Hash-based signatures

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(including slides from Tanja Lange)

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Python snippets for this talk:
`cr.yp.to/talks/2023.02.01/hash-20230201.tar.gz`

Hash functions

The SHA-256 cryptographic hash function

```
$ echo hello
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$
```

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```
$ echo hello
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$ echo hello | sha256sum
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$ echo this is a longer message | sha256sum
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$
```

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```

The `sha256sum` program computes the SHA-256 hash function.
This is a function $H : \{0,1\}^* \to \{0,1\}^{256}$. Each output is 32 bytes.

Exercise: Hash-function input lengths

1. SHA-256 actually requires input to be at most $2^{64} - 1$ bits. Figure out # years for today's fastest CPU to reach this limit.

2. Reading exercise: Is there an input-size limit for SHA-3?

3. Exploitable buffer overflow was announced 2022.10 in some SHA-3 software. Reading exercise: How did this happen?

4. How would you have avoided the buffer overflow?

The SHA-256 cryptographic hash function in Python 3

```
>>> import hashlib
>>> def sha256(x):
...   h = hashlib.sha256()
...   h.update(x)
...   return h.digest()
...
>>> print(sha256(b’hello’).hex())
2cf24dba5fb0a30e26e83b2ac5b9e29e1b161e5c1fa7425e73043362938b9824
>>>
```

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>>> print(sha256(b’hello\n’).hex())
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>>> print(sha256(b’hello\n’).hex())
5891b5b522d5df086d0ff0b110fbd9d21bb4fc7163af34d08286a2e846f6be03
>>> print(sha256(b’hello\n’*1000000).hex())
1a2cce61984891495b00826ef591104a34ff35766bbbcaaff965f766154812ab
>>>
```

Goals of cryptographic hash functions

What do we want from a hash function $H : \{0,1\}^* \to \{0,1\}^n$?

For any string x, think of $H(x)$ as an n-bit fingerprint of x.

Goals:

- $H(x)$ looks totally random;
- nobody can find two different strings x, x' with $H(x) = H(x')$;
- any tiny change from x to x' makes a totally new $H(x')$;
- nobody can compute $H(x)$ without knowing all of x;
- nobody can compute a secret x given only $H(x)$;
- ...

Warning: Some hash goals are difficult to mathematically define.

Generic hardness of preimage resistance

Goal: Given $y \in H(\{0,1\}^*)$,
finding $x \in \{0,1\}^*$ with $H(x) = y$ is hard.

Here y is given, and is known to be the image of some $x \in \{0,1\}^*$.
Typically there are many such x,
but it should be hard to find any.

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Typically there are many such x,
but it should be hard to find any.

Generic attack: Try $\approx 2^n$ random choices of x.
If the output of H is distributed uniformly then
each x has a $1/2^n$ chance of $H(x) = y$.

e.g. $\approx 2^{128}$ tries if $n = 128$: very expensive.

Exercise: multi-target attacks

Given $y_1, y_2, \ldots, y_{2^{20}}$,
how long does it take to find $x_1, x_2, \ldots, x_{2^{20}}$
such that $H(x_1) = y_1$ and $H(x_2) = y_2$ and $\ldots$ and $H(x_{2^{20}}) = y_{2^{20}}$?

Generic hardness of second-preimage resistance

Goal: Given $x \in \{0,1\}^*$, finding $x' \in \{0,1\}^*$
with $x \neq x'$ and $H(x') = H(x)$ is hard.

Here x is given, determining $y = H(x)$.
Typically there are many other $x' \neq x$ with the same image,
but it should be computationally hard to find any.

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Typically there are many other $x' \neq x$ with the same image,
but it should be computationally hard to find any.

Generic attack: Try $\approx 2^n$ random choices of $x' \neq x$.
Same speed as for first preimages.

Generic hardness of collision resistance

Goal: Finding $x, x' \in \{0,1\}^*$
with $x \neq x'$ and $H(x') = H(x)$ is hard.

Attacker has full flexibility to choose any output y.
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Generic hardness of collision resistance

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Attacker has full flexibility to choose any output y.
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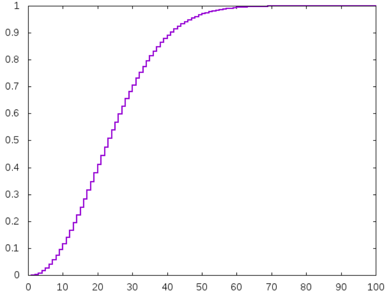
to find two different strings x, x' with the same output.

Generic attack: Try $\approx 2^{n/2}$ random choices of x. This number is much lower than 2^n because there is no restriction on the target.

The "birthday paradox": if one draws
$\approx 1.17\sqrt{m}$ elements at random
from a set of m elements,
then with $\approx 50\%$ probability
one has picked one element twice.



Weaknesses in common cryptographic hash functions

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MD4-specific collision attack (1995) in seconds.
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MD5 (1992 Rivest): $n = 128$, so 2^{64} generic collision attack.
MD5-specific collision attack (2004) in one hour on a cluster.
Current best collision attack (2013) costs 2^{18} H calls.
Chosen-prefix collisions (2008) showed real-world exploitability.

Flame malware (2012) used MD5 collision
to sign fake Windows update.

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SHA-1 (1995 NSA): $n = 160$, so 2^{80} generic collision attack.
Collisions published (2017): `https://shattered.io/`.
Practical attack, chosen-prefix collision (2020):
`https://sha-mbles.github.io/`

The NSA view of cryptographic standardization

"Narrowing the encryption problem to a single, influential algorithm might drive out competitors, and that would reduce the field that NSA had to be concerned about. Could a public encryption standard be made secure enough to protect against everything but a massive brute force attack, but **weak enough to still permit an attack of some nature** using very sophisticated (and expensive) techniques?" (Emphasis added.)

This quote is from an internal NSA history book.

Some unbroken hash functions

SHA-256 (NSA): $n = 256$, so 2^{128} generic collision attack.

SHA-512 (NSA): $n = 512$.

"SHA-2" refers to SHA-256, SHA-512, etc.

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SHA3-512 (Bertoni–Daemen–Peeters–van Assche): $n = 512$.

Another popular SHA-3 finalist, faster than SHA-3 in software: BLAKE. Successors: BLAKE2, BLAKE3.

One-time signatures

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Use a hash function to build a **public-key signature system**.

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Everyone learns signer's public key.

Using secret key, signer can sign any message m,
producing a signed message (m, s).

Everyone can verify (m, s) using signer's public key.

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but that passes verification using signer's public key.

Attacker looks at public key and at signed messages.
Tries modifying the signed messages or creating new messages.

A signature scheme for empty messages: key generation

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```
import os,hashlib

def sha3_256(x):
  h = hashlib.sha3_256()
  h.update(x)
  return h.digest()

def keypair():
  secret = sha3_256(os.urandom(32))
  public = sha3_256(secret)
  return public,secret
```

A signature scheme for <u>empty</u> messages: key generation

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import os,hashlib

def sha3_256(x):
  h = hashlib.sha3_256()
  h.update(x)
  return h.digest()

def keypair():
  secret = sha3_256(os.urandom(32))
  public = sha3_256(secret)
  return public,secret
```

```
>>> import signempty
>>> pk,sk = signempty.keypair()
>>> pk.hex()
'61ba682f03259a276dc2d790ed4863113d5559ad7cdd3c282083b9aa6b170ff
>>> sk.hex()
'4645dd39db47dd18b646a34b8f2dc6afd7fa62cc6faafc2ad3426dc94394335
```

Signing and verifying empty messages

```
def sign(message,secret):
  if not isinstance(message,bytes):
    raise TypeError('message must be a byte string')
  if message != b'':
    raise ValueError('message must be empty')
  signedmessage = secret
  return signedmessage

def open(signedmessage,public):
  if len(signedmessage) != 32:
    raise ValueError('bad signature')
  if sha3_256(signedmessage) != public:
    raise ValueError('bad signature')
  message = b''
  return message
```

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def open(signedmessage,public):
  if len(signedmessage) != 32:
    raise ValueError('bad signature')
  if sha3_256(signedmessage) != public:
    raise ValueError('bad signature')
  message = b''
  return message
```

```
>>> sm = signempty.sign(b'',sk)
>>> signempty.open(sm,pk)
b''
```

A signature scheme for 1-bit messages: keygen, signing

```
import signempty

def keypair():
  p0,s0 = signempty.keypair()
  p1,s1 = signempty.keypair()
  return (p0,p1),(s0,s1)

def sign(message,secret):
  if not isinstance(message,bytes):
    raise TypeError('message must be a byte string')
  if message == b'0':
    return message,signempty.sign(b'',secret[0])
  if message == b'1':
    return message,signempty.sign(b'',secret[1])
  raise ValueError("message must be b'0' or b'1'")
```

A signature scheme for 1-bit messages: verification

```
def open(signedmessage,public):
  if not isinstance(signedmessage[0],bytes):
    raise TypeError('message must be a byte string')
  if signedmessage[0] == b'0':
    signempty.open(signedmessage[1],public[0])
    return b'0'
  if signedmessage[0] == b'1':
    signempty.open(signedmessage[1],public[1])
    return b'1'
  raise ValueError('bad signature')
```

A signature scheme for 1-bit messages: verification

```
def open(signedmessage,public):
  if not isinstance(signedmessage[0],bytes):
    raise TypeError('message must be a byte string')
  if signedmessage[0] == b'0':
    signempty.open(signedmessage[1],public[0])
    return b'0'
  if signedmessage[0] == b'1':
    signempty.open(signedmessage[1],public[1])
    return b'1'
  raise ValueError('bad signature')
```

```
>>> import signbit
>>> pk,sk = signbit.keypair()
>>> sm = signbit.sign(b'1',sk)
>>> signbit.open(sm,pk)
b'1'
```

A signature scheme for 4-bit messages: key generation

```
import signbit

def keypair():
  p0,s0 = signbit.keypair()
  p1,s1 = signbit.keypair()
  p2,s2 = signbit.keypair()
  p3,s3 = signbit.keypair()
  return (p0,p1,p2,p3),(s0,s1,s2,s3)

def sign(m,secret):
  if not isinstance(m,bytes):
    raise TypeError('message must be a byte string')
  if len(m) != 4:
    raise ValueError('message must have length 4')
  sm0 = signbit.sign(m[0:1],secret[0])
  sm1 = signbit.sign(m[1:2],secret[1])
  sm2 = signbit.sign(m[2:3],secret[2])
  sm3 = signbit.sign(m[3:4],secret[3])
  return sm0,sm1,sm2,sm3
```

A signature scheme for 4-bit messages: sign & verify

```
def open(sm,public):
  if len(sm) != 4:
    raise ValueError('signed message must have length 4')
  m0 = signbit.open(sm[0],public[0])
  m1 = signbit.open(sm[1],public[1])
  m2 = signbit.open(sm[2],public[2])
  m3 = signbit.open(sm[3],public[3])
  return m0+m1+m2+m3
```

Do not use one secret key to sign two messages!

```
>>> import sign4bits
>>> pk,sk = sign4bits.keypair()
>>> sm0111 = sign4bits.sign(b’0111’,sk)
>>> sign4bits.open(sm0111,pk)
b’0111’
>>> sm1101 = sign4bits.sign(b’1101’,sk)
>>> sign4bits.open(sm1101,pk)
b’1101’
```

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>>> pk,sk = sign4bits.keypair()
>>> sm0111 = sign4bits.sign(b’0111’,sk)
>>> sign4bits.open(sm0111,pk)
b’0111’
>>> sm1101 = sign4bits.sign(b’1101’,sk)
>>> sign4bits.open(sm1101,pk)
b’1101’
>>> forgery = sm1101[:2]+sm0111[2:]
>>> sign4bits.open(forgery,pk)
b’1111’
```

Lamport's 1-time signature system

Sign arbitrary-length message by signing its 256-bit hash:

```
def hashbits(message):
  h = sha3_256(message)
  return [(b'0',b'1')[1&(h[i//8]>>(i%8))] for i in range(256)]

def keypair():
  keys = [signbit.keypair() for n in range(256)]
  return zip(*keys)

def sign(message,secret):
  hbits = hashbits(message)
  sigs = [signbit.sign(hbits[i],secret[i]) for i in range(256)]
  return sigs,message

def open(sm,public):
  if len(sm[0]) != 256:
    raise ValueError('wrong signature length')
  message = sm[1]
  hbits = hashbits(message)
  for i in range(256):
    if hbits[i] != signbit.open(sm[0][i],public[i]):
      raise ValueError('bit %d of hash does not match'%i)
  return message
```

Can we build shorter signatures?

Each Lamport signature has 256 `signbit` signatures.
Each `signbit` signature has 1 `signempty` signature.
Each `signempty` signature has one hash output (32 bytes).
Total 256 hash outputs (8192 bytes).

For a 4-bit message: 4 hash outputs (128 bytes).

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For a 4-bit message: 4 hash outputs (128 bytes).

Idea for doing better, just 1 hash output for a 4-bit message:

- Define

$$H^i(x) = H(H^{i-1}(x)) = \underbrace{H(H(\cdots(H}_{i \text{ times}}(x)))).$$

- Pick random sk, compute pk $= H^{16}(\text{sk})$.
- For message $m \in \{0, 1, \ldots, 15\}$ reveal $s = H^m(\text{sk})$ as signature.
- To verify check that pk $= H^{16-m}(s)$.

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- To verify check that pk $= H^{16-m}(s)$.

This is the weak Winternitz signature system.

Weak Winternitz

```
def keypair():
  secret = sha3_256(os.urandom(32))
  public = secret
  for i in range(16): public = sha3_256(public)
  return public,secret

def sign(m,secret):
  if not isinstance(m,int) or m<0 or m>15:
    raise ValueError('message must be in {0,1,...,15}')
  s = secret
  for i in range(m): s = sha3_256(s)
  return s,m

def open(sm,public):
  if not isinstance(sm[1],int) or sm[1]<0 or sm[1]>15:
    raise ValueError('message must be in {0,1,...,15}')
  c = sm[0]
  for i in range(16-sm[1]): c = sha3_256(c)
  if c != public: raise ValueError('bad signature')
  return sm[1]
```

Why this is “weak” Winternitz

This is insecure even if you sign only 1 message!

```
>>> import weak_winternitz
>>> pk,sk = weak_winternitz.keypair()
>>> sm7 = weak_winternitz.sign(7,sk)
>>> H = weak_winternitz.sha3_256
>>> weak_winternitz.open(sm7,pk)
7
>>> forgery = H(sm7[0]),8
>>> weak_winternitz.open(forgery,pk)
8
>>> forgery2 = H(forgery[0]),9
>>> weak_winternitz.open(forgery2,pk)
9
>>>
```

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9
>>>
```

Fix: Strong Winternitz uses weak Winternitz twice,
running one chain forward, one chain backward.
(Exercise: this is safe with H^{15} instead of H^{16} in weak Winternitz.)

Strong Winternitz

```
import weak_winternitz

def keypair():
  keys = [weak_winternitz.keypair() for n in range(2)]
  return zip(*keys)

def sign(m,secret):
  if not isinstance(m,int) or m<0 or m>15:
    raise ValueError('message must be in {0,1,...,15}')
  sign0 = weak_winternitz.sign(m,secret[0])
  sign1 = weak_winternitz.sign(15-m,secret[1])
  return sign0[0],sign1[0],m

def open(sm,public):
  if not isinstance(sm[2],int) or sm[2]<0 or sm[2]>15:
    raise ValueError('message must be in {0,1,...,15}')
  weak_winternitz.open((sm[0],sm[2]),public[0])
  weak_winternitz.open((sm[1],15-sm[2]),public[1])
  return sm[2]
```

Full Winternitz, using base 2^8

Write 256-bit message (or 256-bit hash of actual message) in base 2^8 as $(m_0, m_1, \ldots, m_{31})$.

Put $c = \sum_{0 \le i < 32} (2^8 - m_i)$. Note that $c \le 2^{13}$.

Write c in base 2^8 as (c_0, c_1).

Sign with chains of lengths $m_0, m_1, \ldots, m_{31}, c_0, c_1$.

Signature has just 34 hash values. Lamport used 256 hash values.

Exercise: varying the Winternitz base

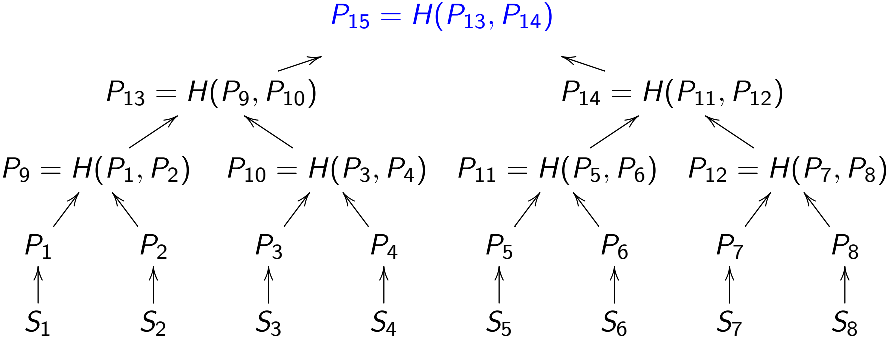
How does Winternitz work in base 2^5 for signing 256 bits?
How does this compare to base 2^8? Efficiency metrics:

- How many bytes are in the signature?
- How many bytes are in the public key?
- How many bytes are in the secret key?
- How many hash-function computations are used in signing?
- How many hash-function computations are used in verifying?

Many-time signatures

Merkle's (e.g.) 8-time signature system

Hash 8 one-time public keys into a single Merkle public key P_{15}.



$$P_{15} = H(P_{13}, P_{14})$$

$$P_{13} = H(P_9, P_{10}) \qquad P_{14} = H(P_{11}, P_{12})$$

$$P_9 = H(P_1, P_2) \quad P_{10} = H(P_3, P_4) \quad P_{11} = H(P_5, P_6) \quad P_{12} = H(P_7, P_8)$$

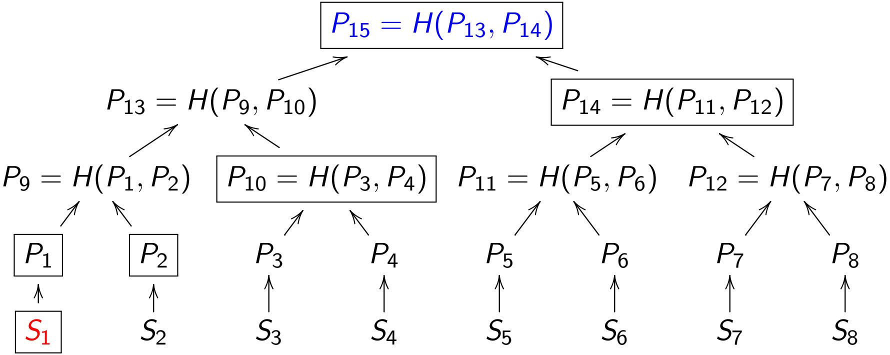
$$P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_5 \quad P_6 \quad P_7 \quad P_8$$

$$S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5 \quad S_6 \quad S_7 \quad S_8$$

$S_i \rightarrow P_i$ can be Lamport or Winternitz one-time signature system.
Each such pair (S_i, P_i) may be used only once.

Signature in 8-time Merkle hash tree

Signature of first message: $(\texttt{sign}(m, S_1), P_1, P_2, P_{10}, P_{14})$.



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$\boxed{P_1}$ $\boxed{P_2}$ P_3 P_4 P_5 P_6 P_7 P_8

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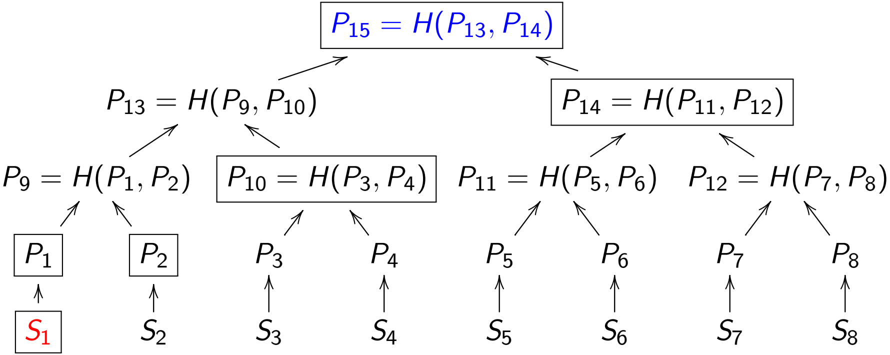

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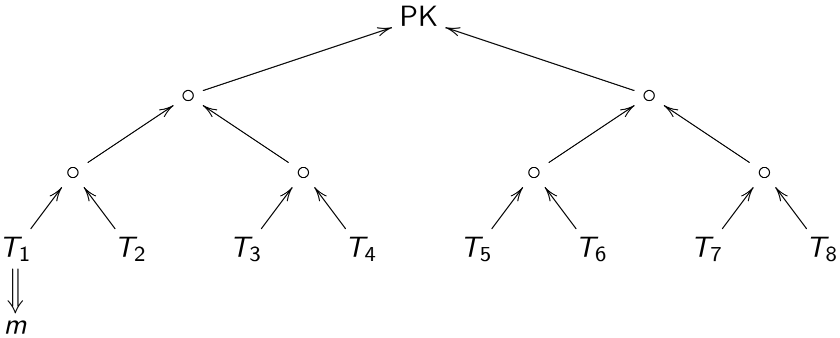
Verify signature $\texttt{sign}(m, S_1)$ with public key P_1.
Link P_1 against public key P_{15} by computing $P'_9 = H(P_1, P_2)$, $P'_{13} = H(P'_9, P_{10})$, and comparing $H(P'_{13}, P_{14})$ with P_{15}.
Reject if $H(P'_{13}, P_{14}) \neq P_{15}$ of if the signature verification failed.

Basic data flow



T_i are one-time signature keys.
$\uparrow$ indicates input to hash function.
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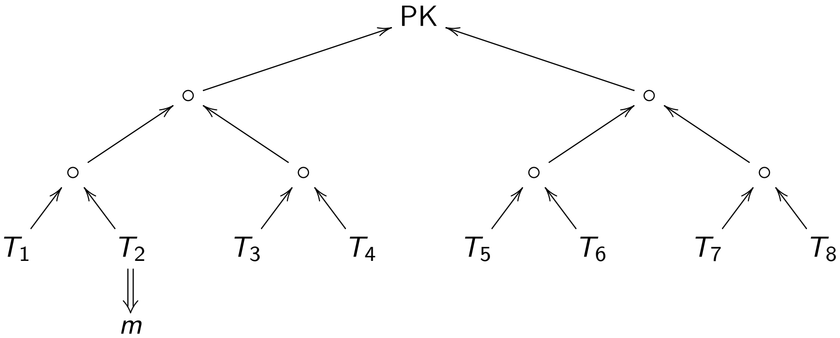

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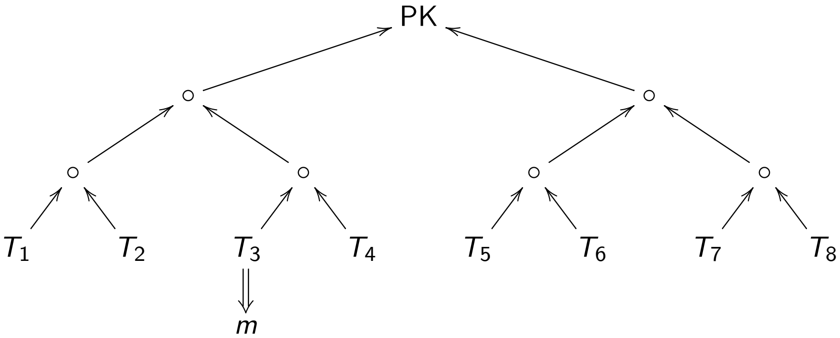

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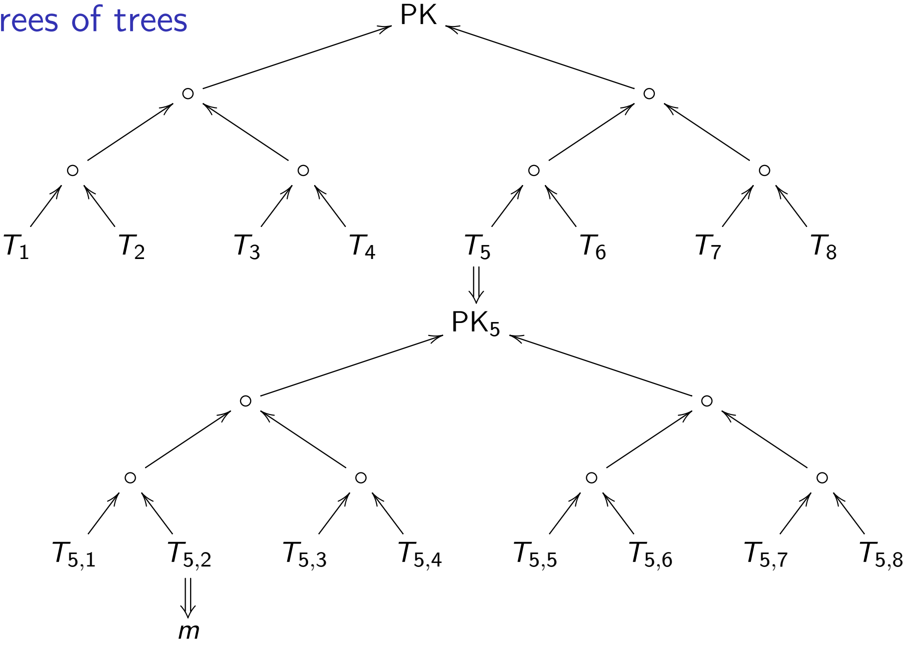
Trees of trees



Hash-based signatures today

Stateful vs. stateless

All of the signature systems so far in this talk
require keeping track of number of messages signed.

Adam Langley: "for most environments it's a huge foot-cannon."
Counting number of messages might not seem difficult, but
what happens if you *copy* the signature state $(\mathrm{sk}, \#\mathrm{messages})$?
Copying is normal: backups, virtual-machine cloning, etc.

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Basic ideas: huge trees (1987 Goldreich), keys on demand (Levin).

600 KB: Goldreich's signature using good 1-time signature scheme.

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41 KB: SPHINCS-256 signature (2014
Bernstein–Hopwood–Hülsing–Lange–Niederhagen–
Papachristodoulou–Schneider–Schwabe–Wilcox-O’Hearn).

More optimizations, more tradeoff options: SPHINCS+.

The hash perspective on post-quantum cryptography

The three major types of post-quantum public-key cryptography:

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  Modern versions with many optimizations:
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  Modern versions with many optimizations:
    - LMS: slightly more streamlined than XMSS.
    - XMSS: easier security review than LMS.
- Dangerous post-quantum public-key cryptosystems:
  use not just hash functions but also structured math problems.

Standardization

- CFRG has published RFCs for XMSS and LMS.
- NIST has copied the XMSS and LMS standards, and has announced that it will standardize SPHINCS+.
- ISO SC27 JTC1 WG2 is working on standard for stateful hash-based signatures.

More information: `https://sphincs.org`.

See also Tanja Lange's course page for more videos and slides for hash-based signatures and more PQC.