McEliece verification

Daniel J. Bernstein
“Who cares? Big keys are unusable!”

Let's look at the facts:

• 1MB is very fast on a modern network.

Are Netflix and YouTube unusable?

• Google's key can be used to protect any number of ciphertexts to/from Google.

• $1 \text{ key} + 10^{6} \text{ ciphertexts}$ for McEliece is several times less network traffic than $1 \text{ key} + 10^{6} \text{ ciphertexts}$ for lattices.

• McEliece deployment is underway: e.g., McEliece is already used in some end-to-end secure-messaging systems and the Mullvad and Rosenpass VPNs.
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Nearly 50 papers attacking one-wayness of McEliece have produced only minor attack speedups since 1978: asymptotically 0% change in pre-quantum security levels. Post-quantum: like AES. The attack surface is thoroughly explored and well understood.
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New: CryptAttackTester includes full attack circuits + analyses passing systematic tests.
McEliece attack challenges

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Observed speeds match algorithm analyses. Security levels are remarkably stable.
Classic McEliece implementations

Official software for Classic McEliece is distributed via SUPERCOP benchmarking framework. Four implementations for each parameter set, all passing TIMECOP:
- **ref**: portable, prioritizing simplicity.
- **vec**: portable, 64-bit vectorization.
- **sse**: Intel/AMD, 128-bit vectorization.
- **avx**: Intel/AMD, 256-bit vectorization.


Daniel J. Bernstein, McEliece verification
Checklist for software verification

Want to verify that each internal operation works correctly for all possible inputs:

- SHAKE256 on constant-length inputs.
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- Sorting integer arrays in constant time.
- “Control bits” for permutations.
- Arithmetic in binary fields (e.g., $\mathbb{F}_{2^{8192}}$).
- Constant-time row reduction of matrices.
- Constant-time decoding of Goppa codes.

Plus: Put everything together into “keygen, enc, dec always work”. Automate the entire process to handle many implementations.
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Want secret permutation of \{0, 1, \ldots, 8191\}. Solution: sort 8192 secret 32-bit integers and their indices; restart if there are collisions.
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 sorting.cr yp.to includes fast constant-time \(N\)-input sorting built from min/max (“sorting networks”) for int32; automated verif with angr + DAG analysis. Classic McEliece also uses int16, int64.
Verified formulas for control bits

Can permute 8192 items in constant time via sorting. Simpler, faster: “Control bits” specify

- swap 0 with 1? swap 2 with 3? etc.;
- swap 0 with 2? swap 1 with 3? etc.;
- swap 0 with 4? swap 1 with 5? etc.;
- and so on: 1, 2, 4, 8, . . . , 8, 4, 2, 1.

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cr.yp.to/papers.html#controlbits presents a proof of fast formulas mapping any given permutation to control bits.
Proof is computer-verified using HOL Light.
Verified formulas for decoding

$mceliece8192128$ secrets: deg-128 irred poly $g \in \mathbb{F}_{8192}[x]$; distinct $s_0, \ldots, s_{8191} \in \mathbb{F}_{8192}$.
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“Goppa codeword”: bits \( c_0, \ldots, c_{8191} \) with

\[ \sum_i c_i s_i^d / g(s_i) = 0 \quad \text{for each} \quad d \in \{0, 1, \ldots, 127\} . \]
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[cr.yp.to/papers.html#goppadecoding]: minicourse on decoding formulas used in the Classic McEliece software. New: Proofs are computer-verified in HOL Light and Lean.
The end is in sight

What I’m working on: More code-analysis tools, automatically matching up stages in the Classic McEliece keygen/enc/dec specification to segments of machine code.

HOL Light already includes a model of basic machine instructions; angr already includes a model of instructions through AVX2.

Binary-field mult is challenging to optimize, but the optimized code is easy to verify: simply trace bilinear operations on bits.