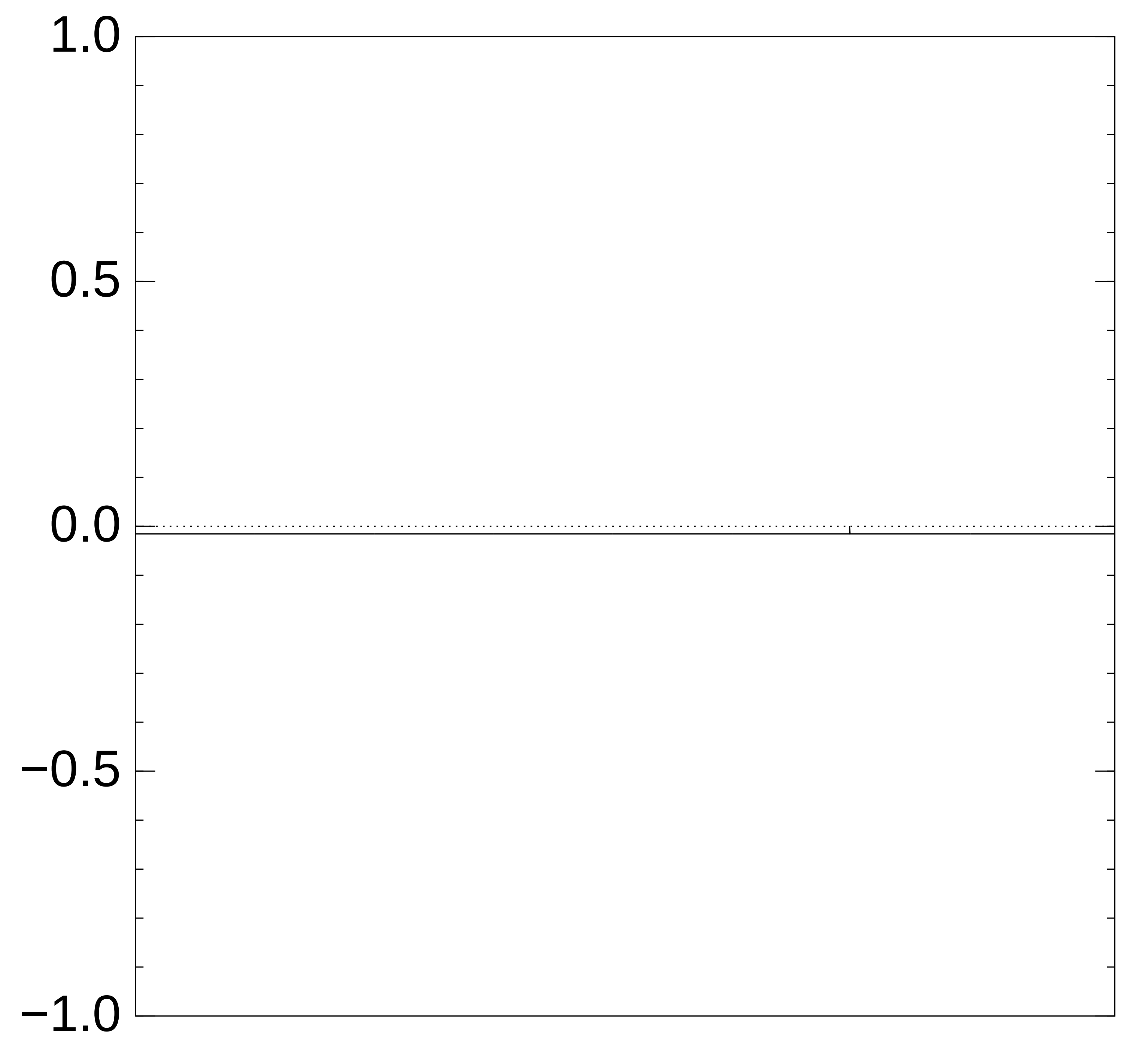
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat steps 1 and 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Graph of $q \mapsto a_q$ for an example with $n = 12$ after $100 \times$ (Step 1 + Step 2):



Very bad stopping point.

om uniform superposition
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Set $a \leftarrow b$ where
a_q if $f(q) = 0$,
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steps 1 and 2
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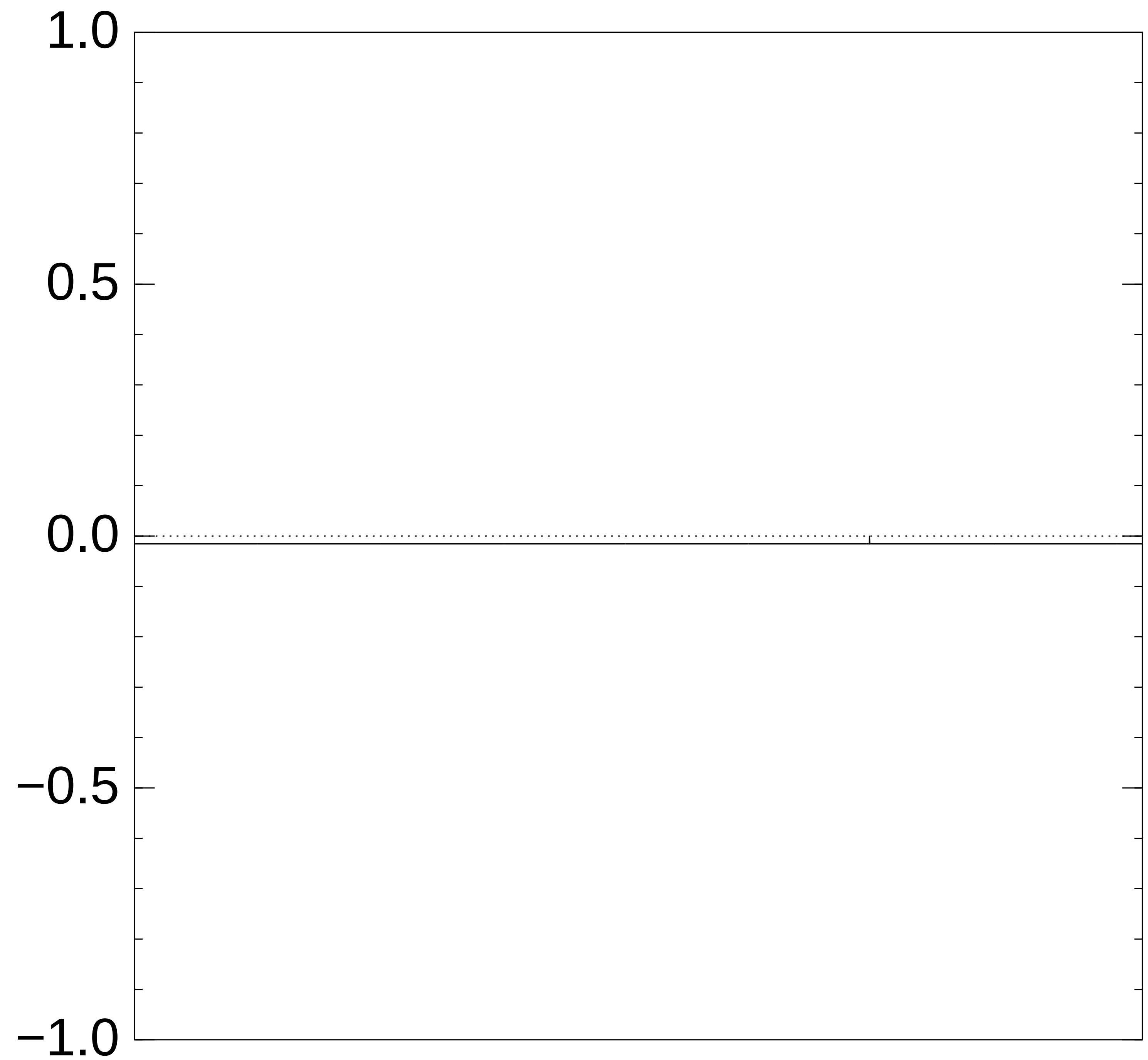

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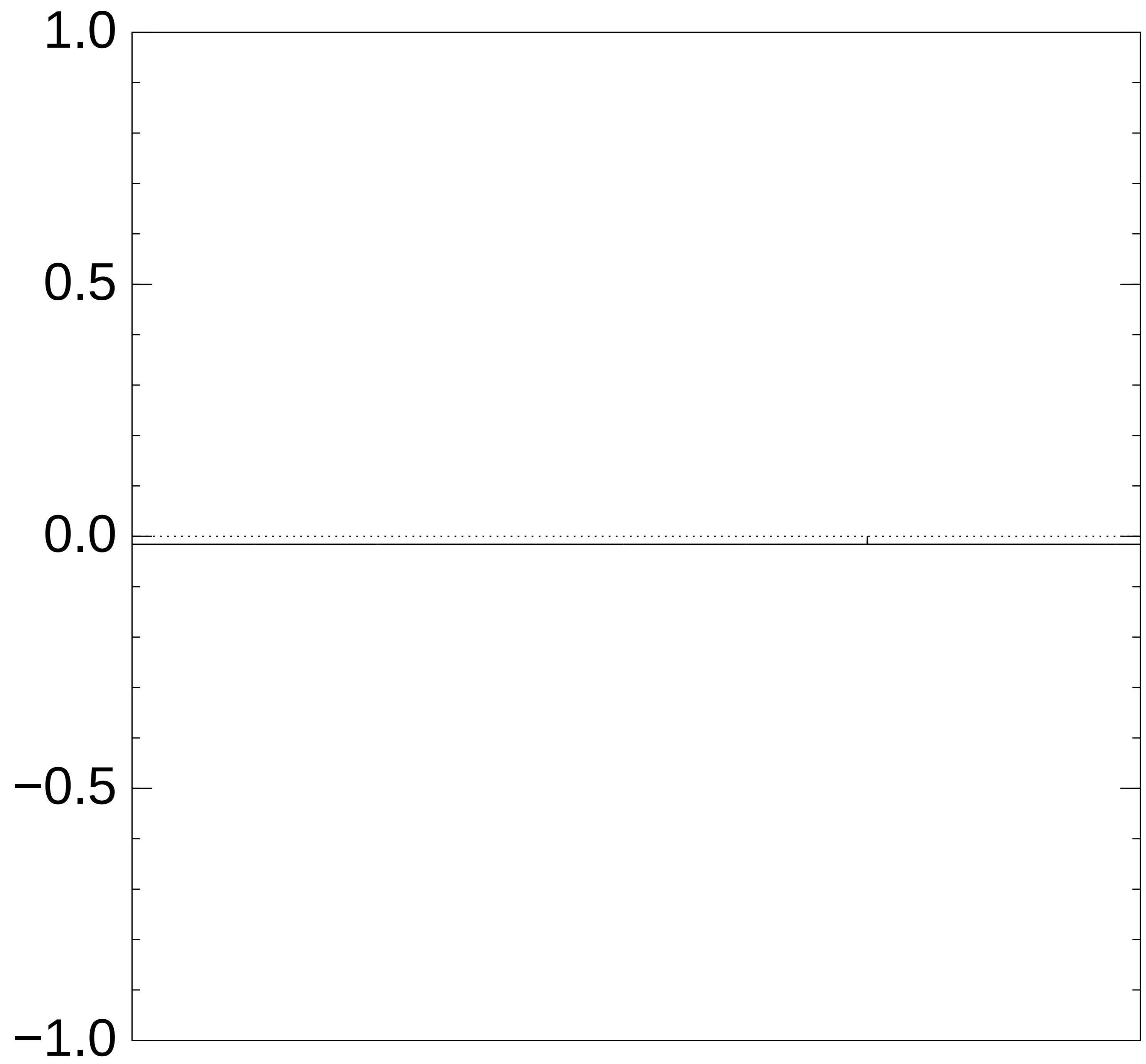

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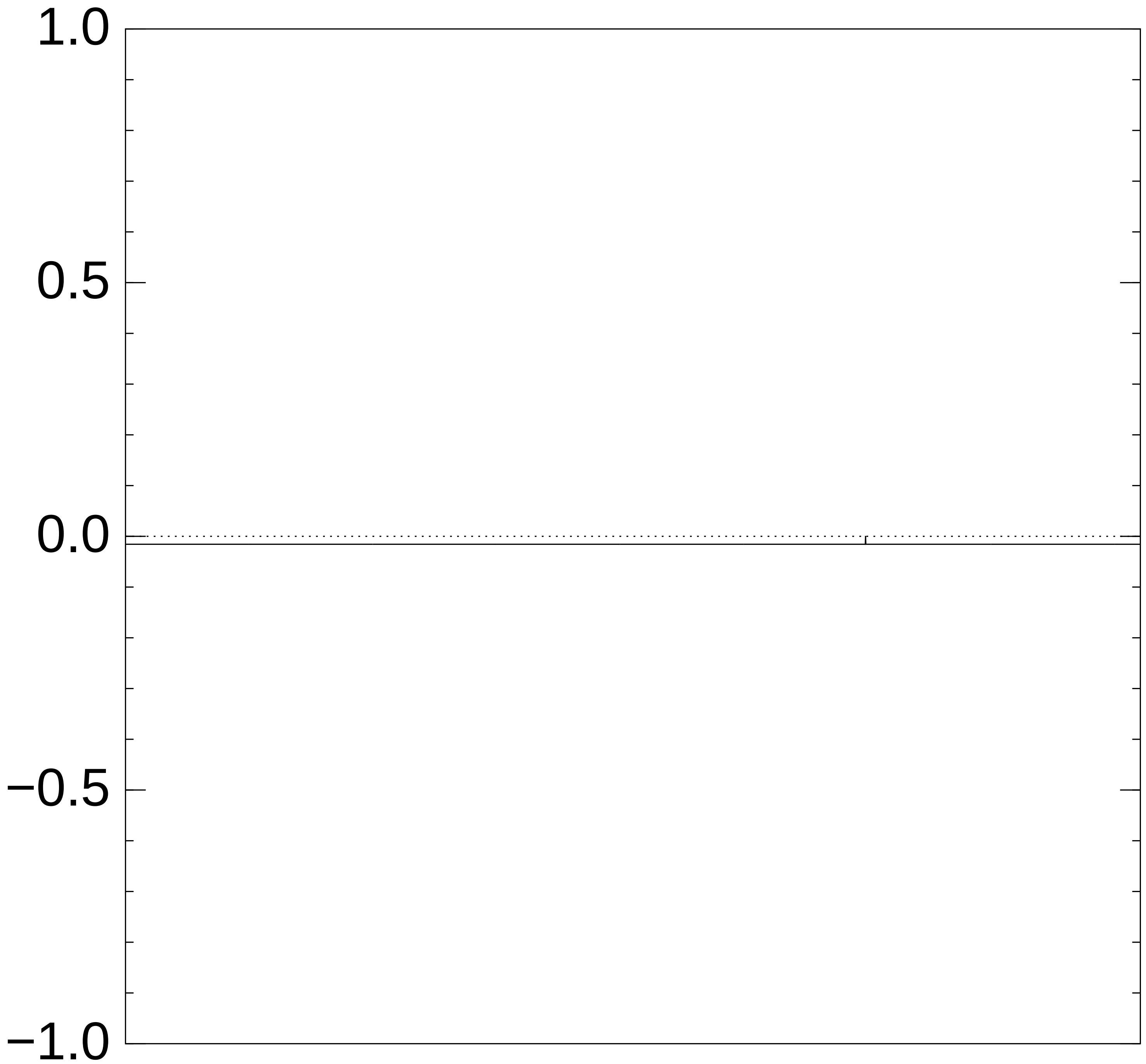

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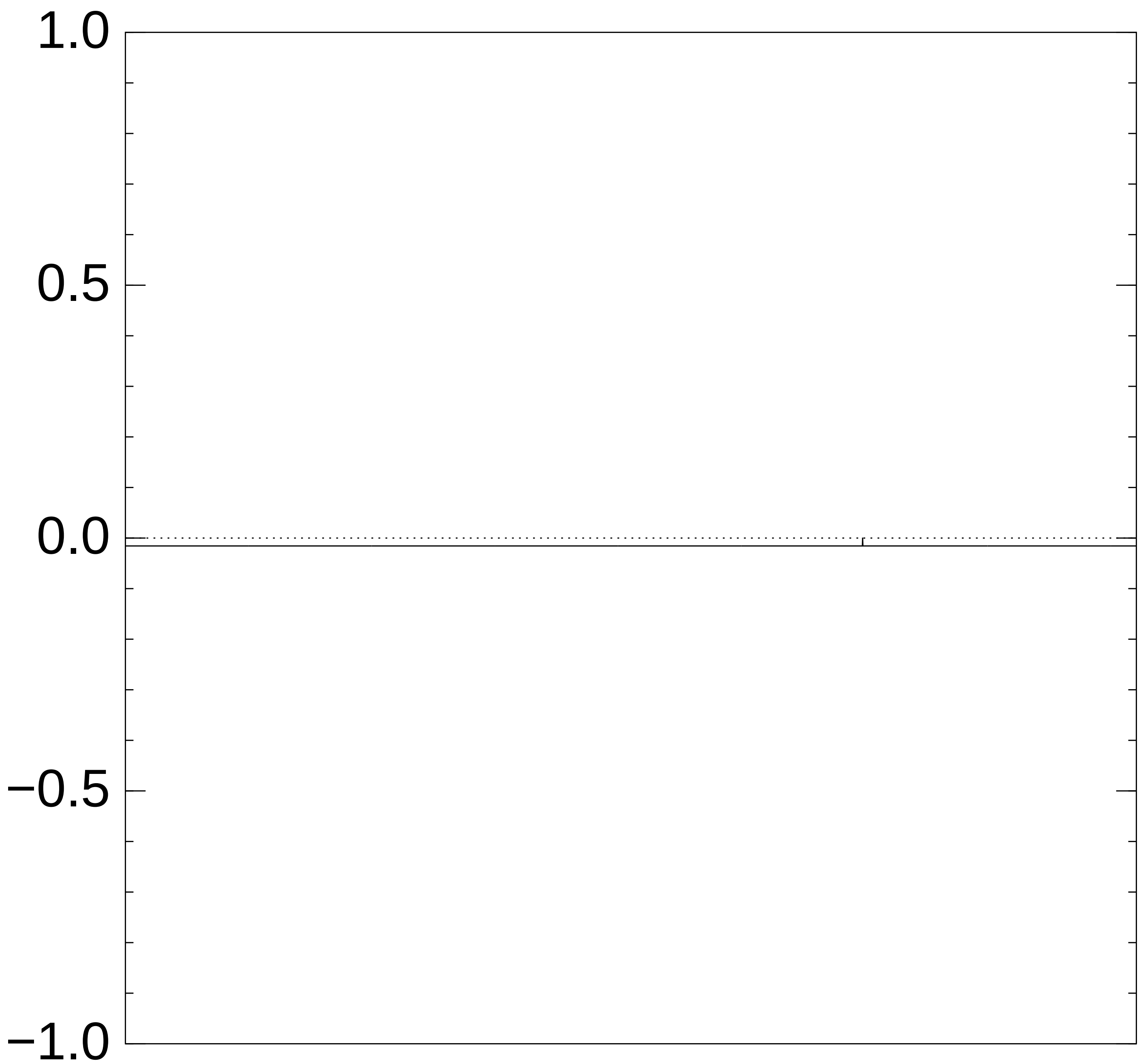

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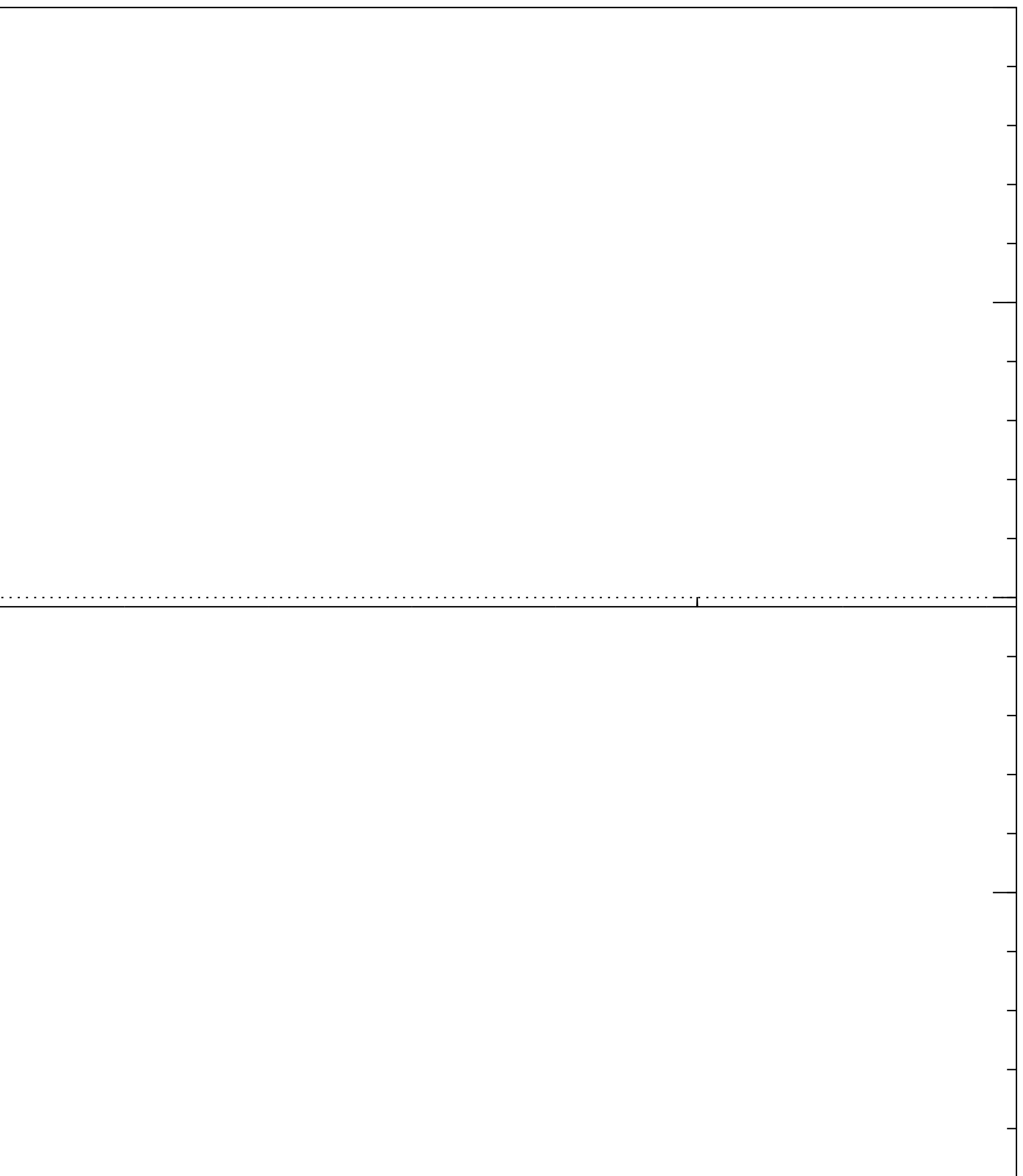

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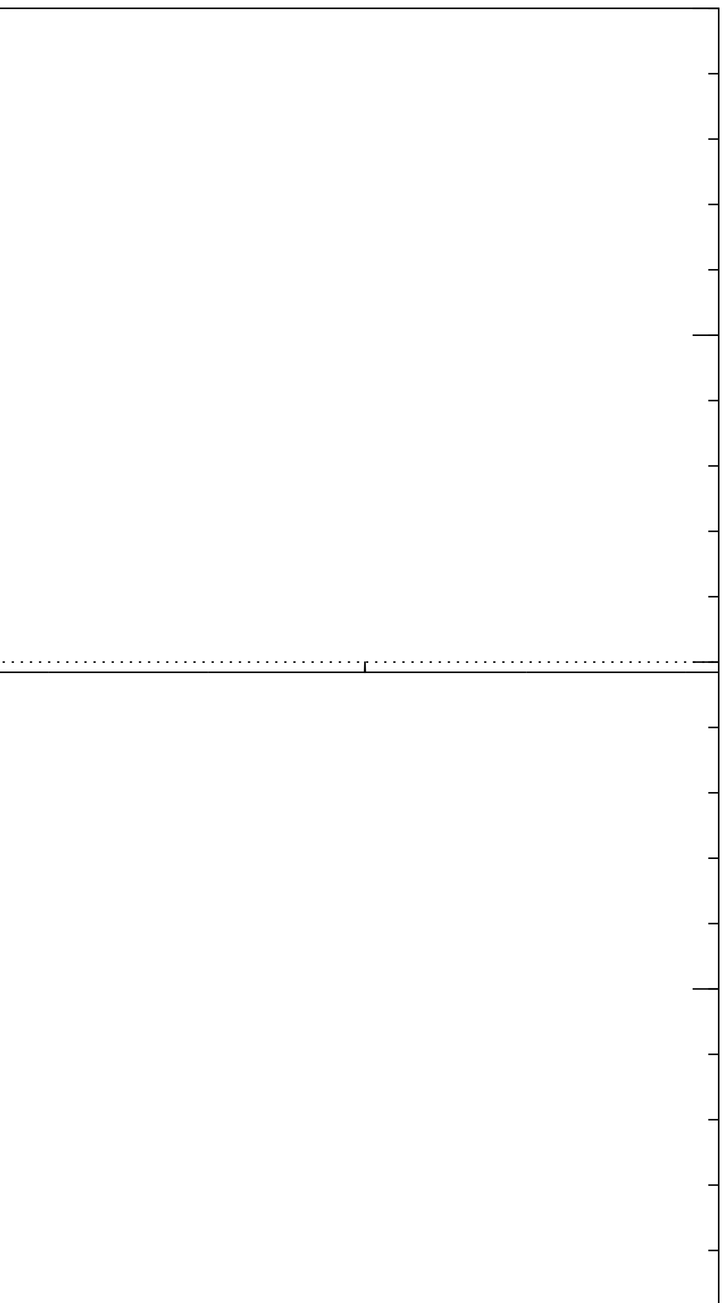

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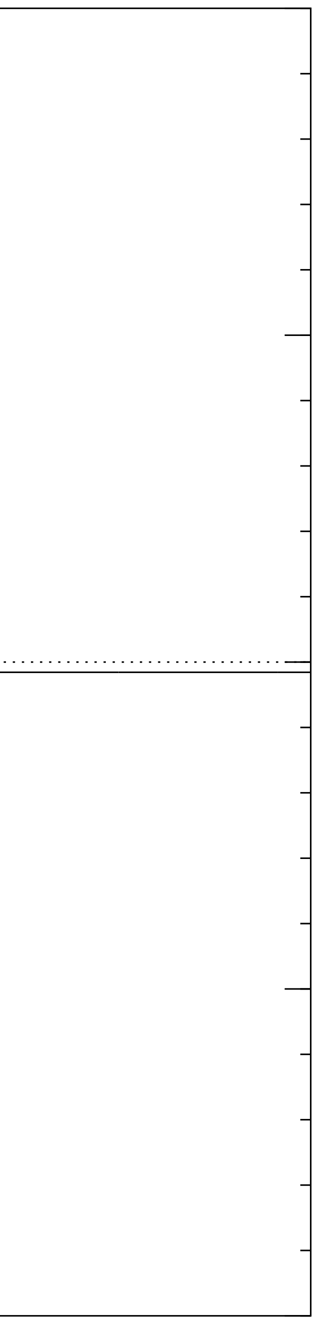

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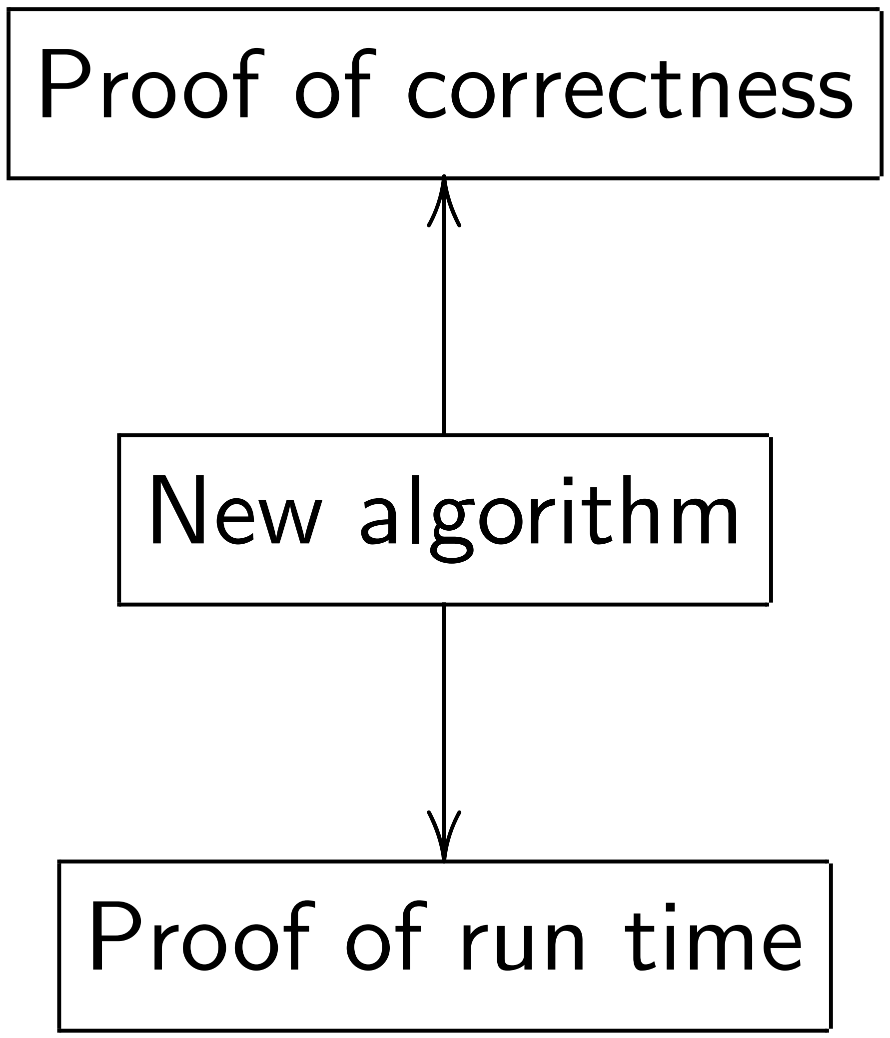

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3. Fast **trapdoor simulation.**
Simulator (like prover) knows
more than the algorithm does.
Tung Chou has implemented this,
found errors in two publications.