

Sorting integer arrays:
security, speed, and verification

D. J. Bernstein

University of Illinois at Chicago,
Ruhr-University Bochum

Bob's laptop screen:

From: Alice

Thank you for your
submission. We received
many interesting papers,
and unfortunately your

Bob assumes this message is
something Alice actually sent.

But today's "security" systems
fail to guarantee this property.
Attacker could have modified
or forged the message.

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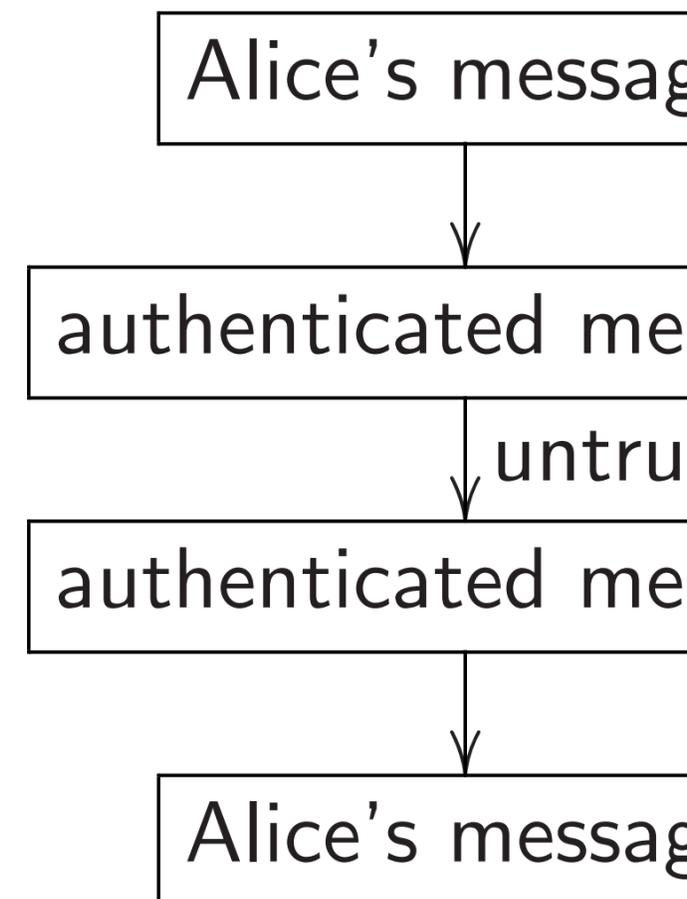
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Cryptographic solution

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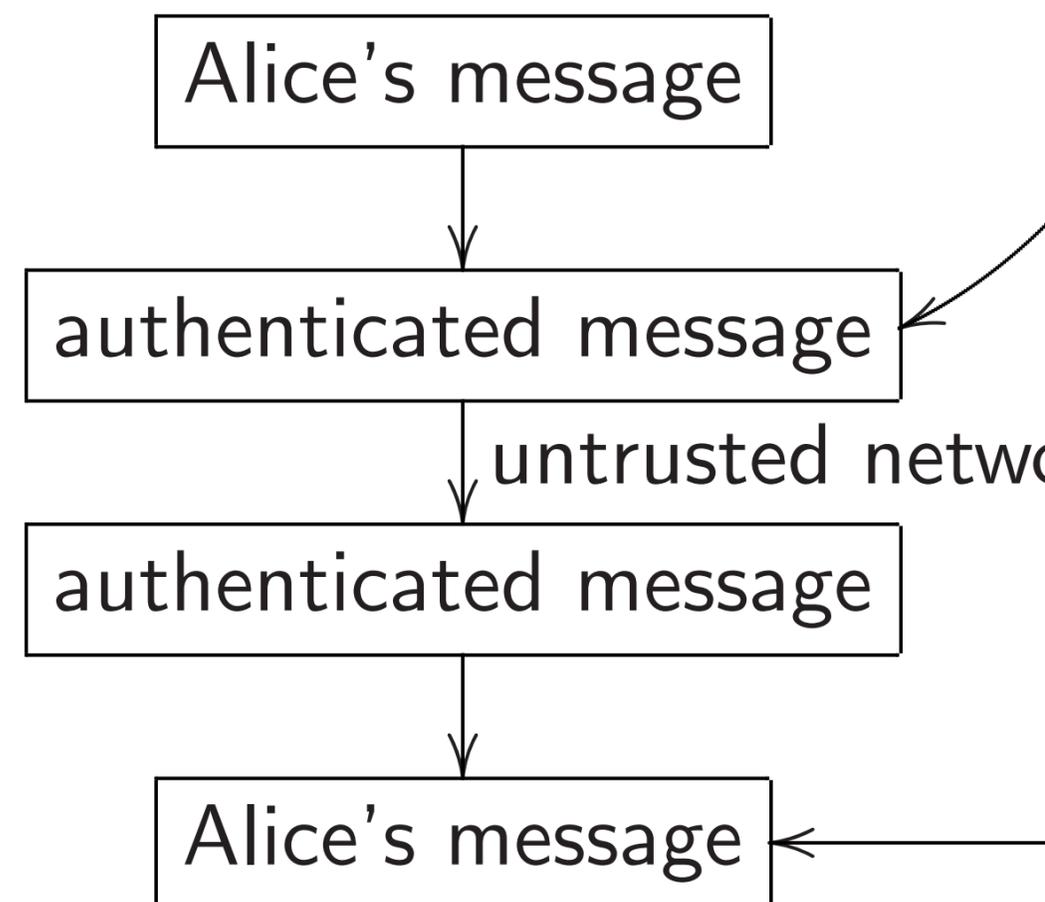
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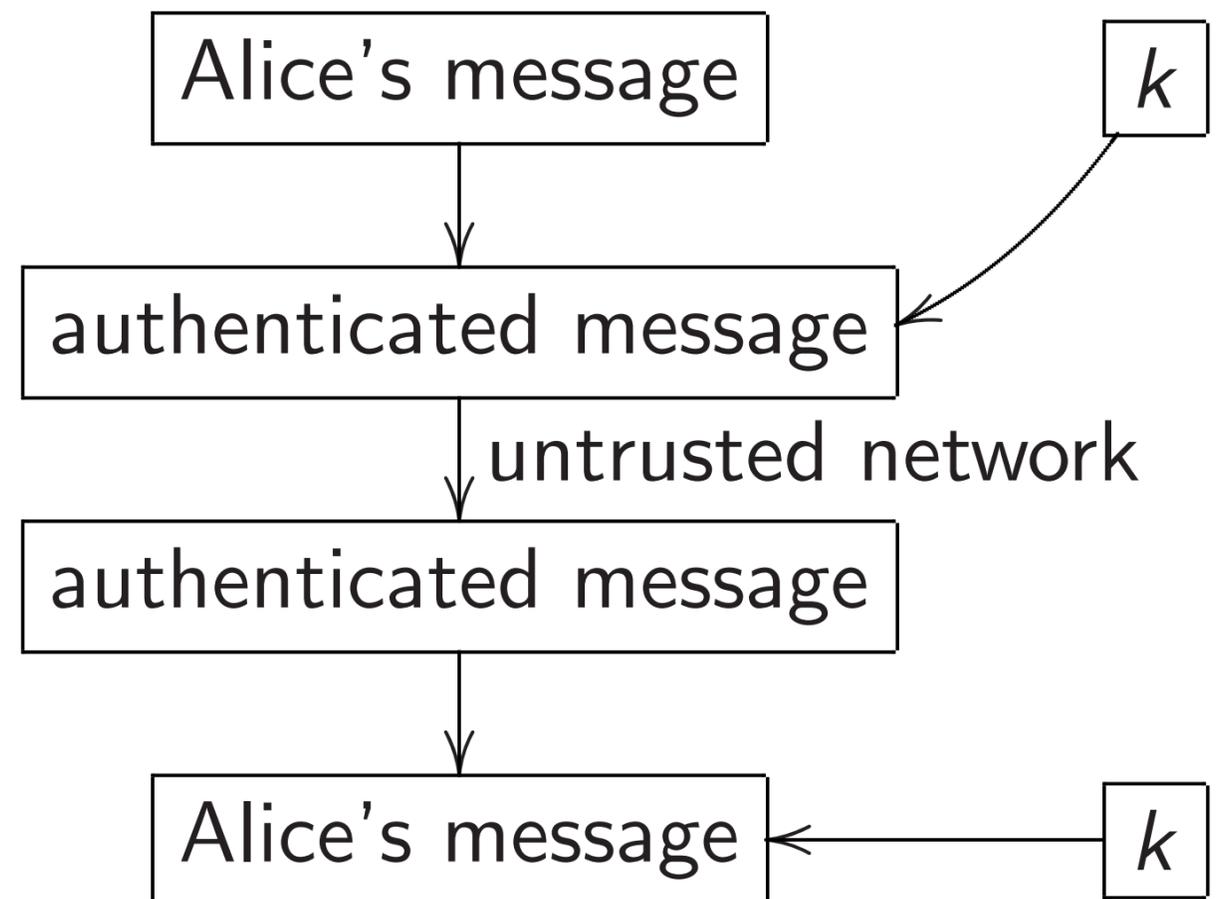
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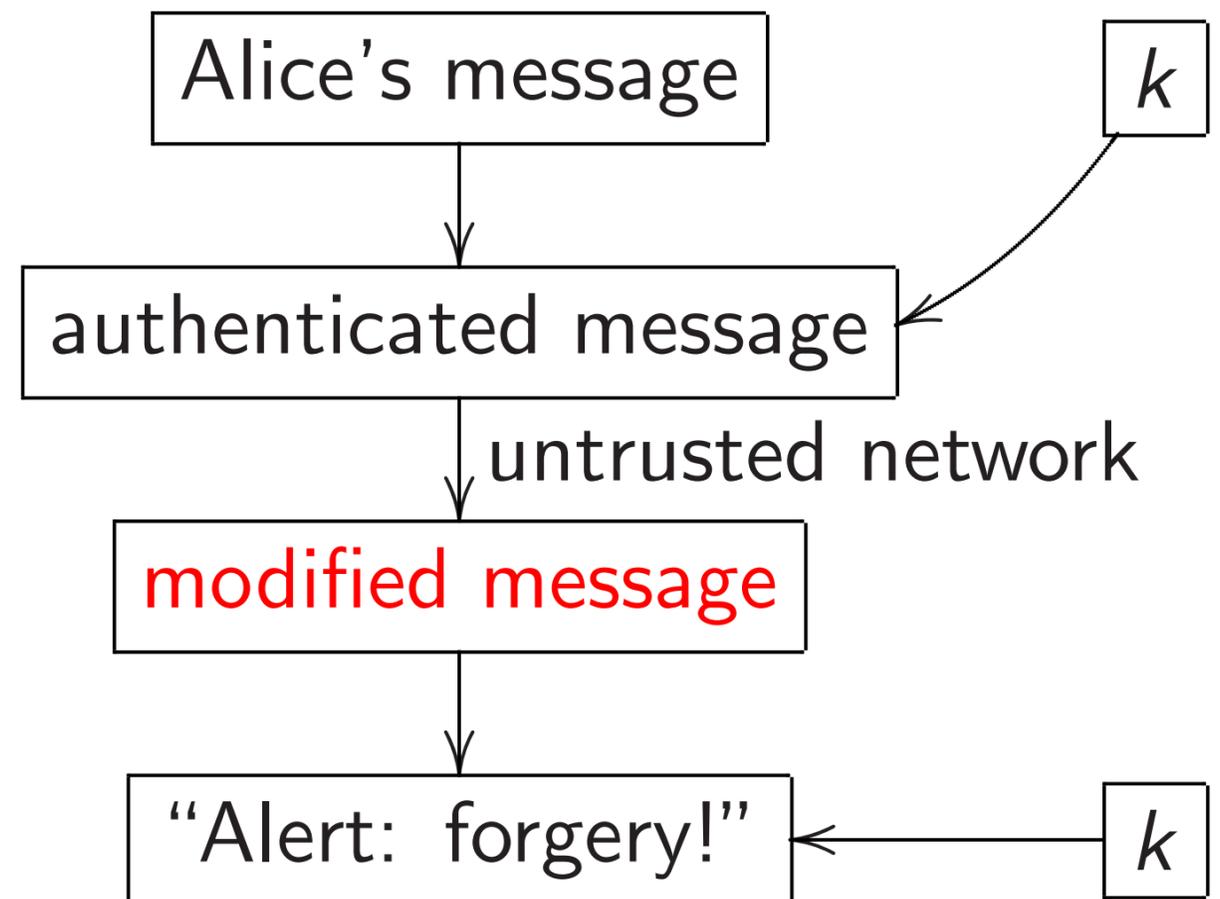
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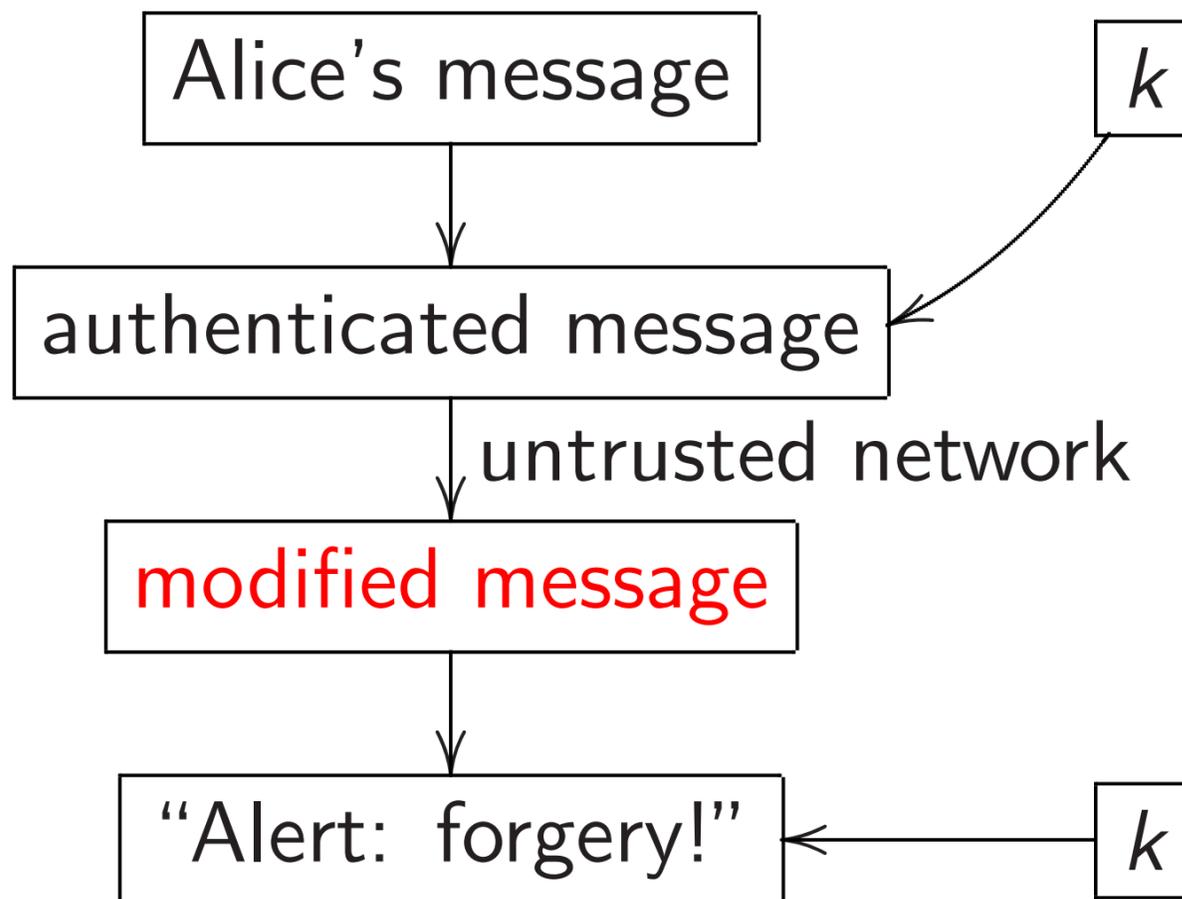
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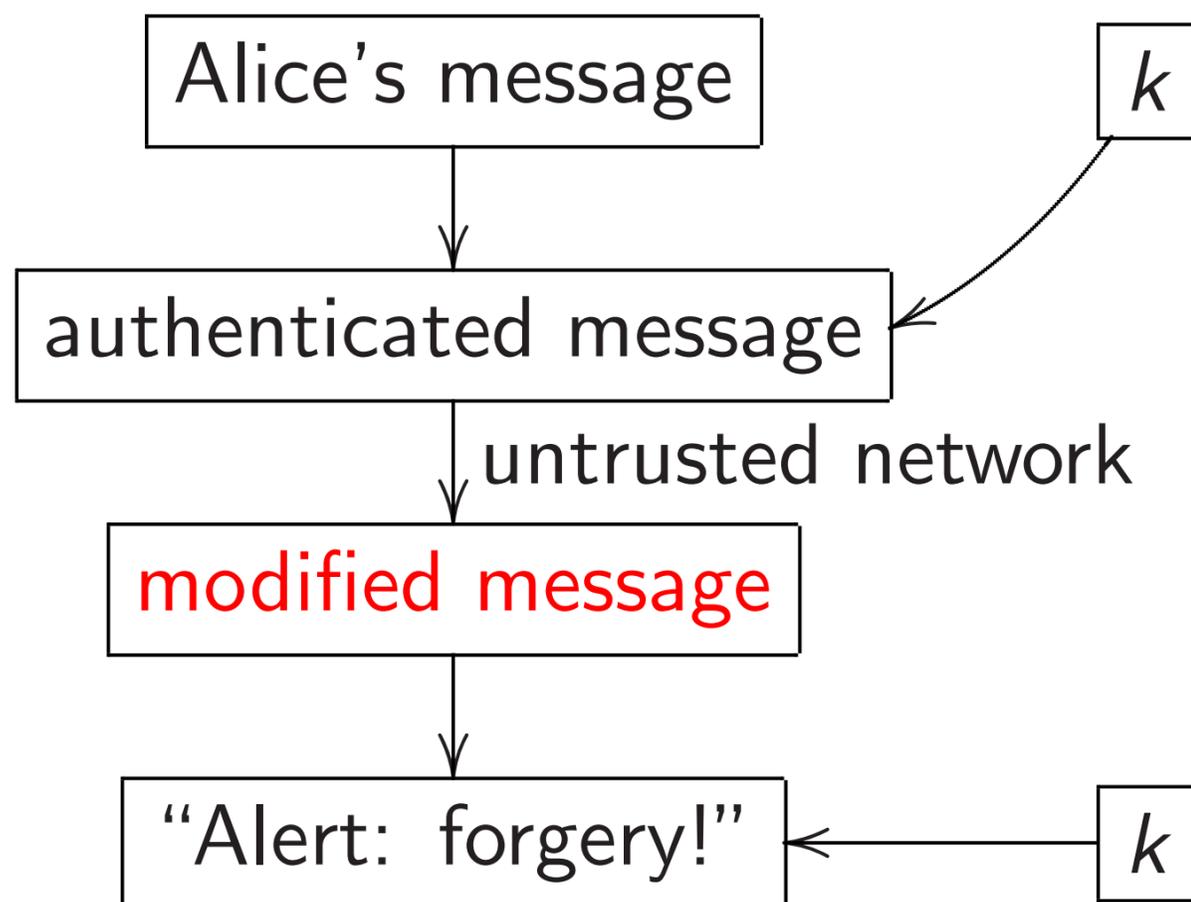
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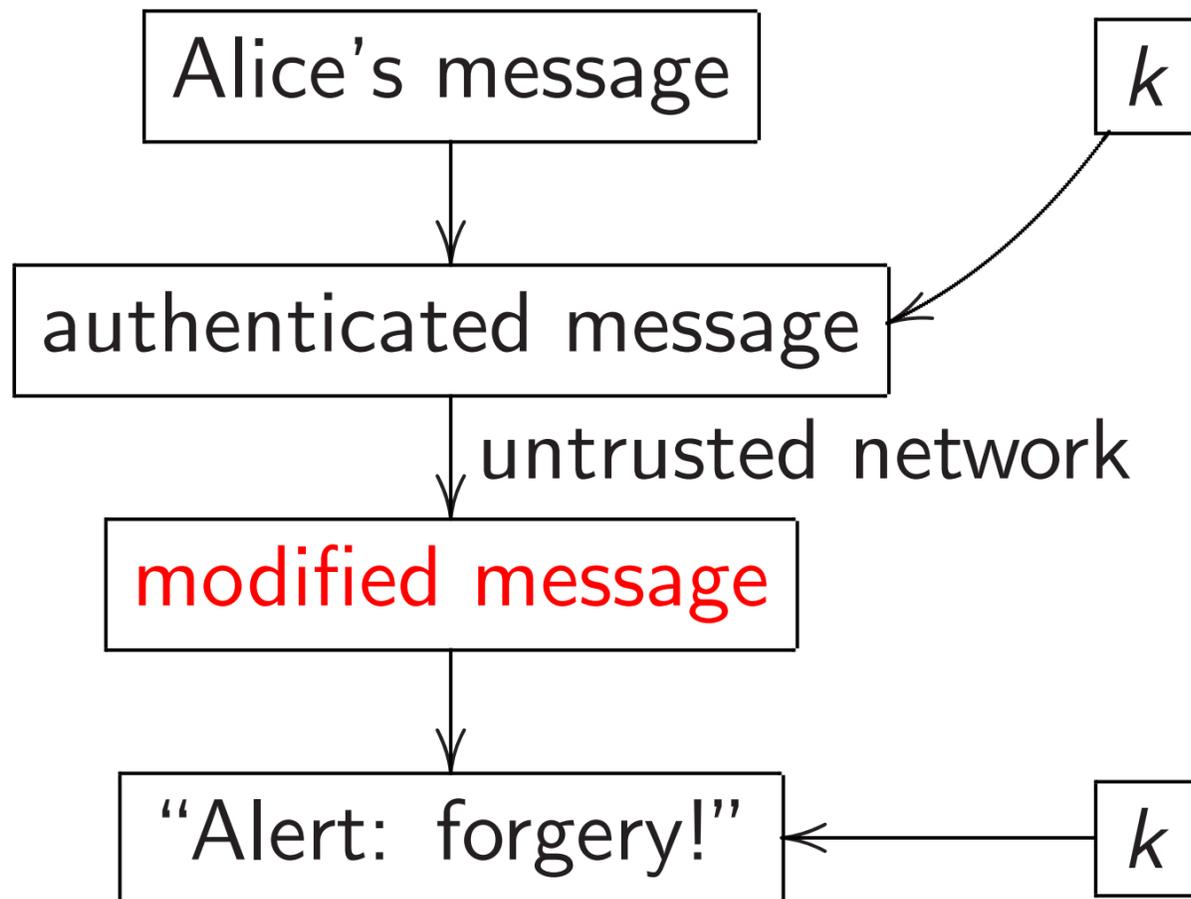
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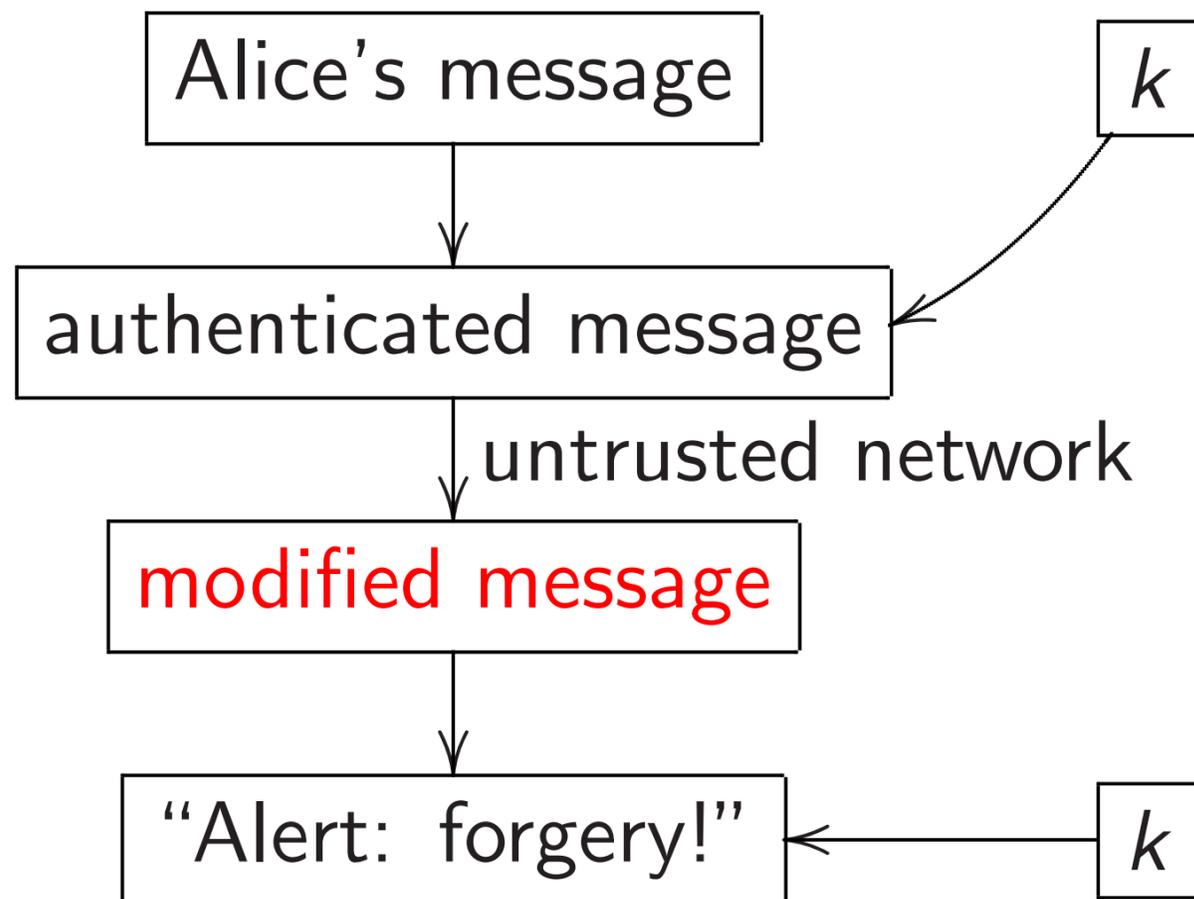
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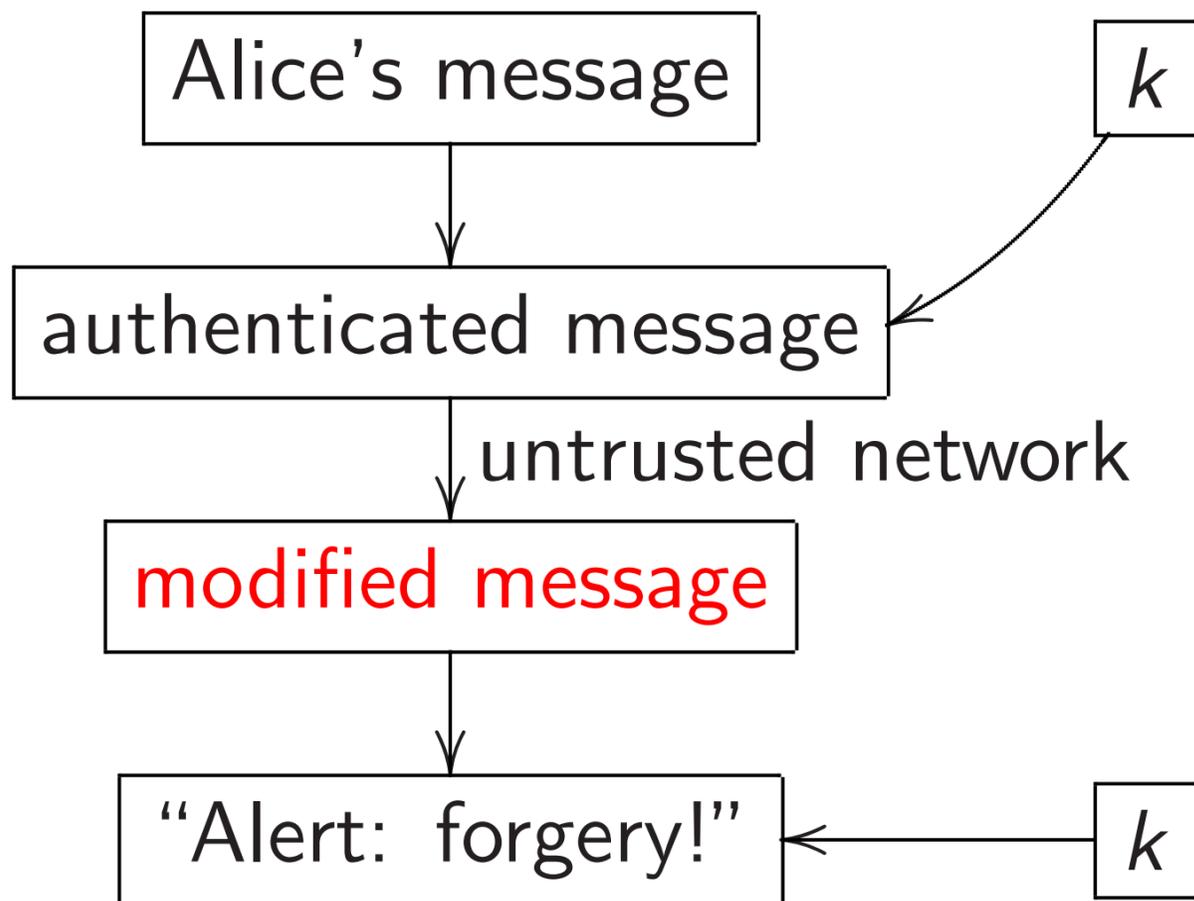
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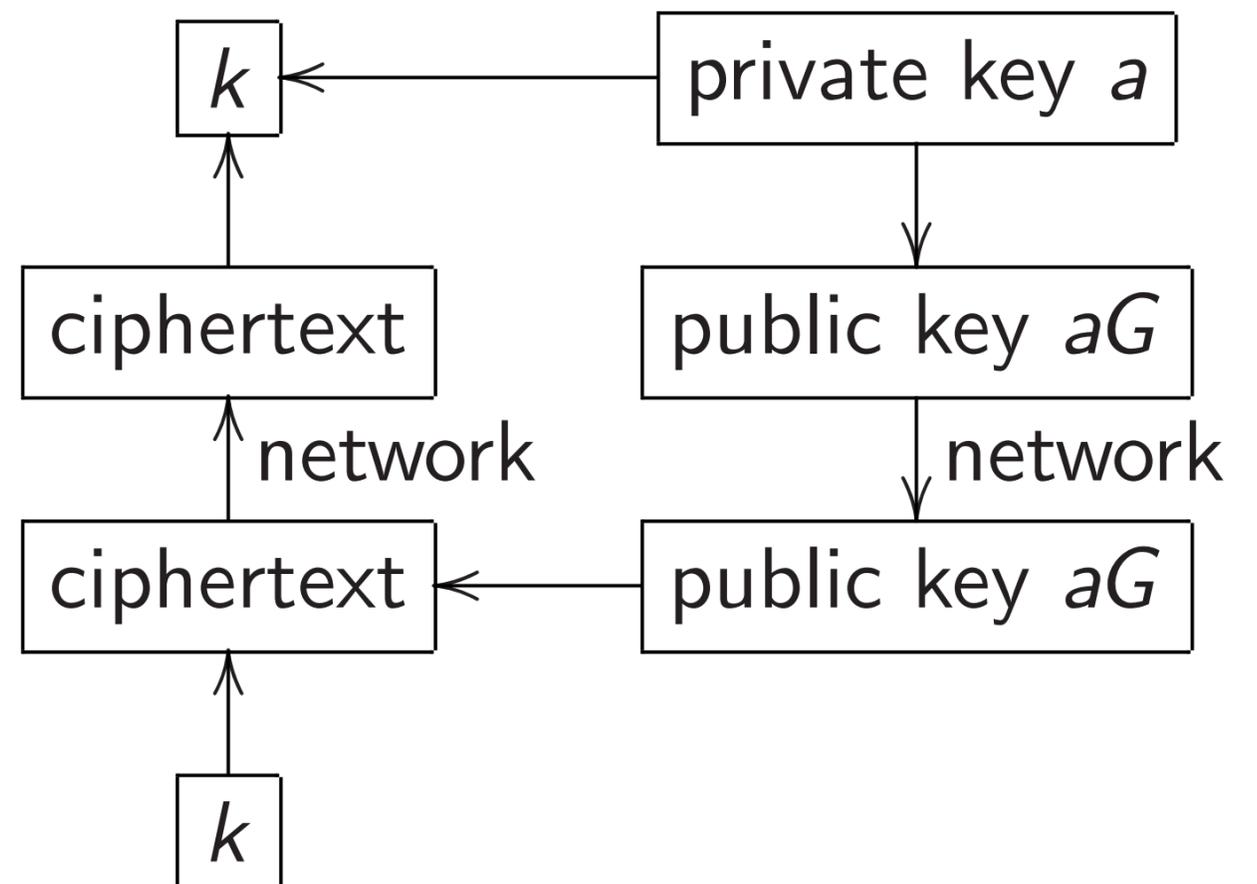


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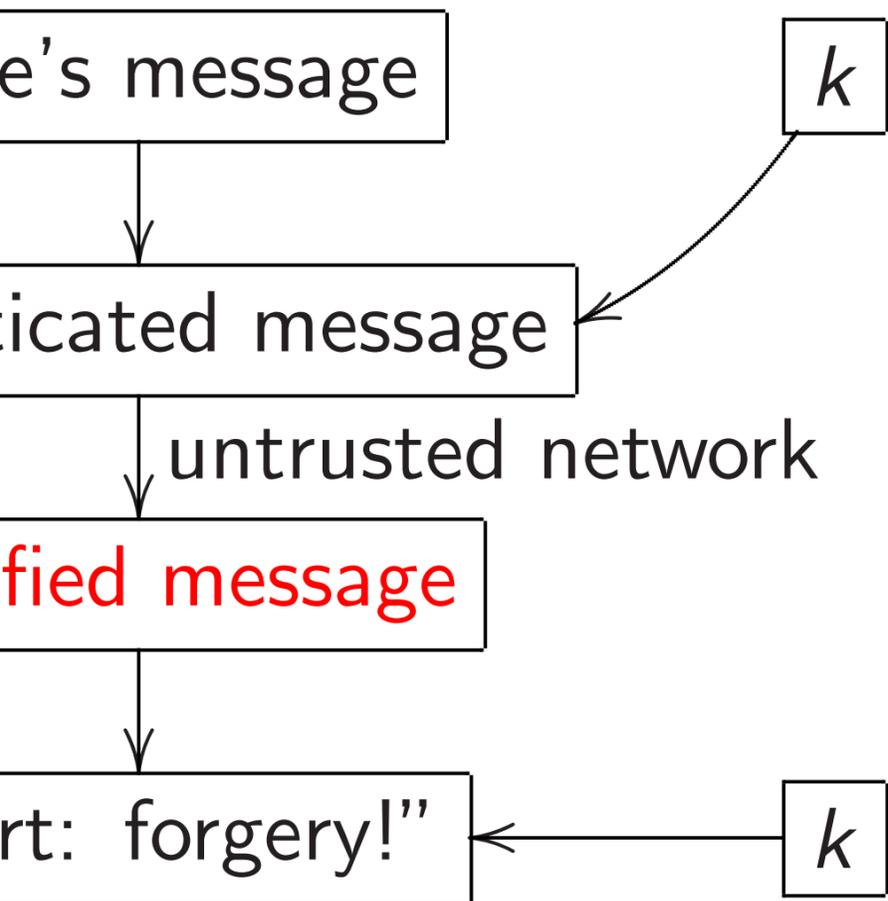
Public-key encryption.



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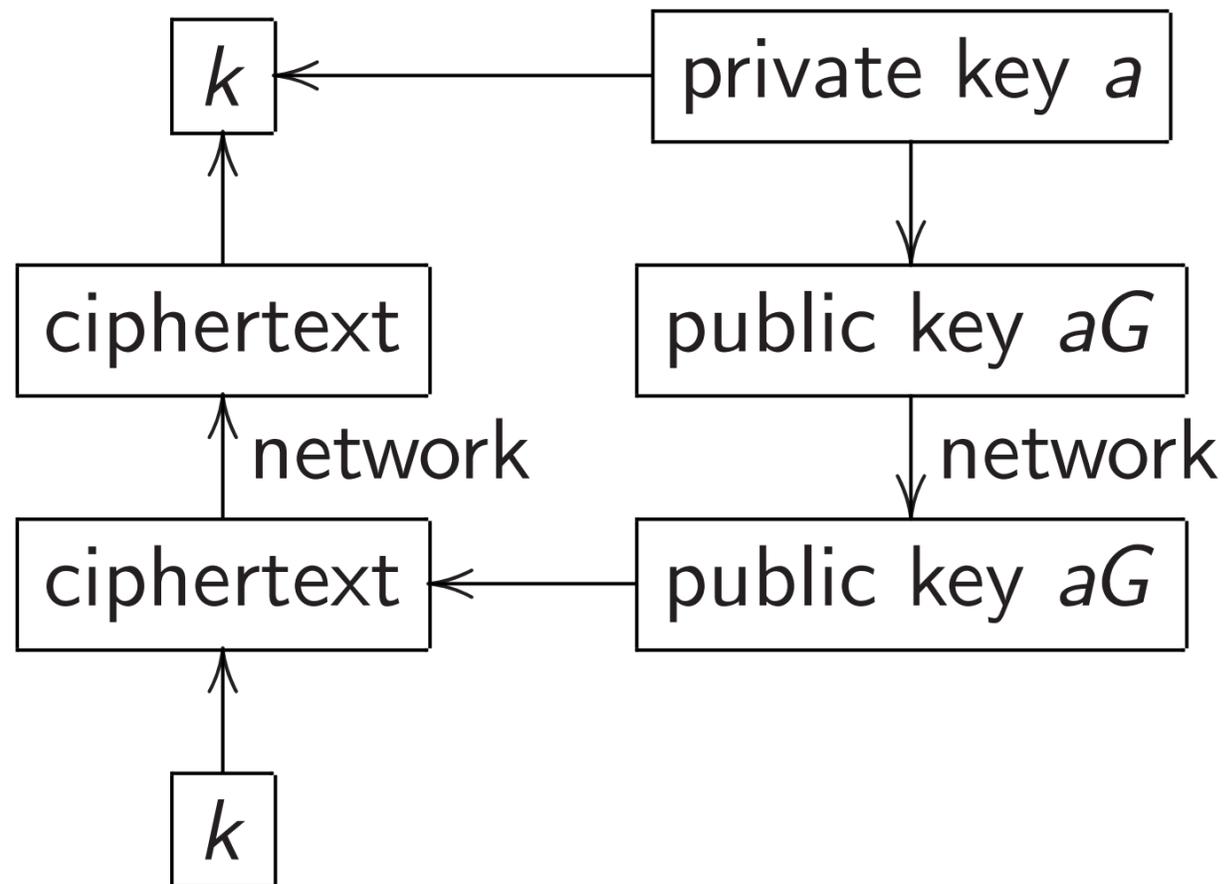
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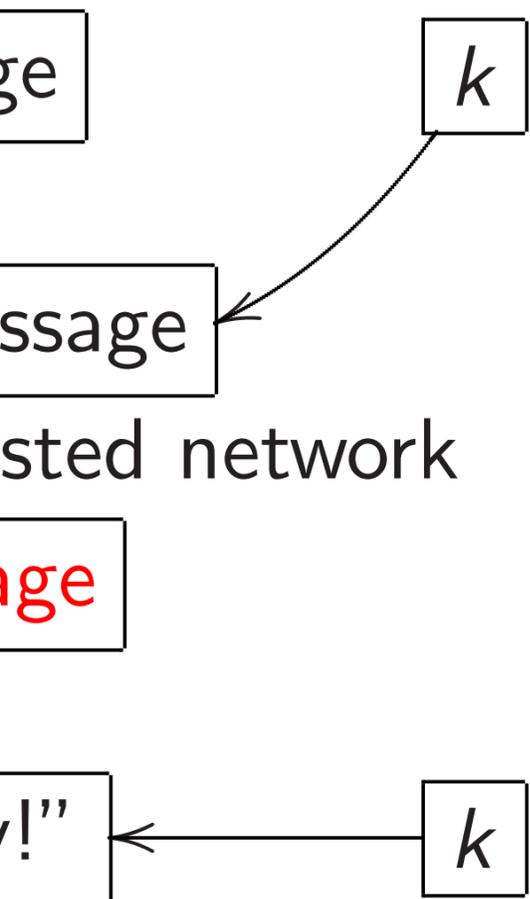
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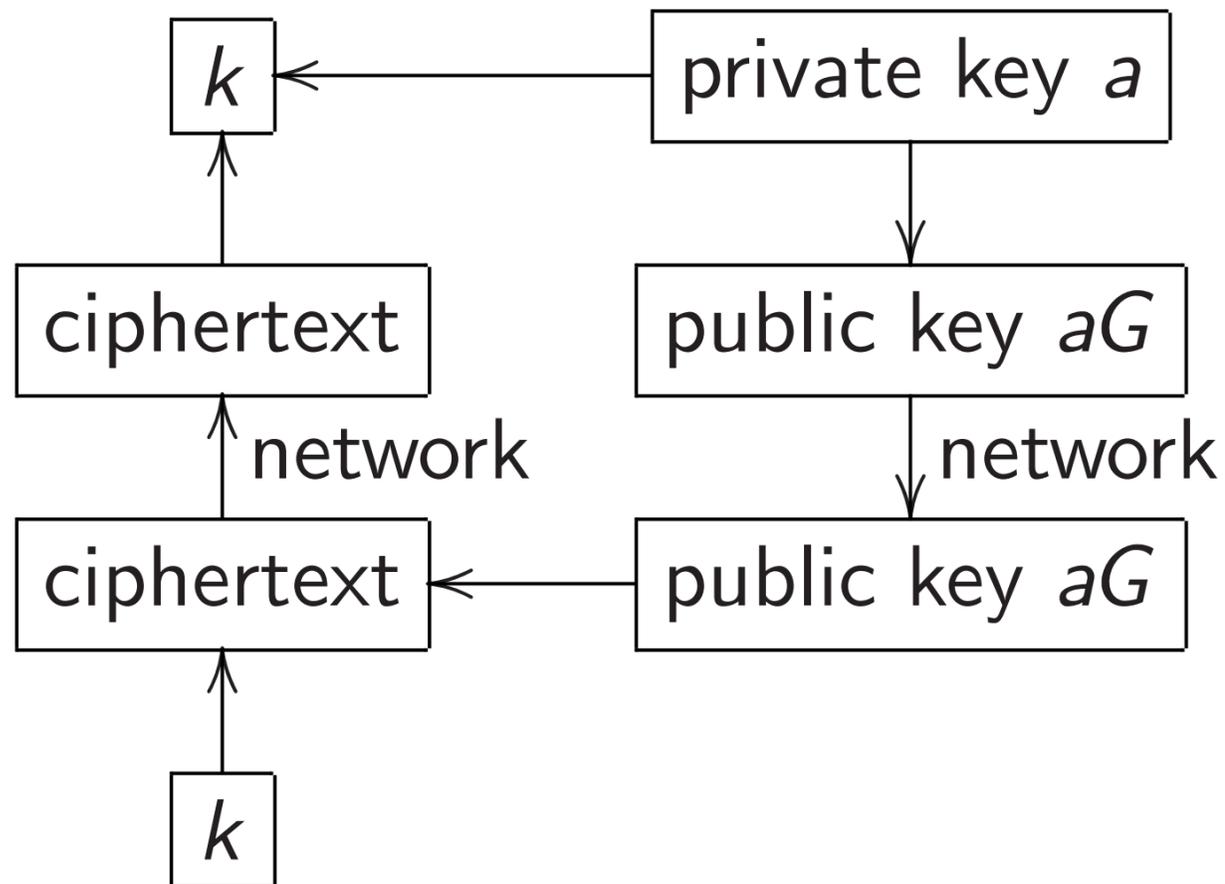
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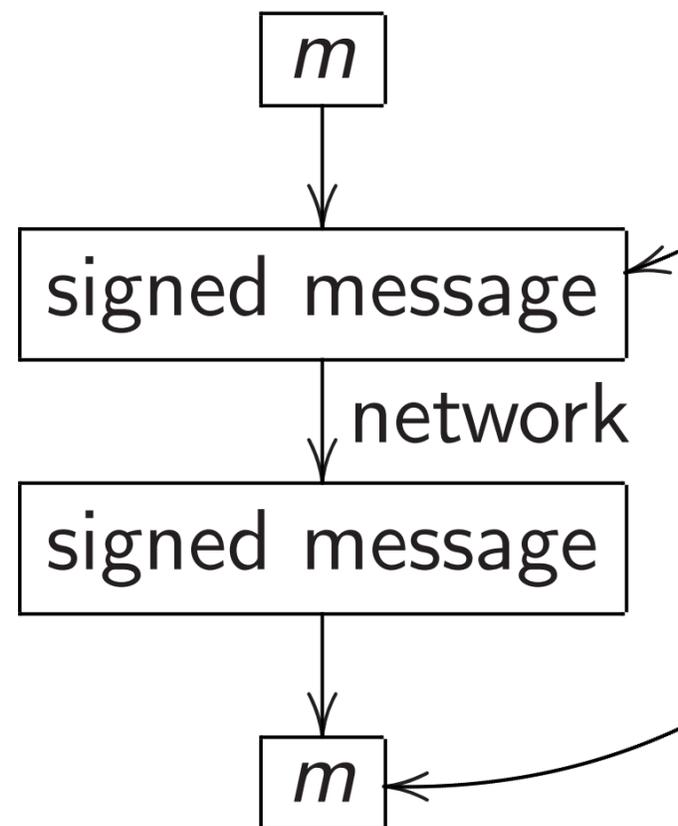
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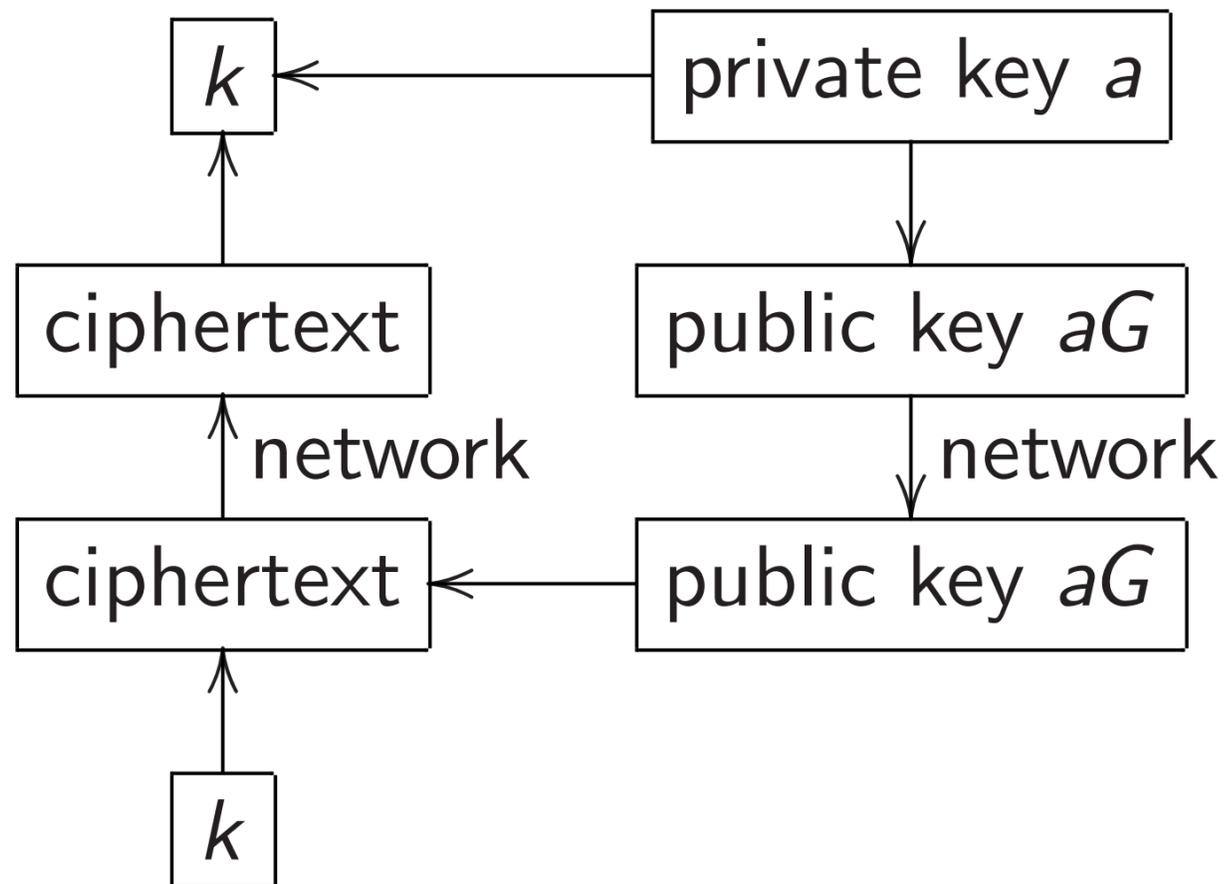
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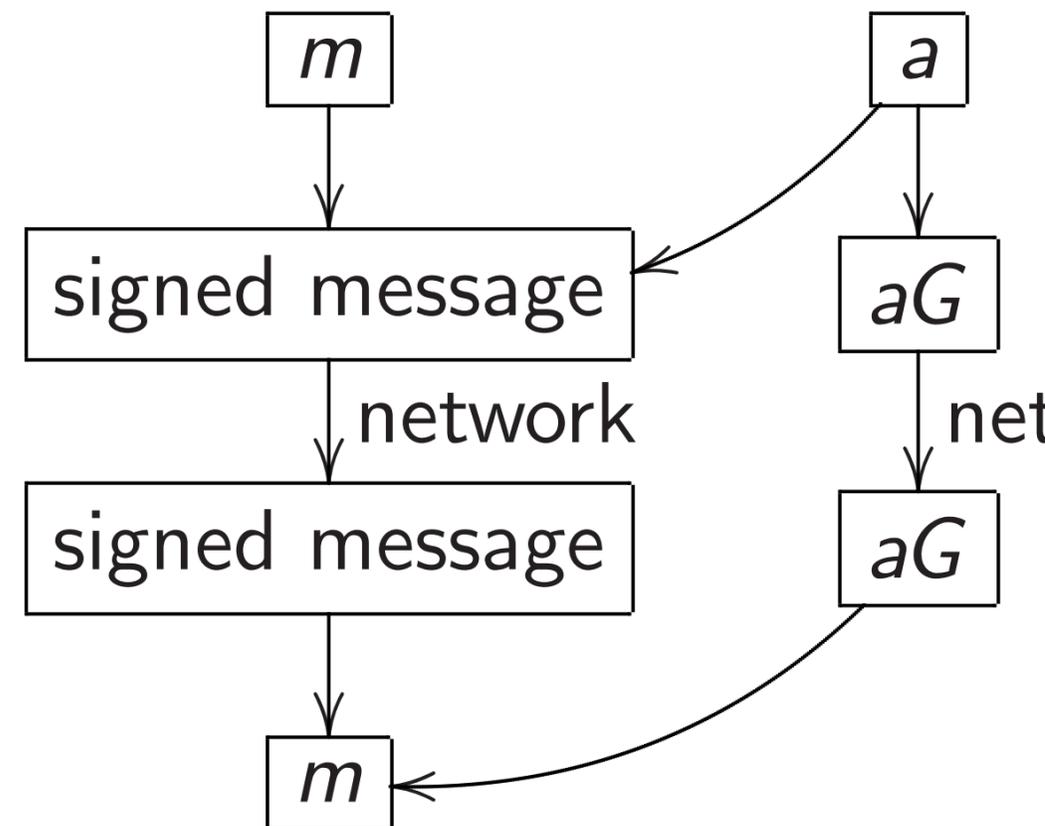
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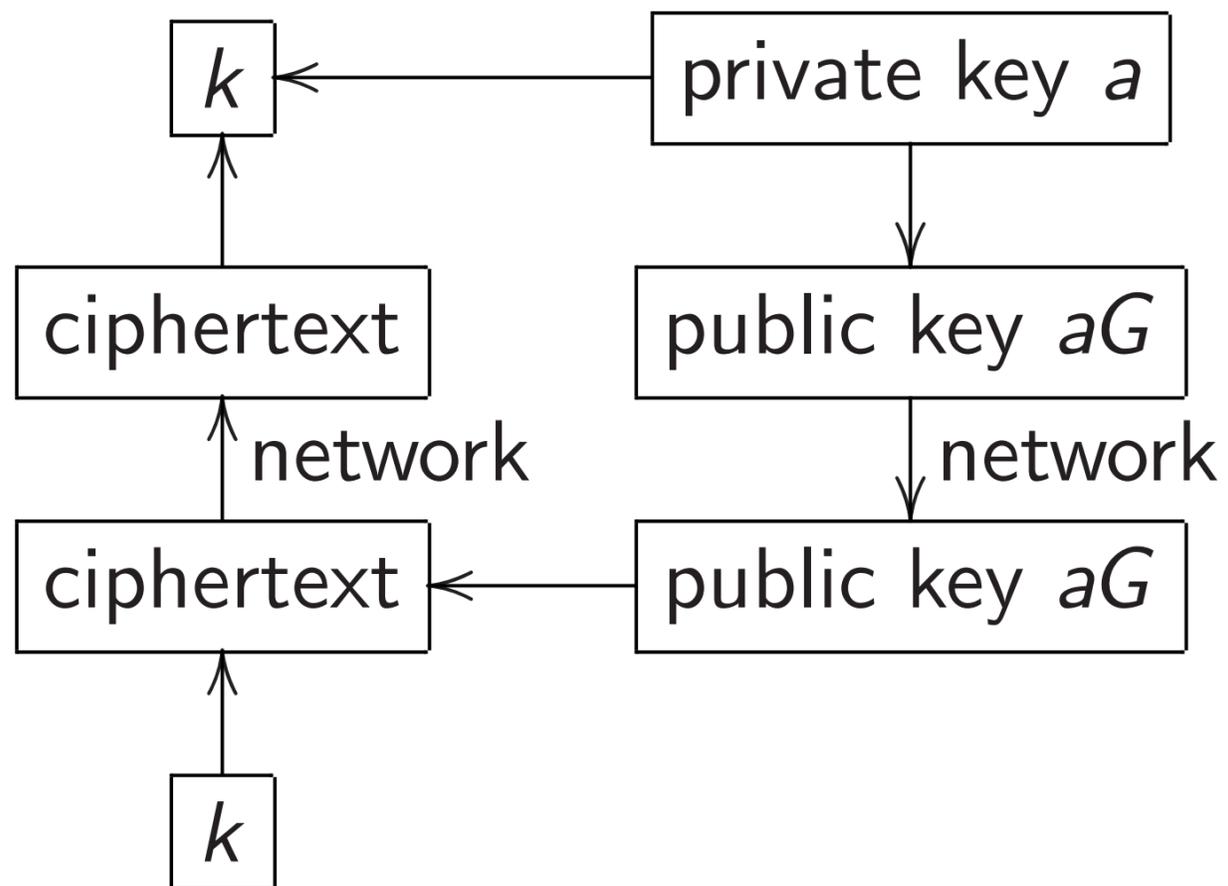


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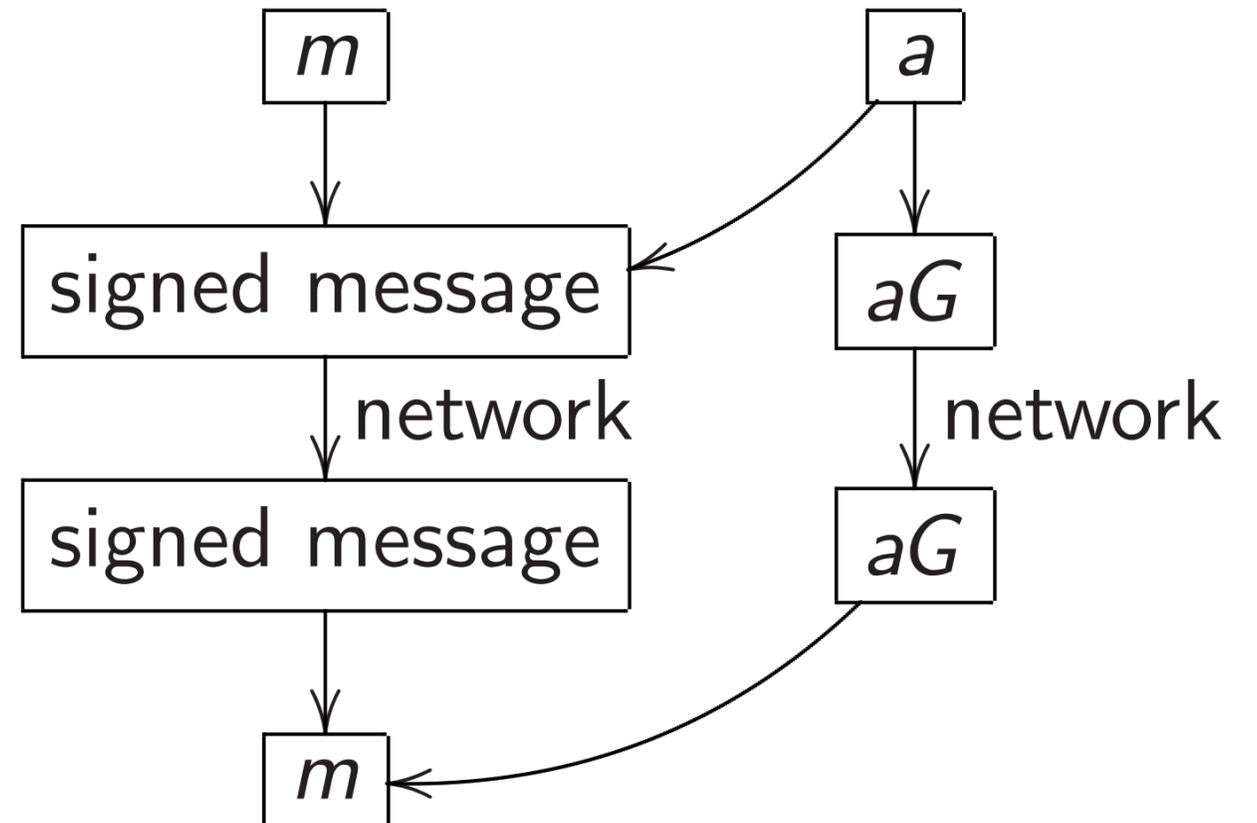
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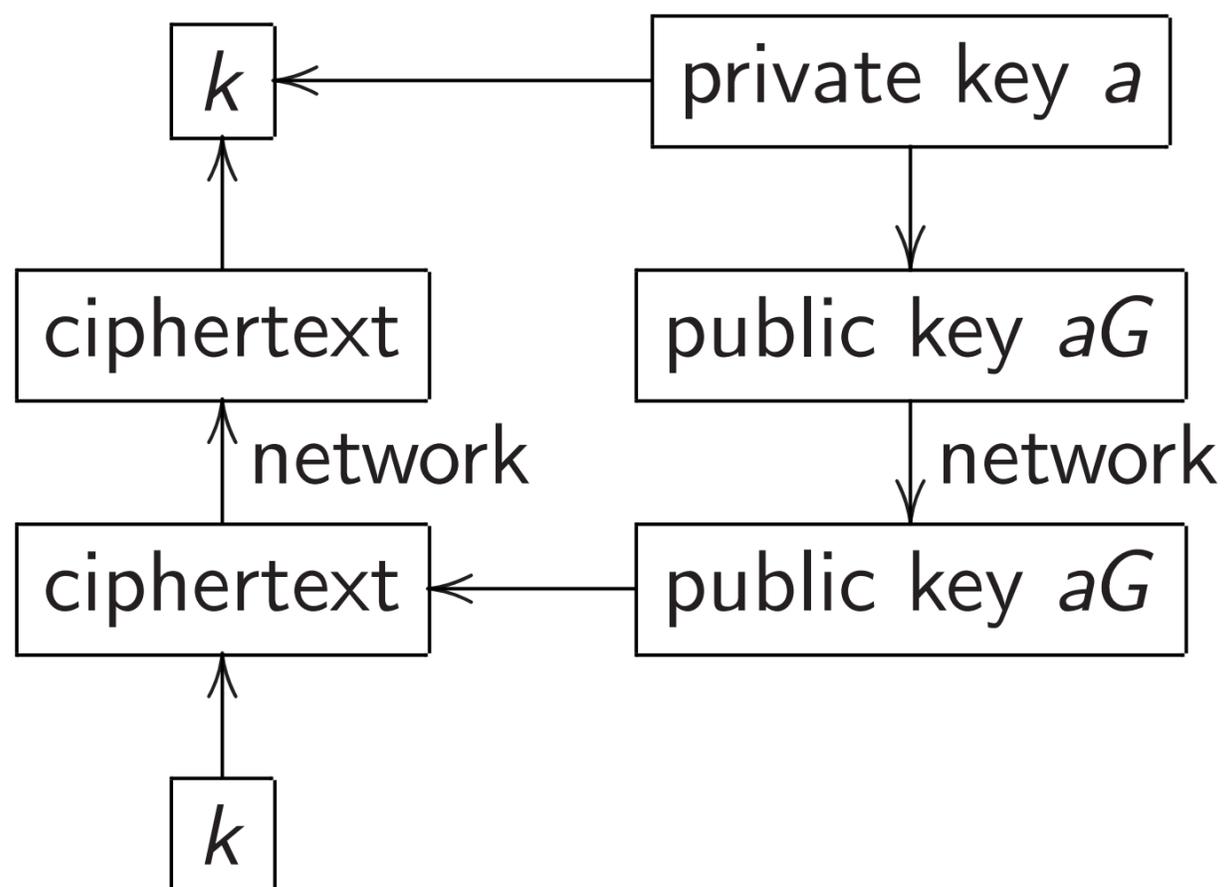


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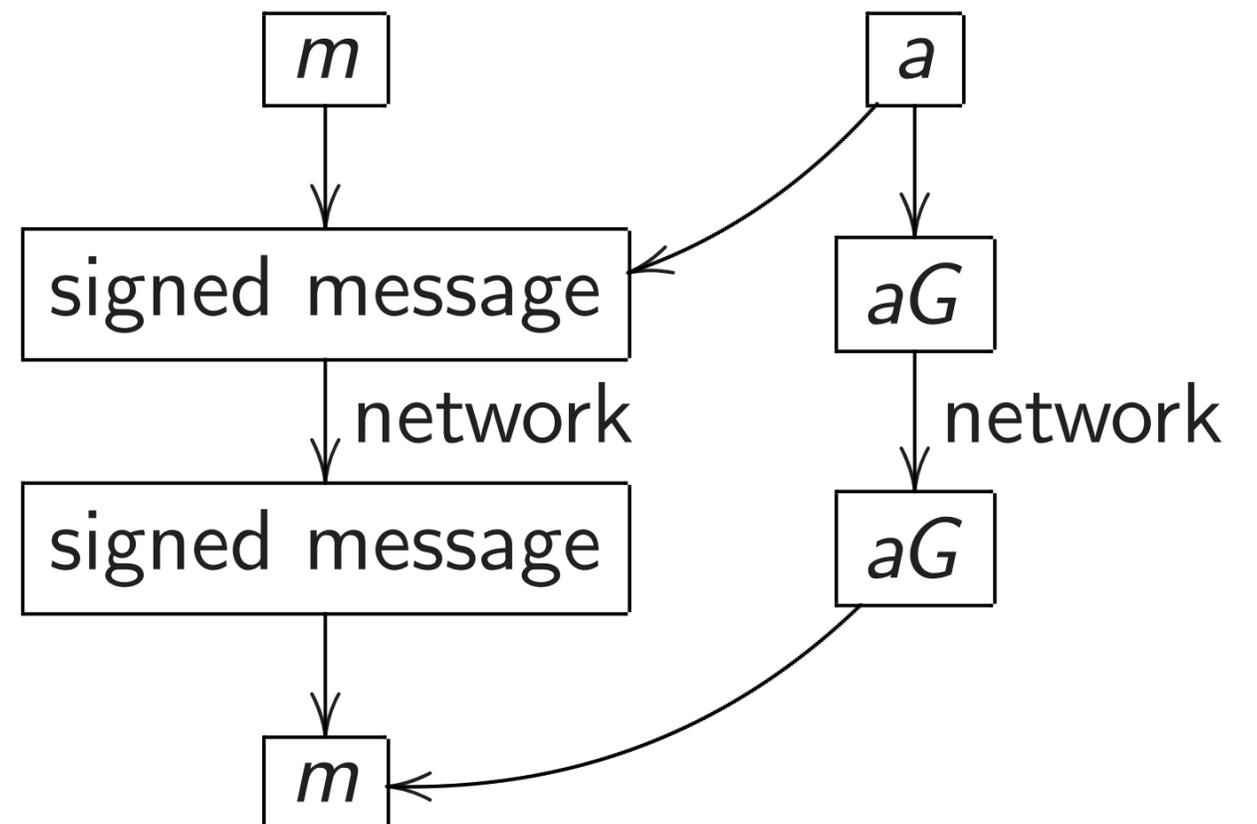
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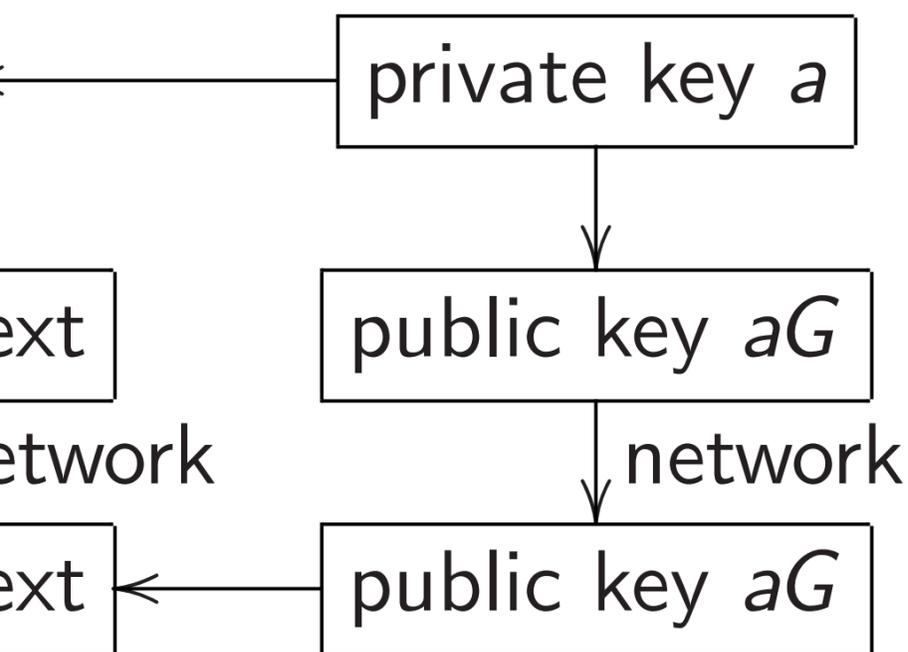
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Cryptography requires TCB to protect secrecy of keys, even if user has no other secrets.

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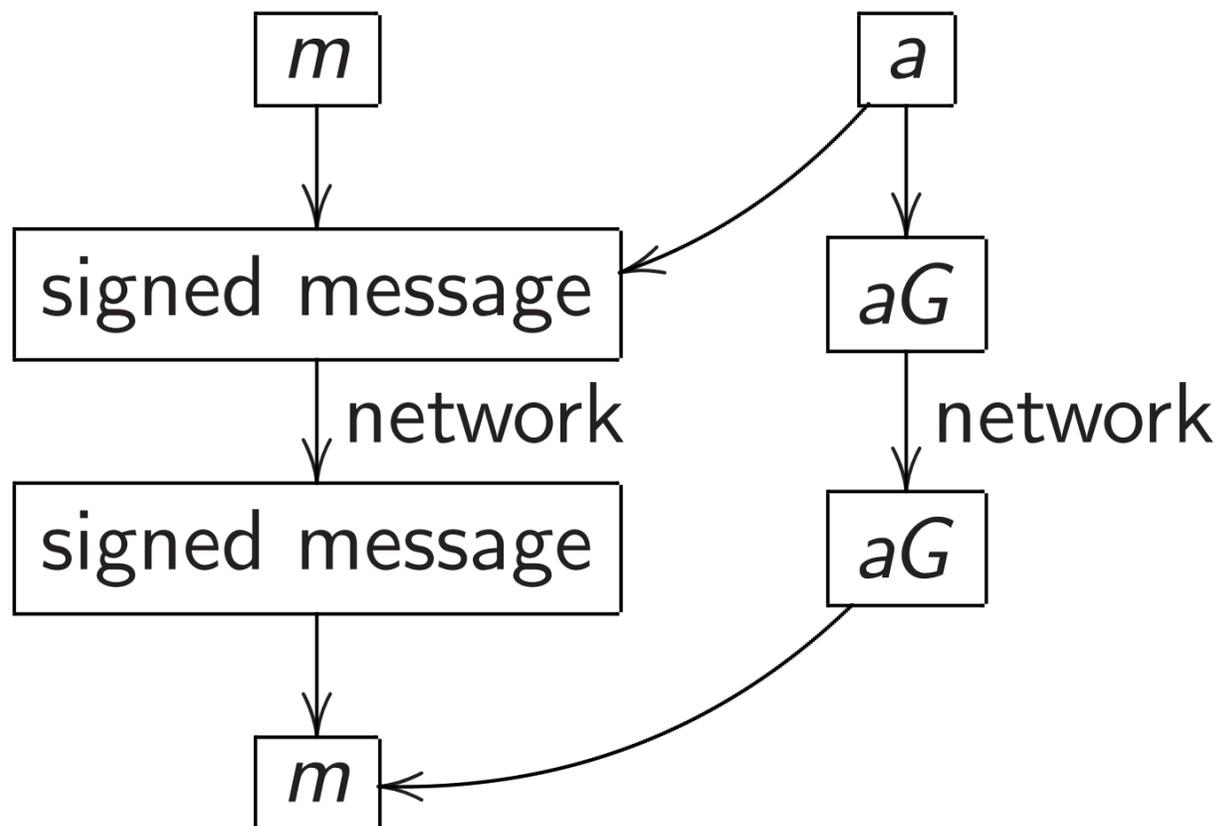
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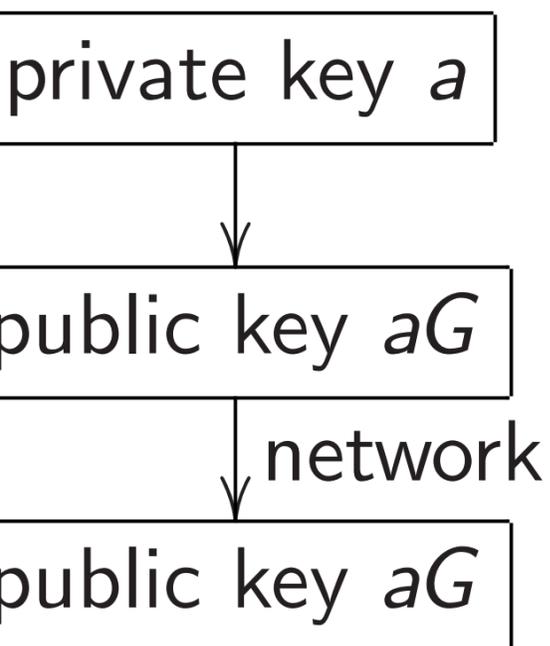
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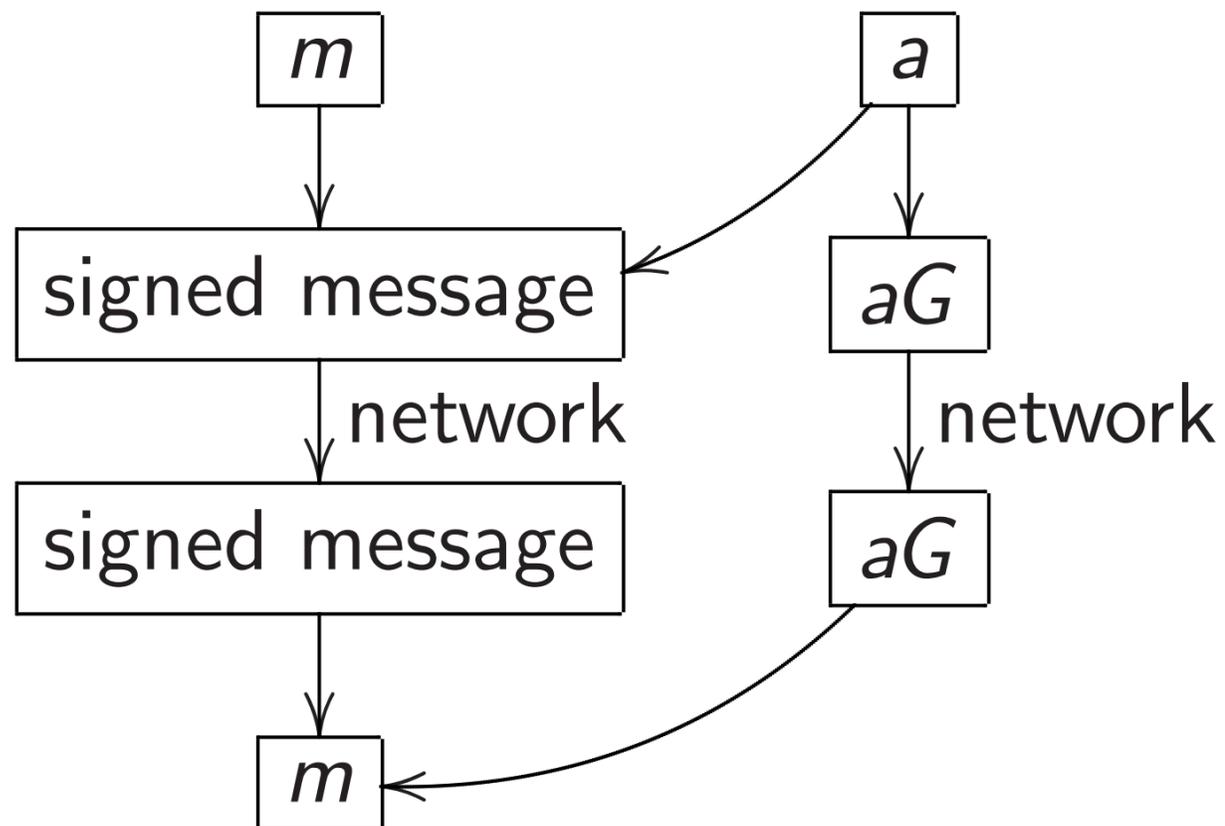
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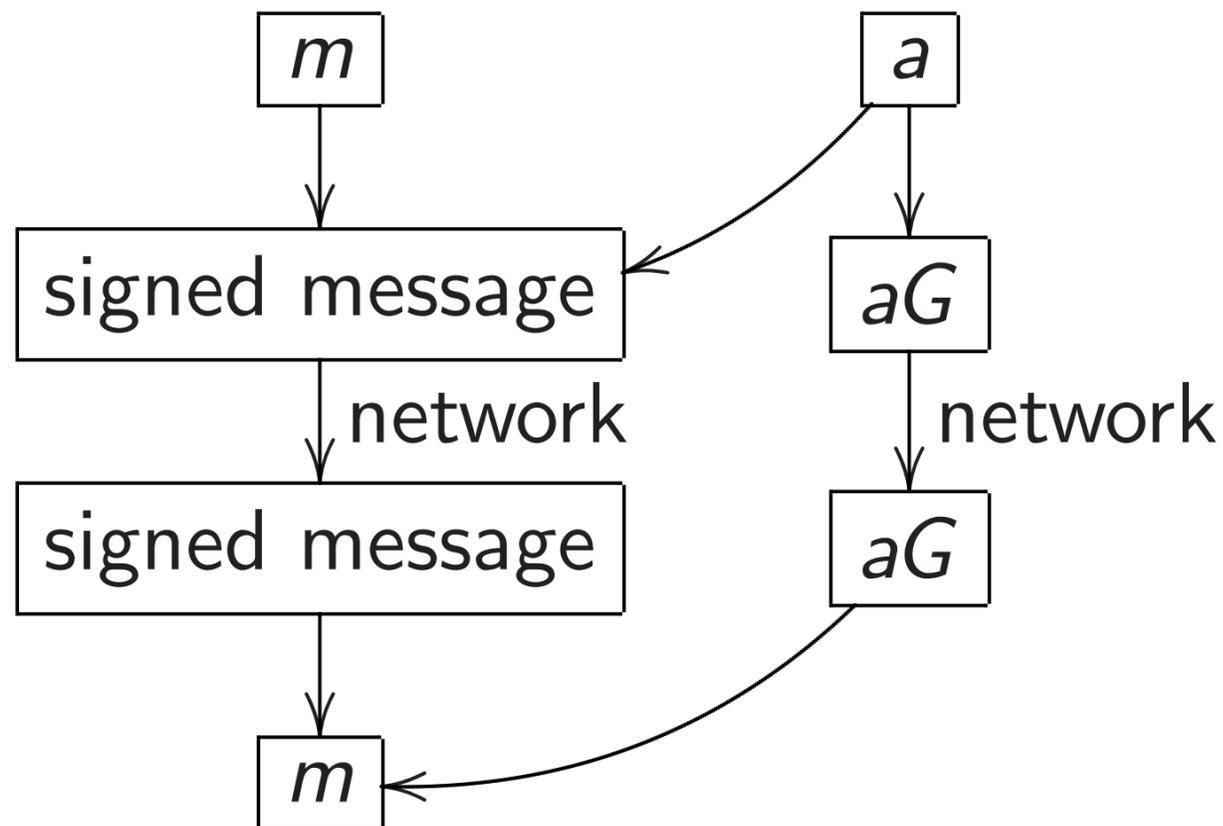
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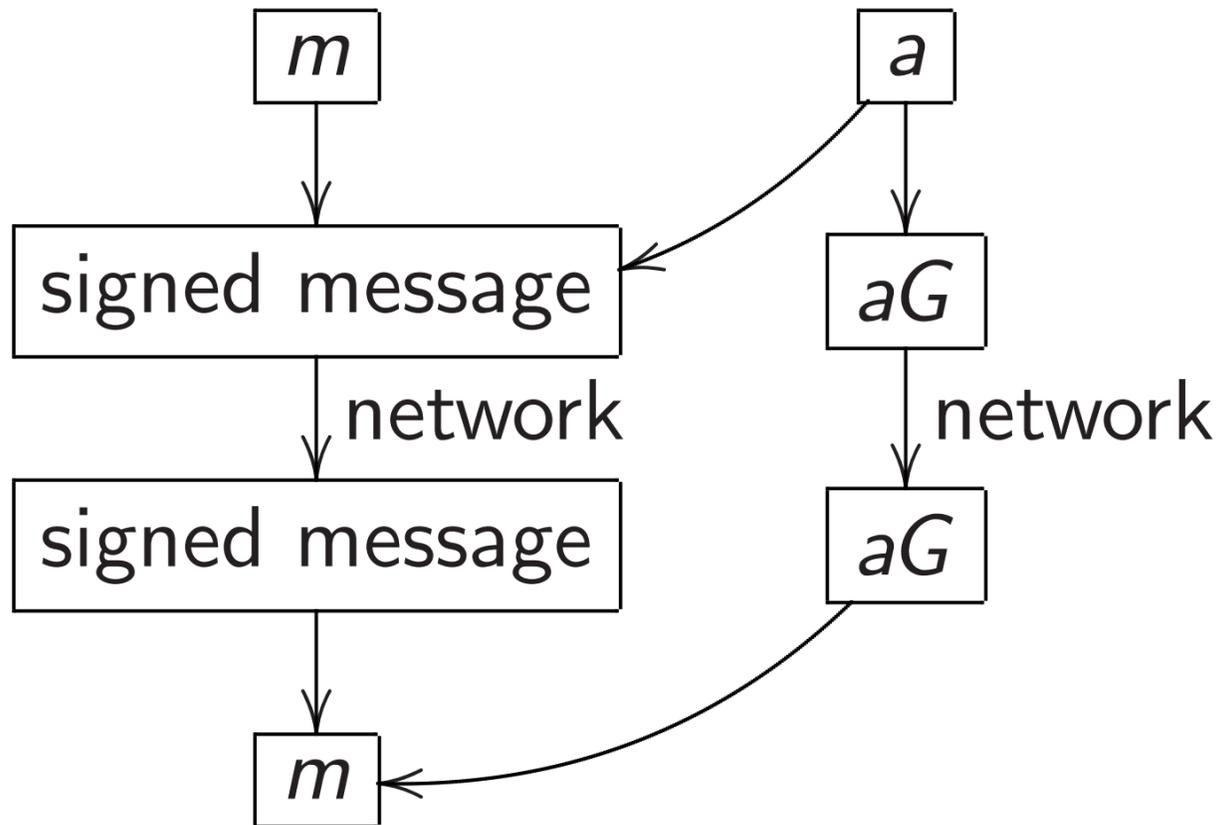
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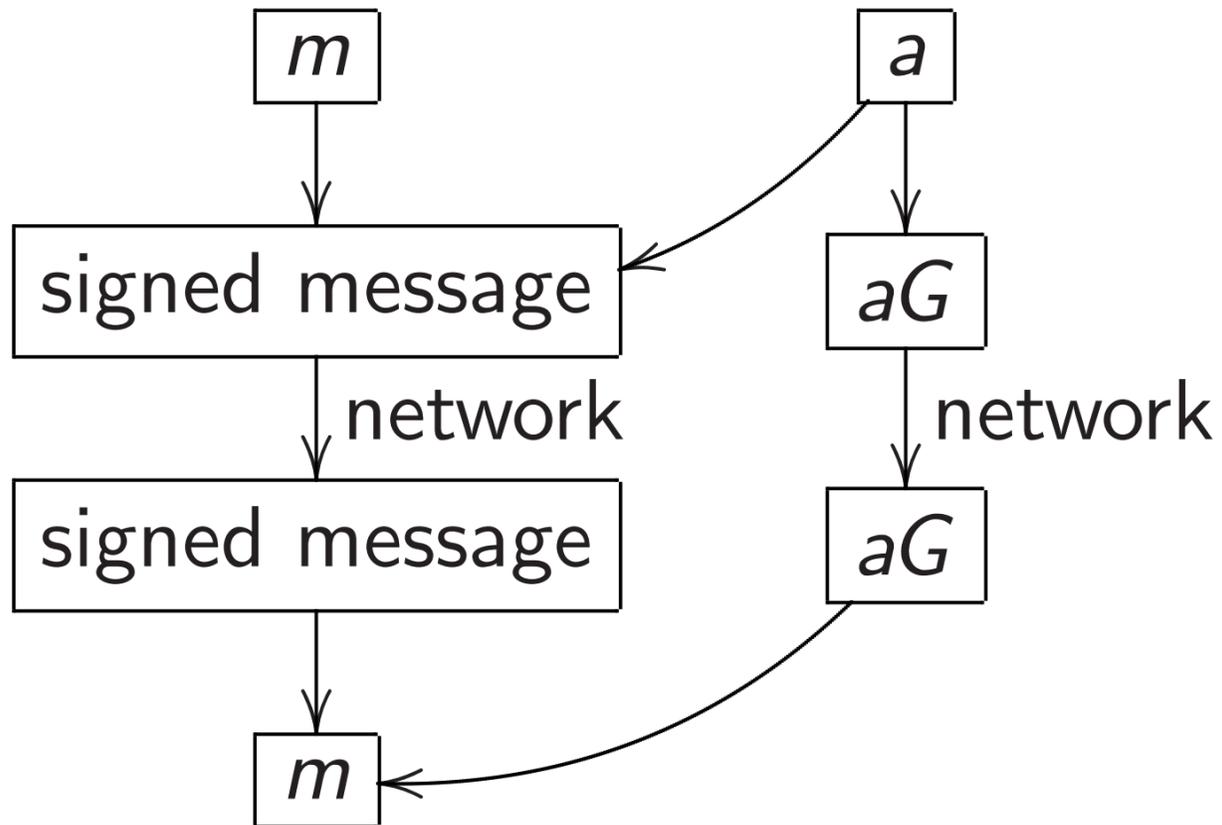
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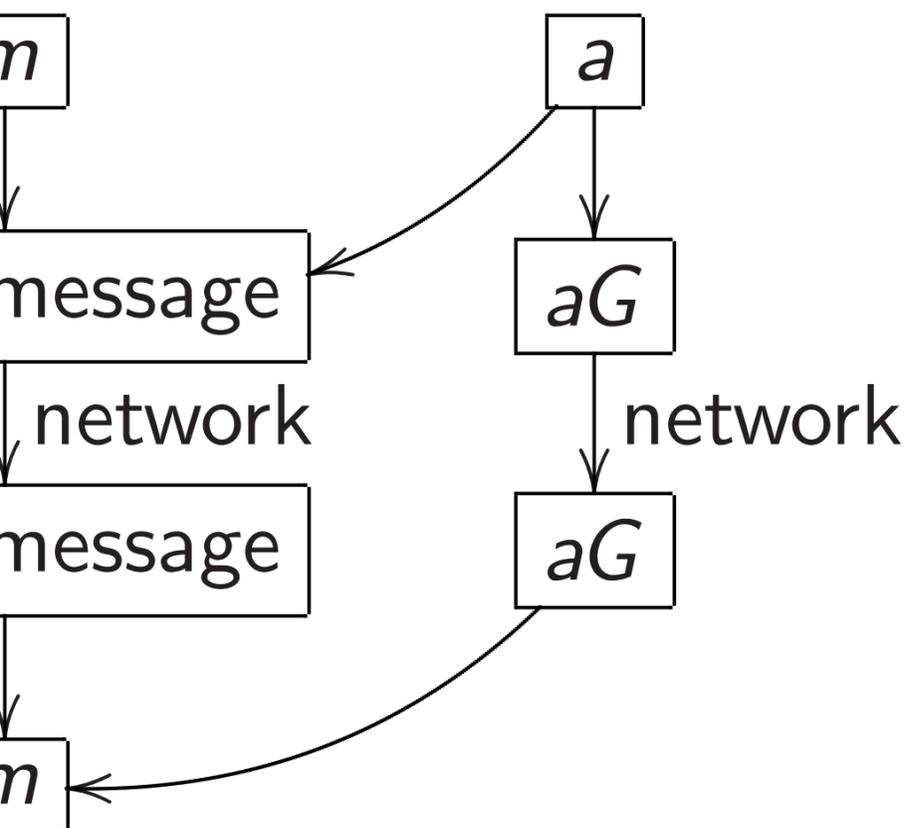
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Schaik–Milburn–Österlund–Frigo–
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secret secrecy of keys,

user has no other secrets.

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Constant-time software

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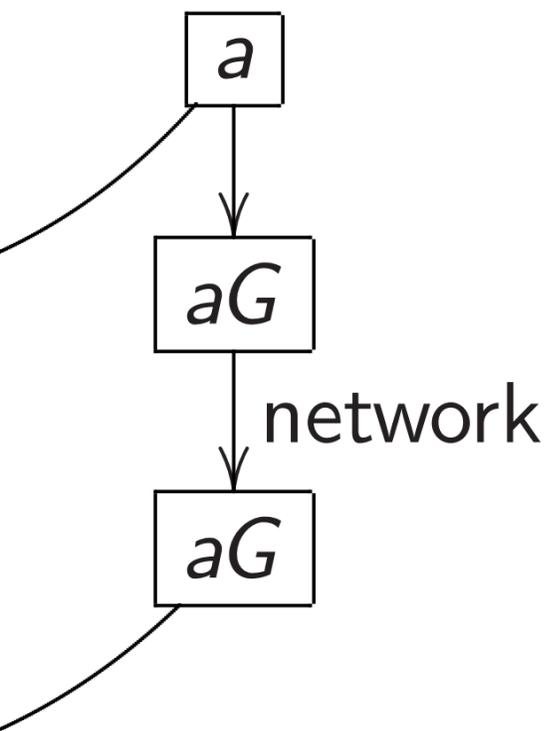
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No confidence in future security.

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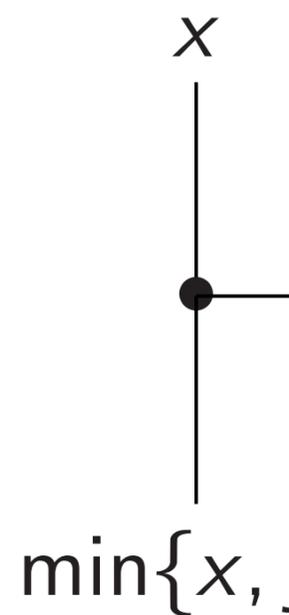
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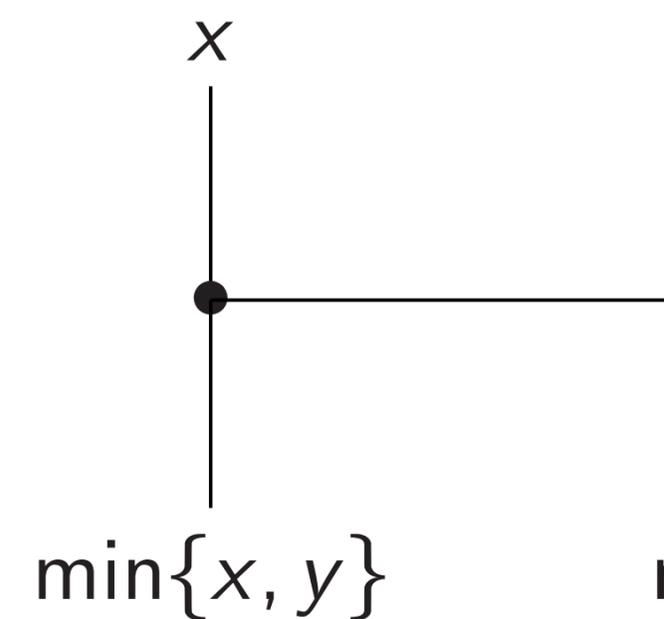
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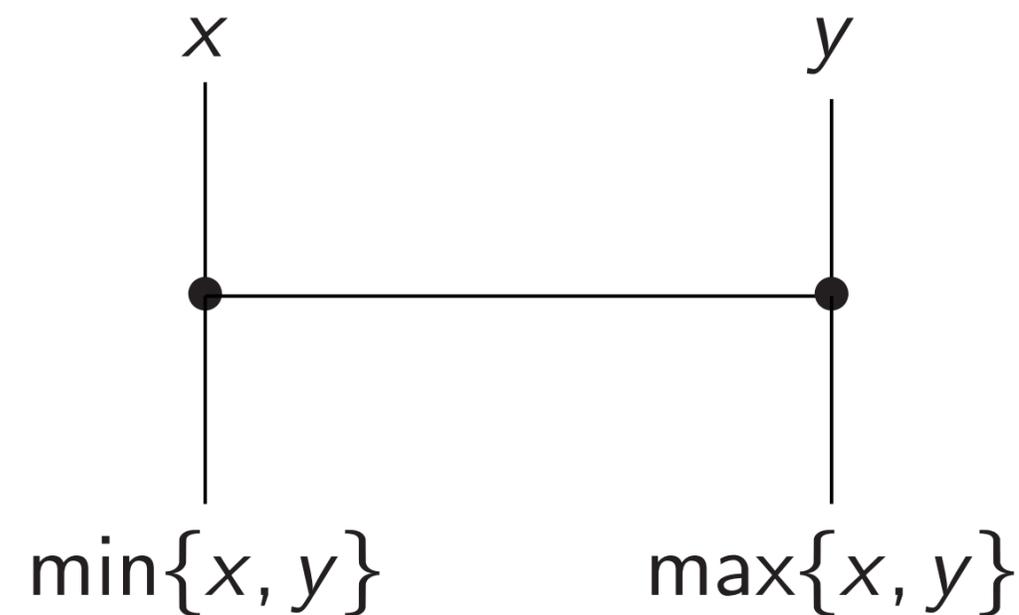
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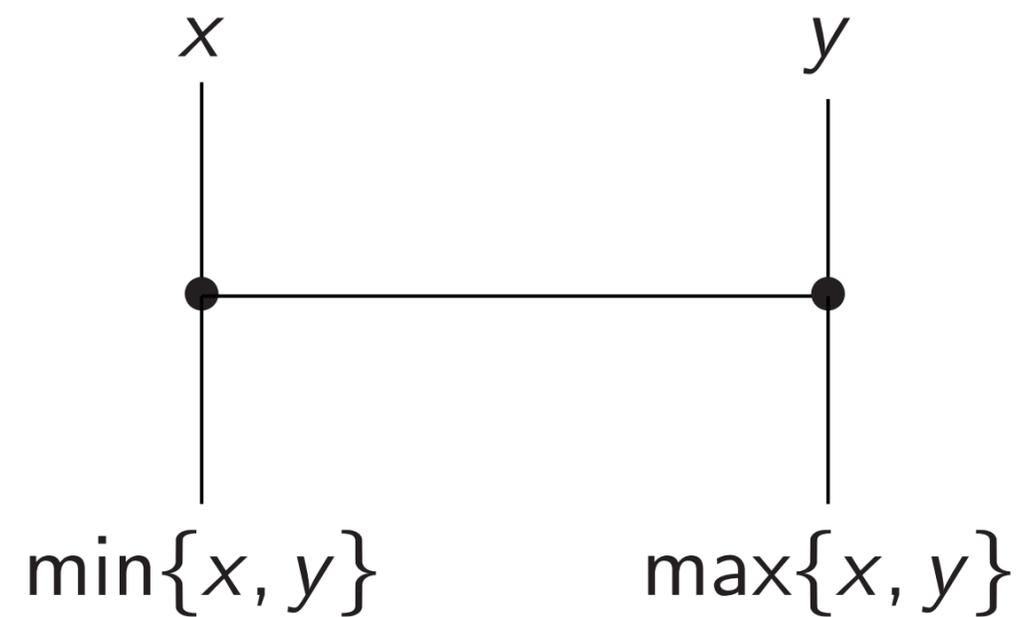
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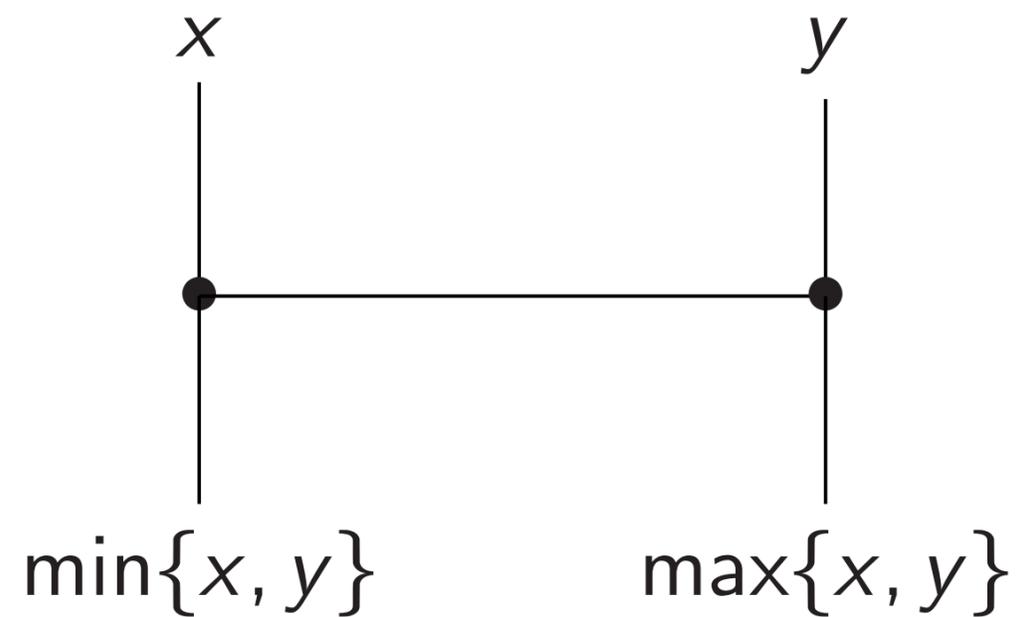
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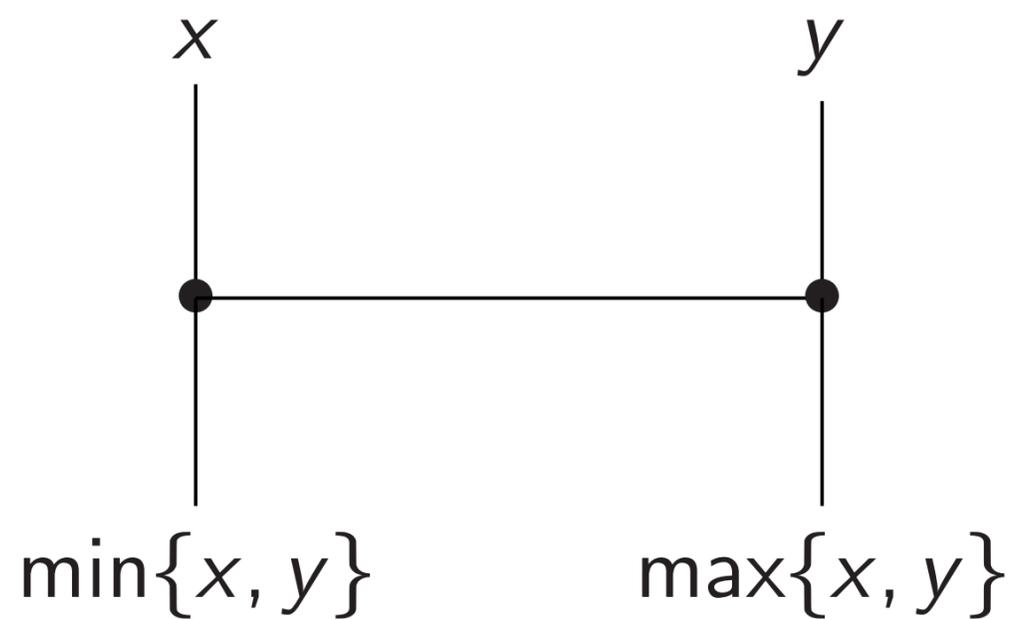
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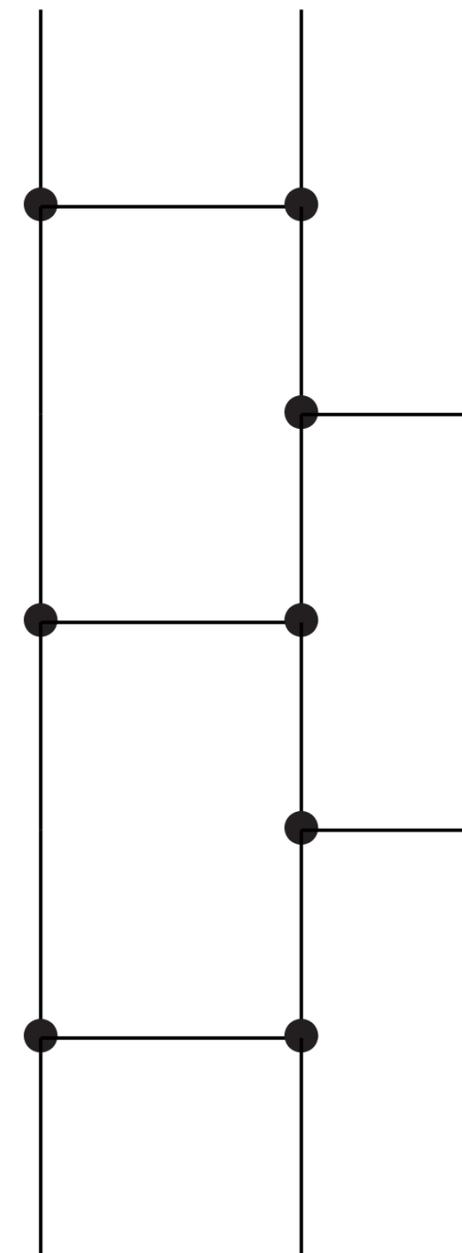


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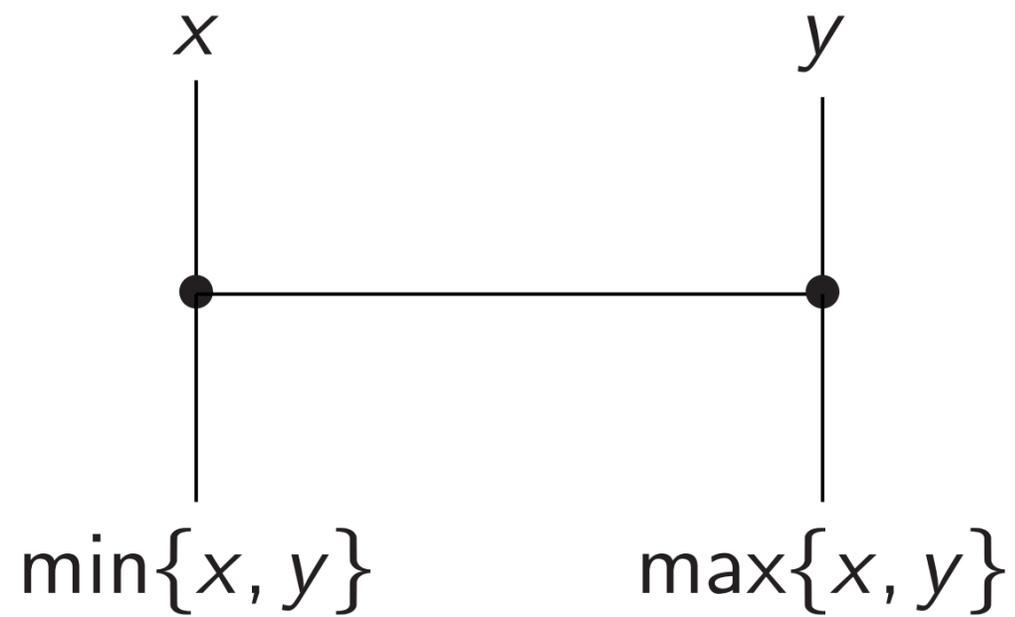
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Combine comparators
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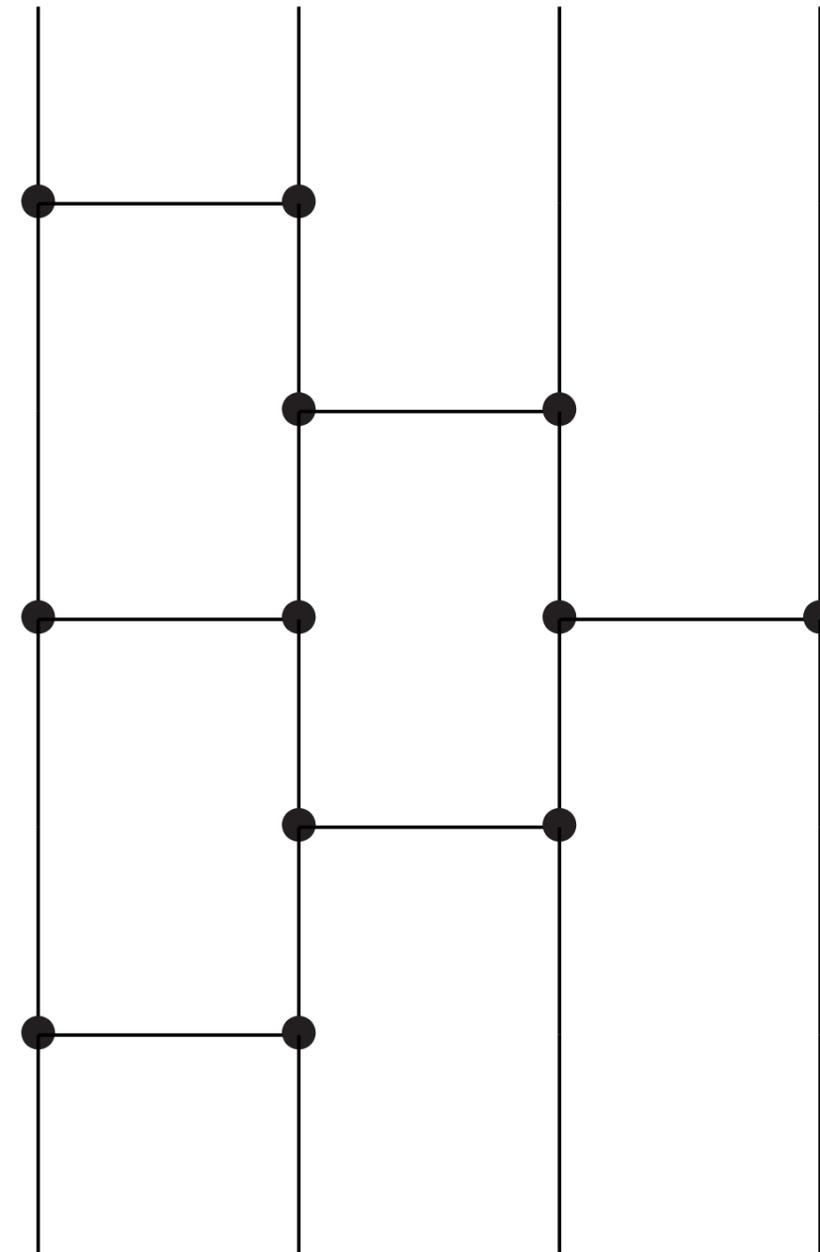
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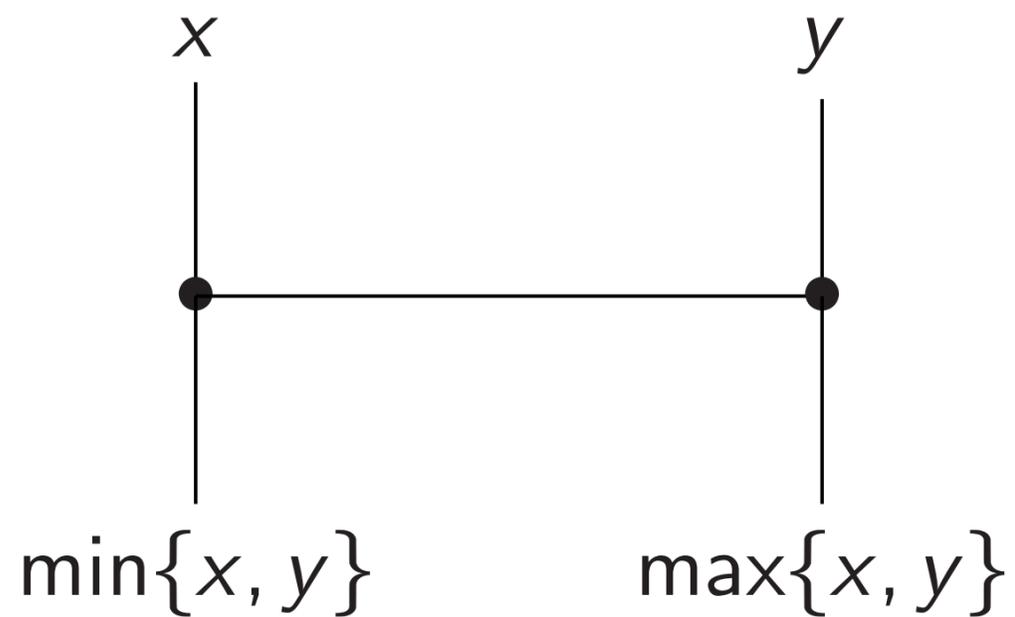
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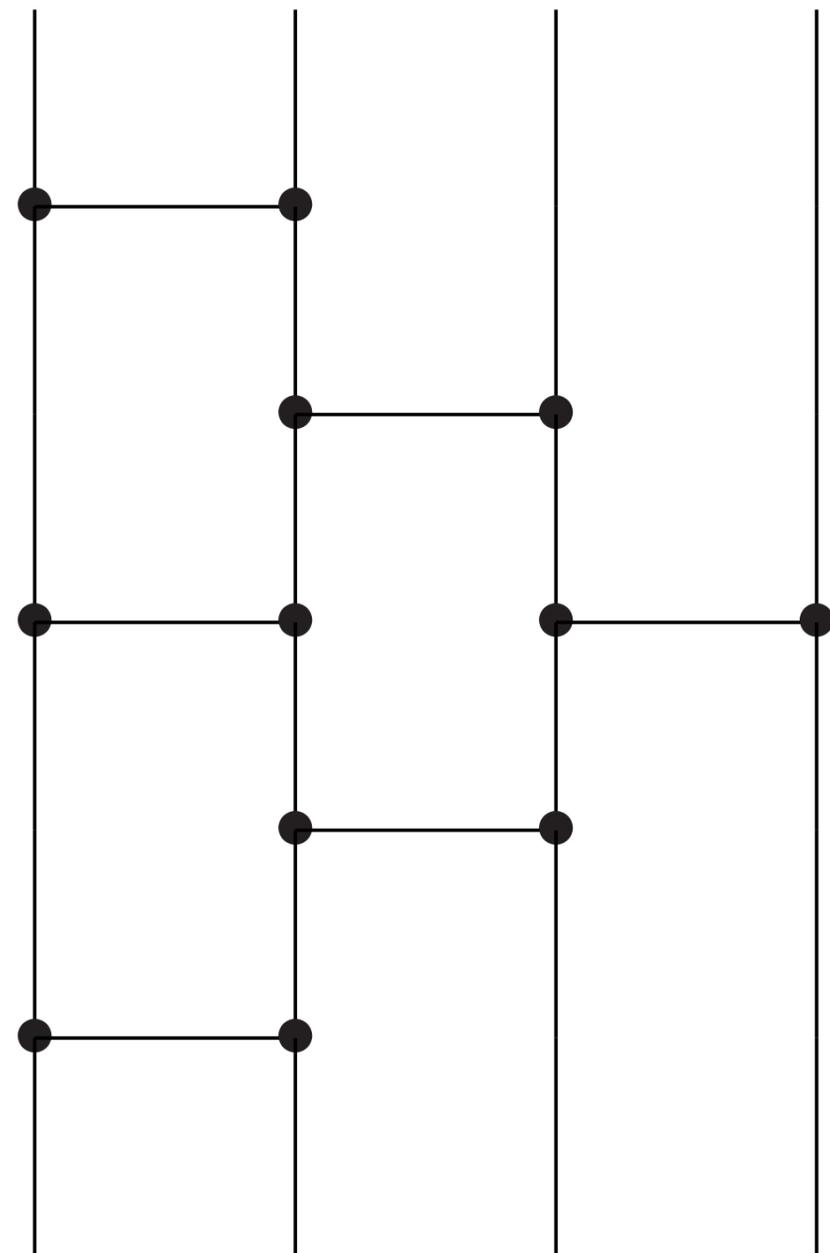
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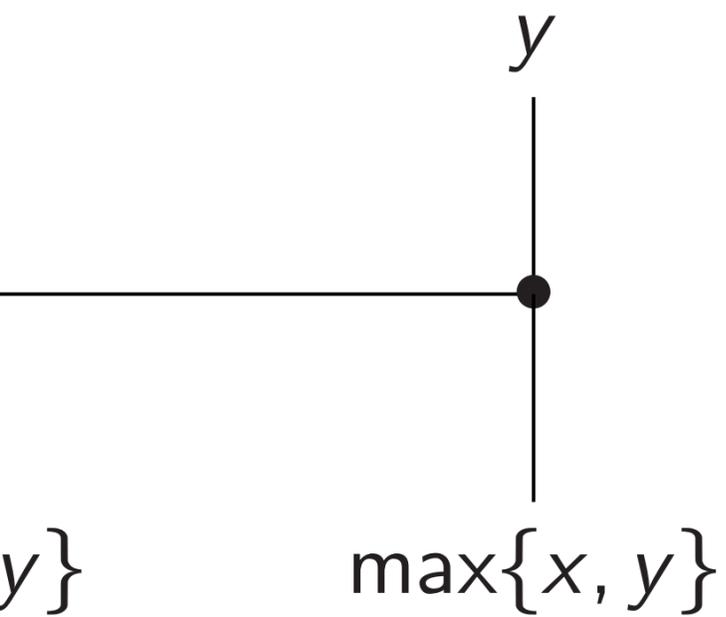
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Example of a sorting network:



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Comparator sorting 2 integers.



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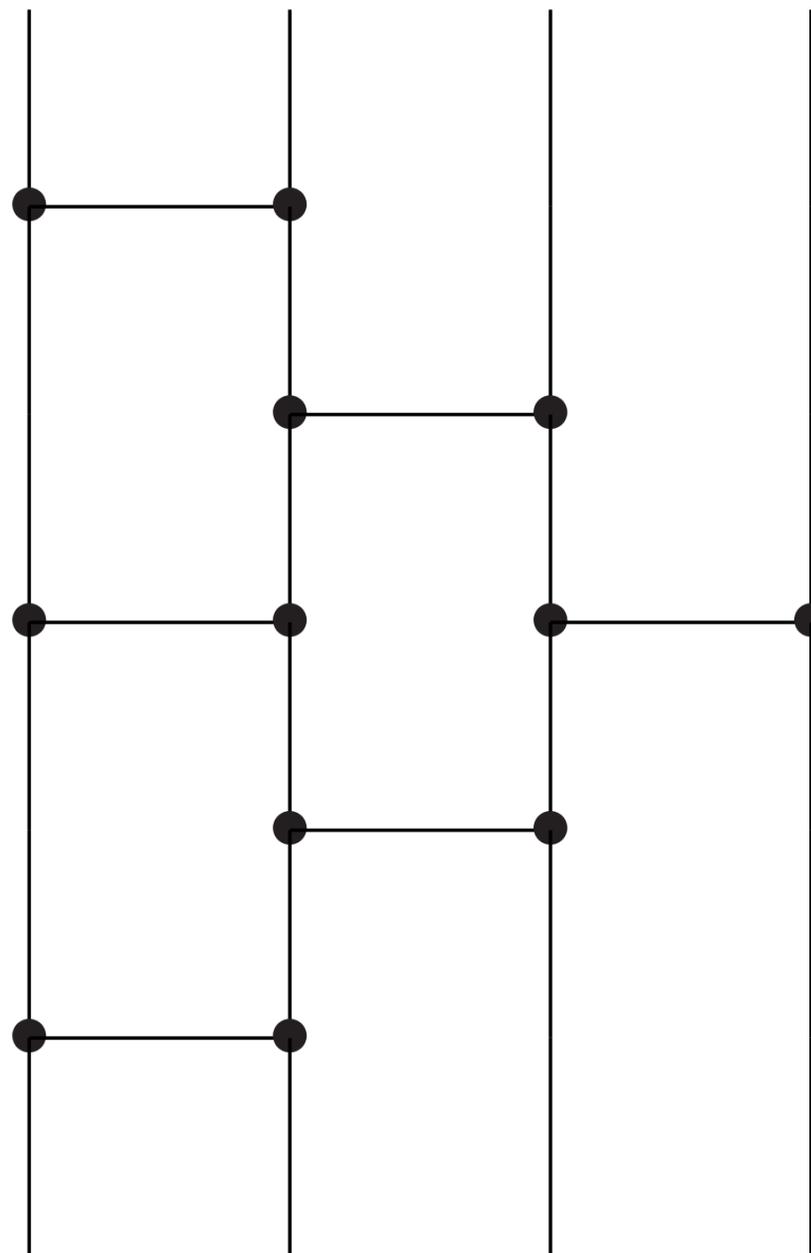
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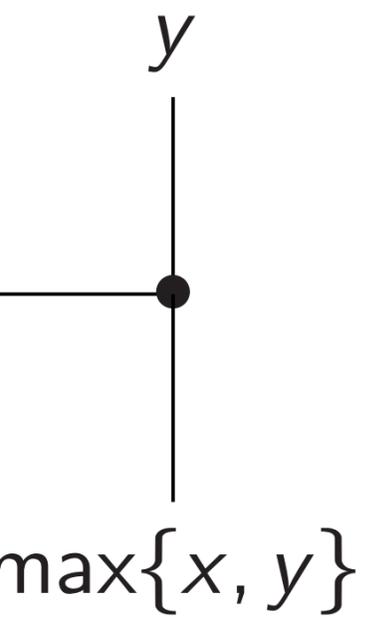
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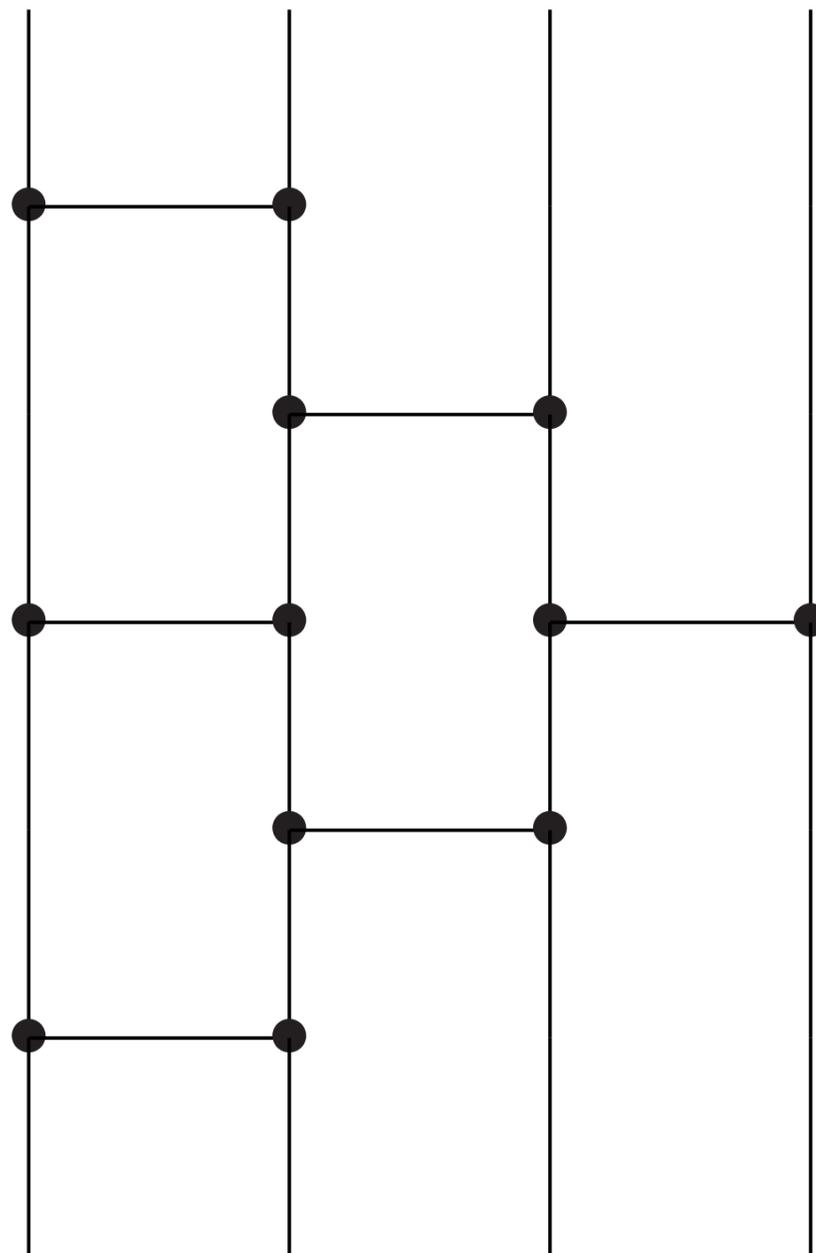
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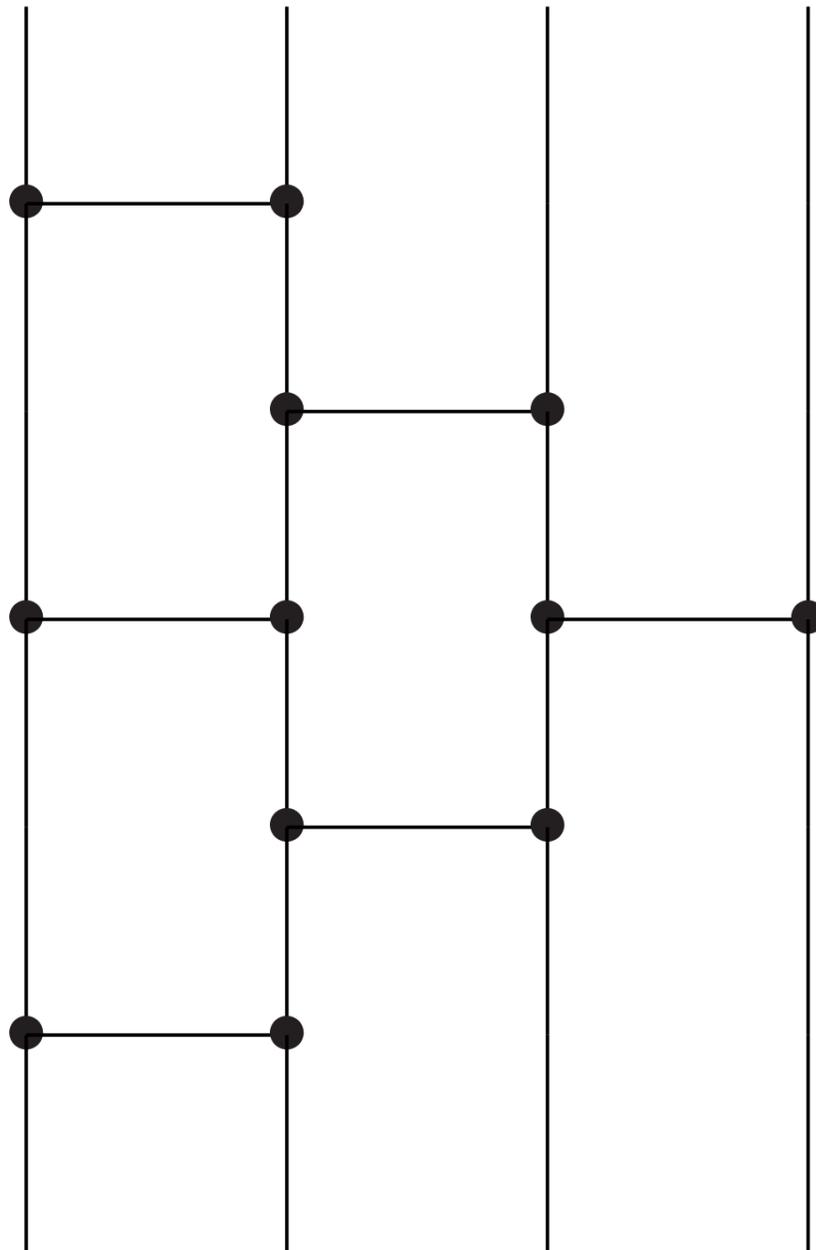


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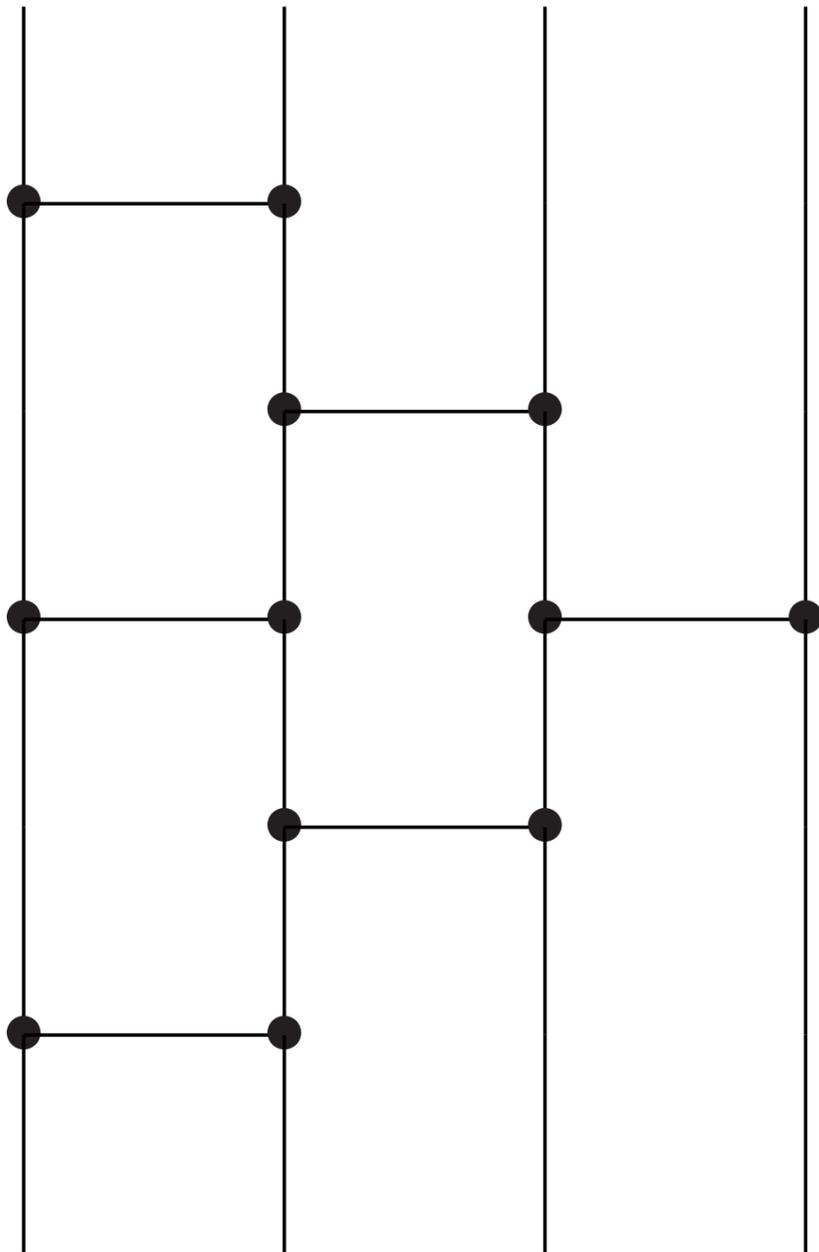


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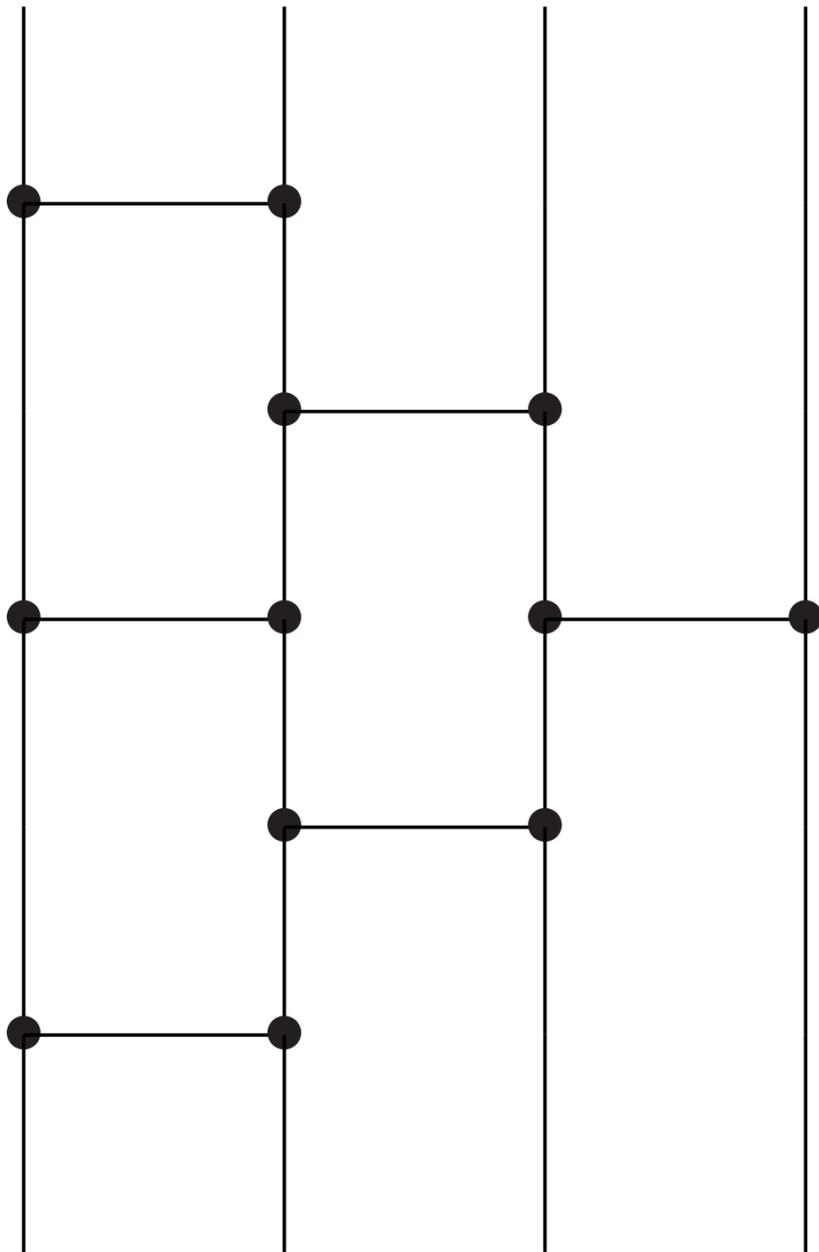
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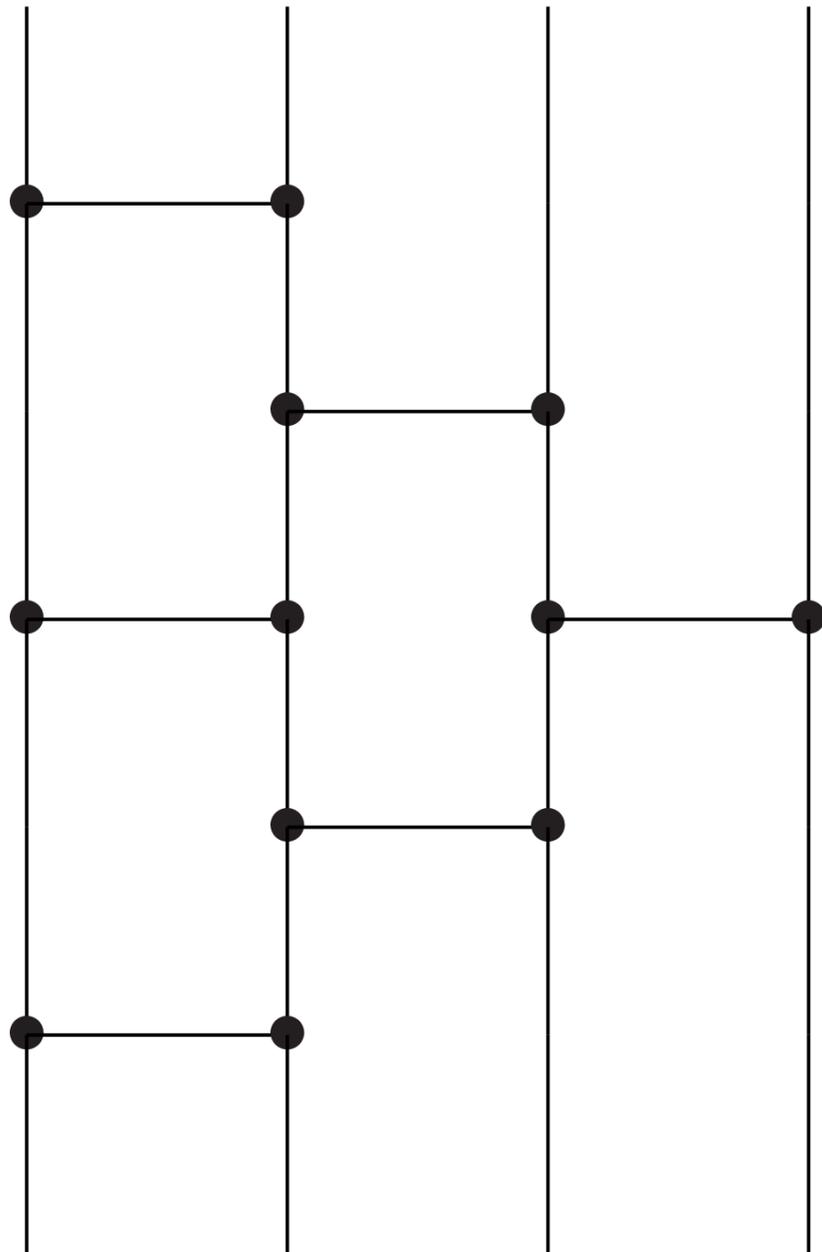


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But $(n^2 - n)/2$ comparators produce complaints about performance as n increases.

Combine comparators into a **sorting network** for more inputs.

Example of a sorting network:



Positions of comparators in a sorting network are independent of the input. Naturally constant-time.

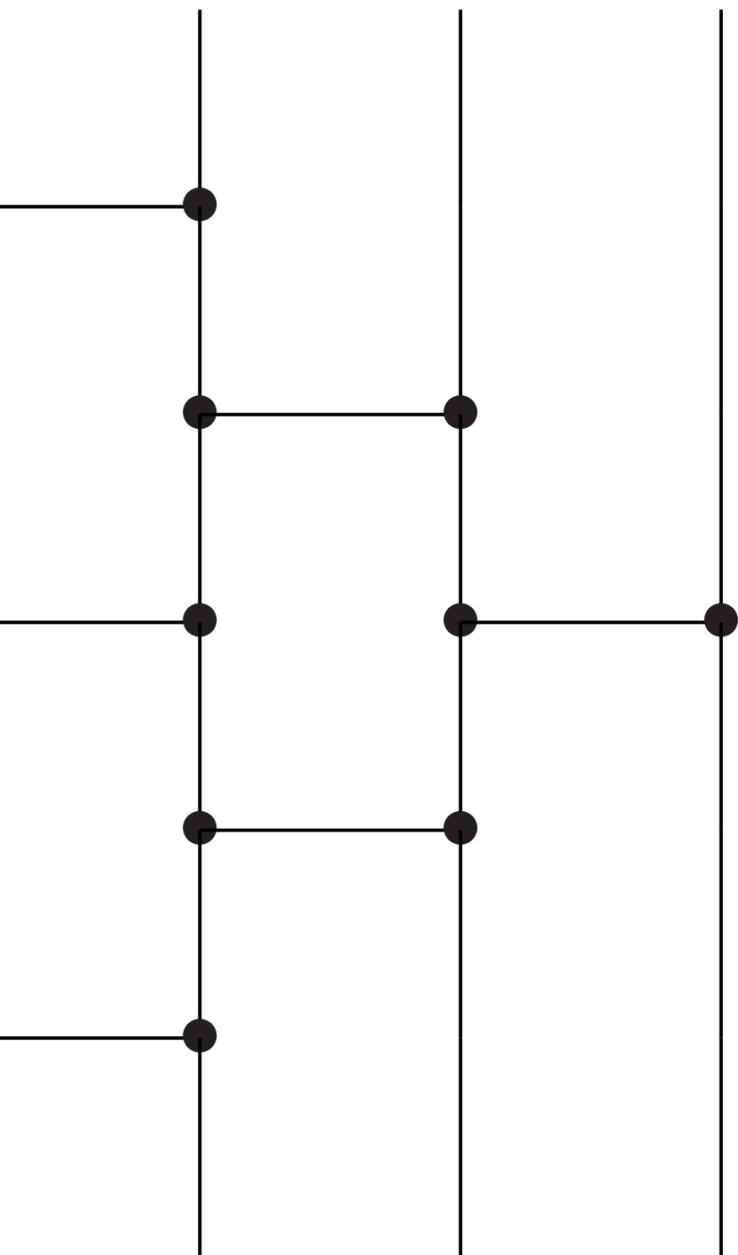
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Speed is a serious issue in the post-quantum competition.

“Cost” is evaluation criterion; “we’d like to stress this once again on the forum that we’d really like to see more platform-optimized implementations”; etc.

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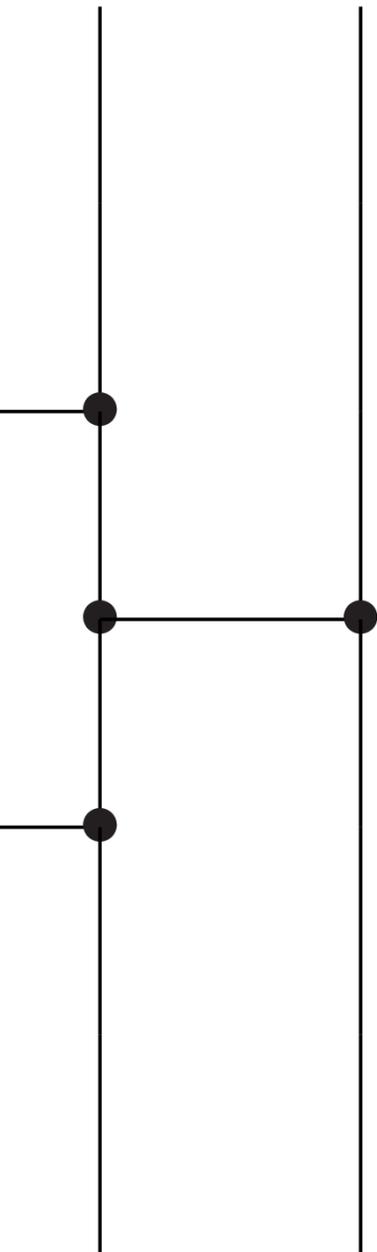
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void int  
{ int64  
  if (n  
    t = 1  
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```
void int32_sort(
{ int64 t,p,q,i;
  if (n < 2) ret
  t = 1;
  while (t < n -
  for (p = t;p >
    for (i = 0;i
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    for (i = 0
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    }
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void int32_sort(int32 *x,
{ int64 t,p,q,i;
  if (n < 2) return;
  t = 1;
  while (t < n - t) t +=
  for (p = t;p > 0;p >>=
    for (i = 0;i < n - p;
      if (!(i & p))
        minmax(x+i,x+i+p)
  for (q = t;q > p;q >>
    for (i = 0;i < n -
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Conditional branch: much slower.

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Sorting software is in the TCB.

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Test the sorting software on many random inputs, increasing inputs, decreasing inputs. Seems to work.

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History: Many security problems involve occasional inputs where TCB works incorrectly.

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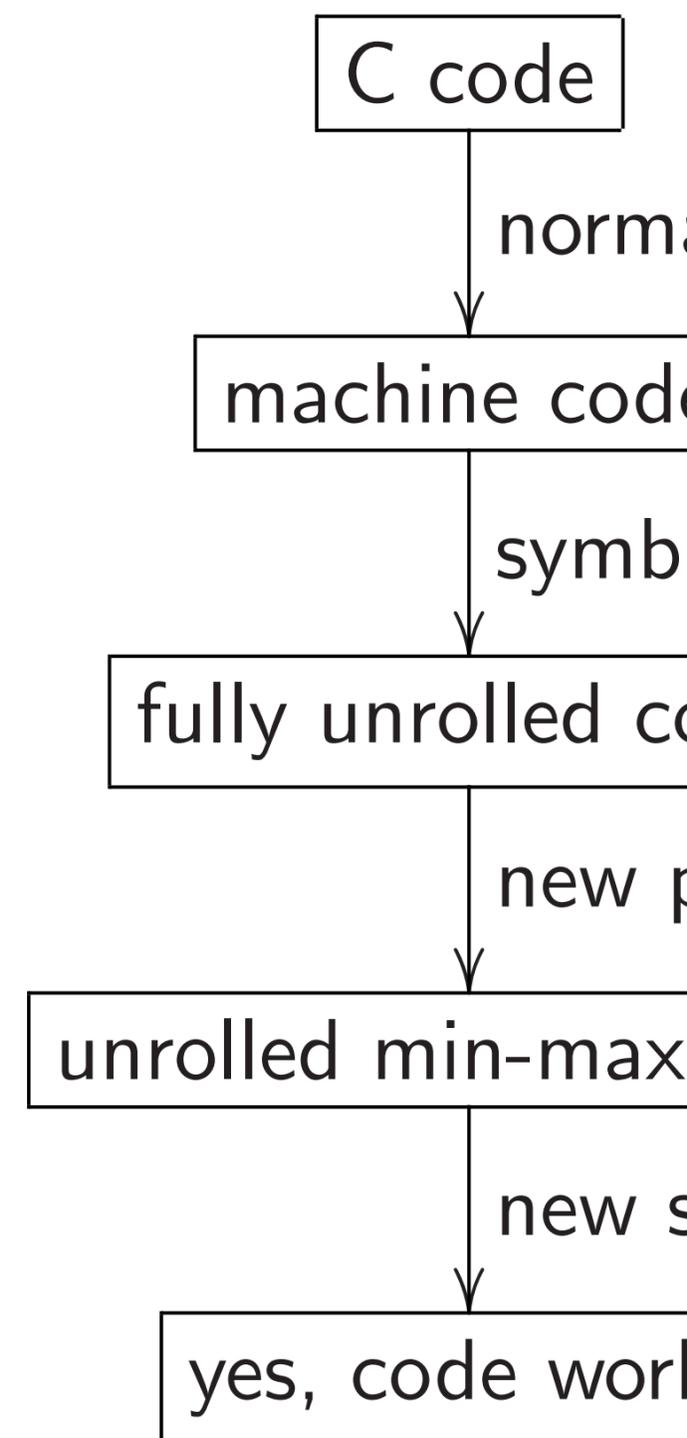
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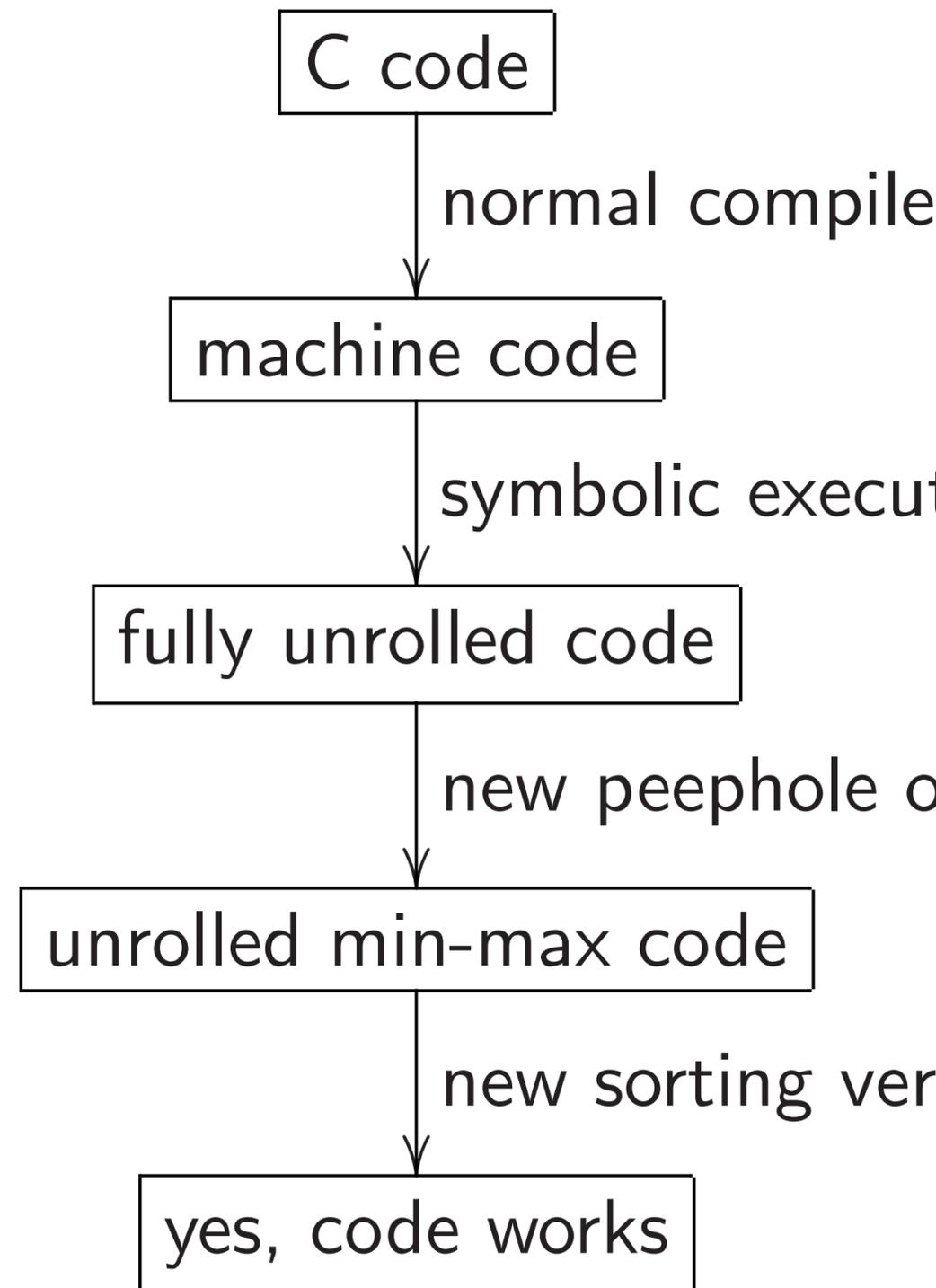
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History: Many security problems involve occasional inputs where TCB works incorrectly.

For each used n (e.g., 768):



Verification

Sorting software is in the TCB.

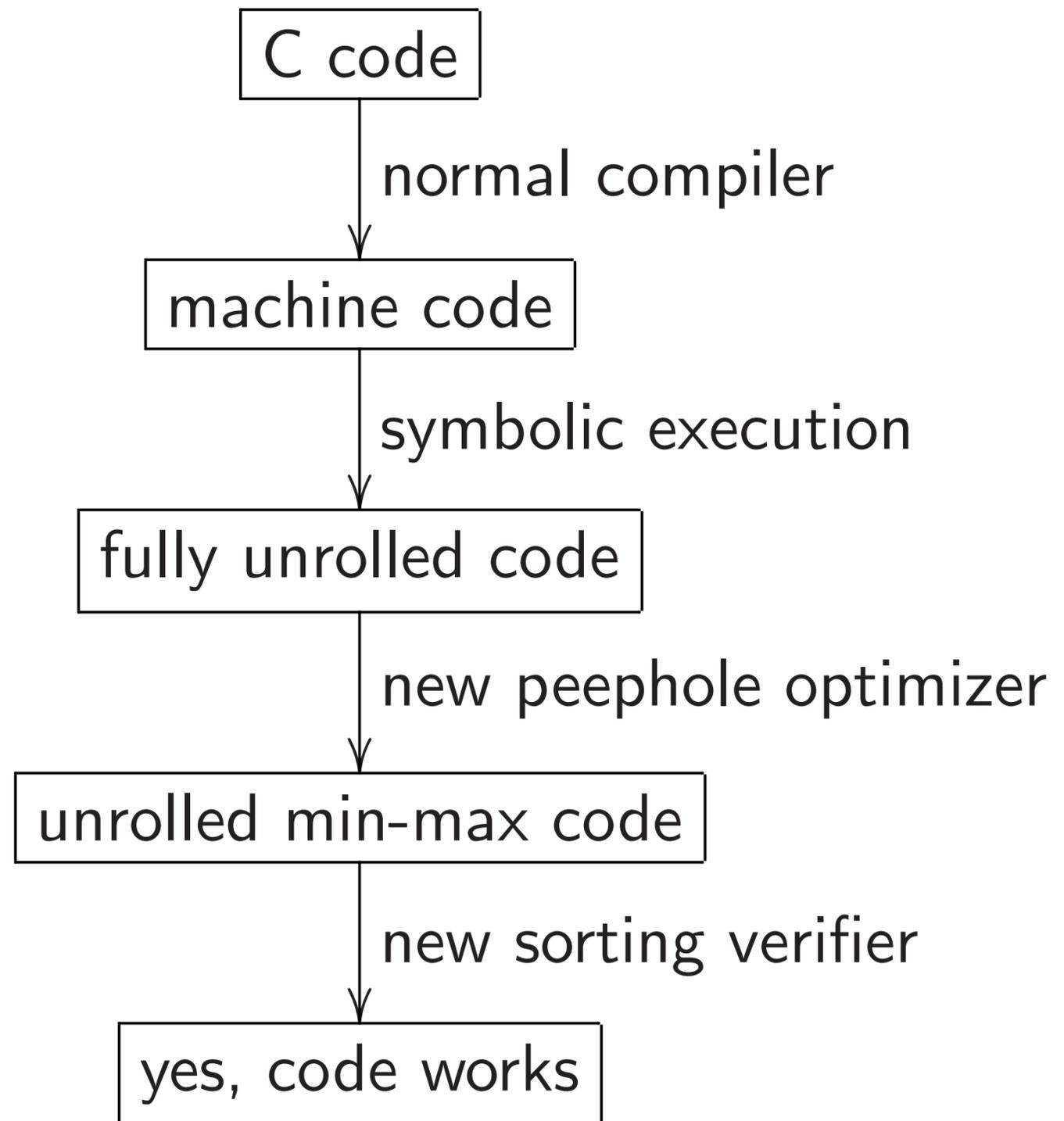
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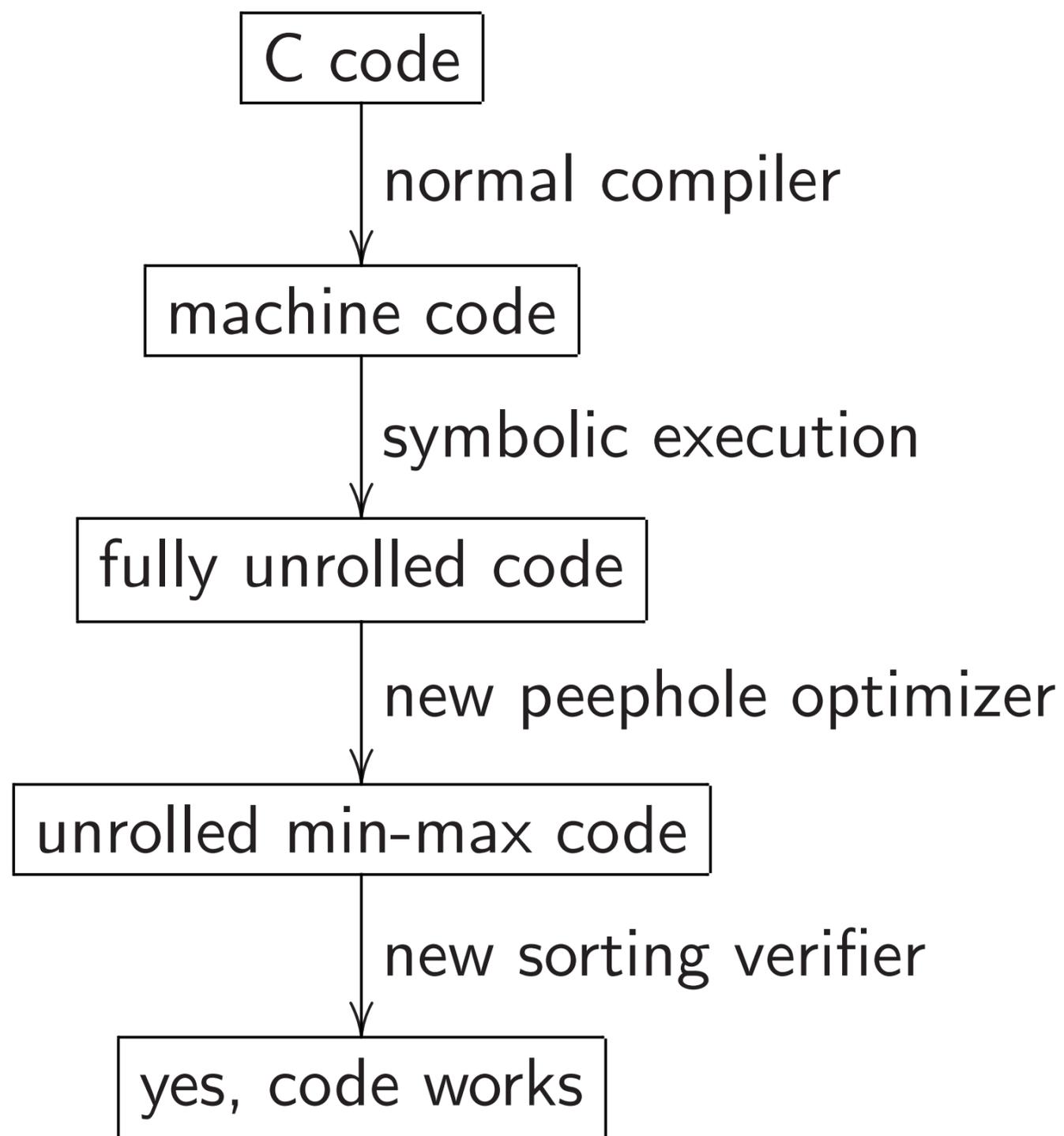
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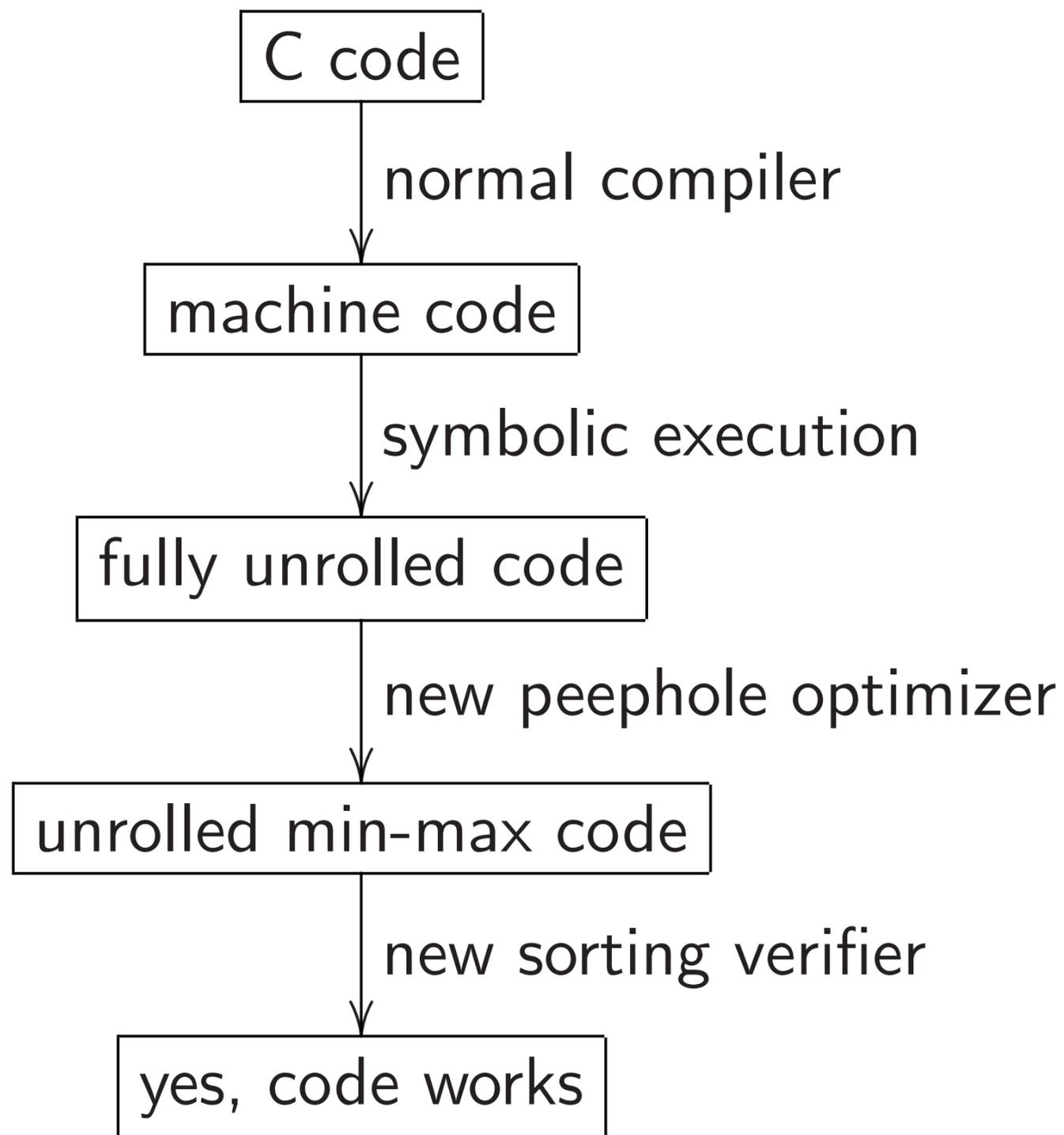
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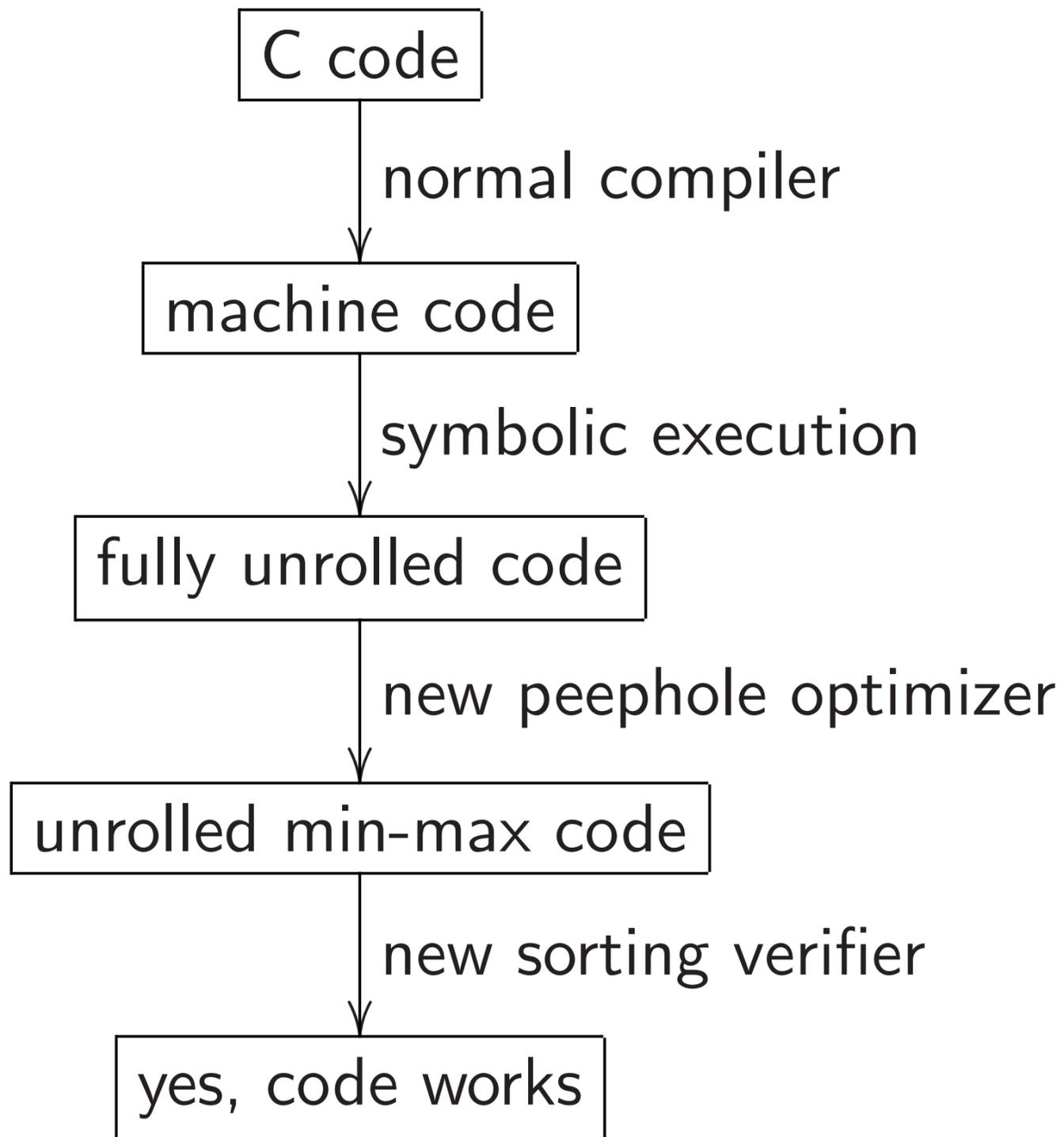
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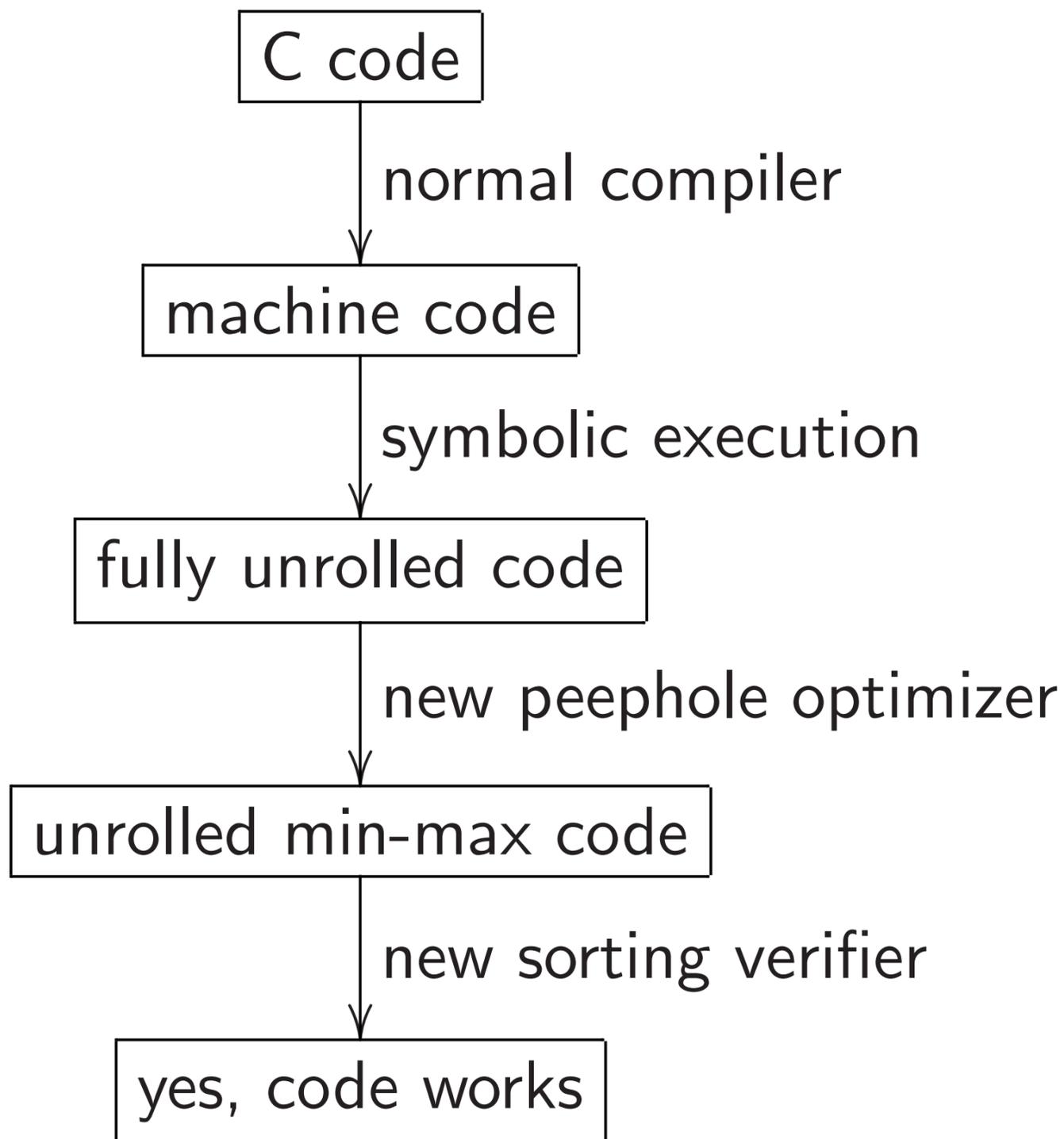
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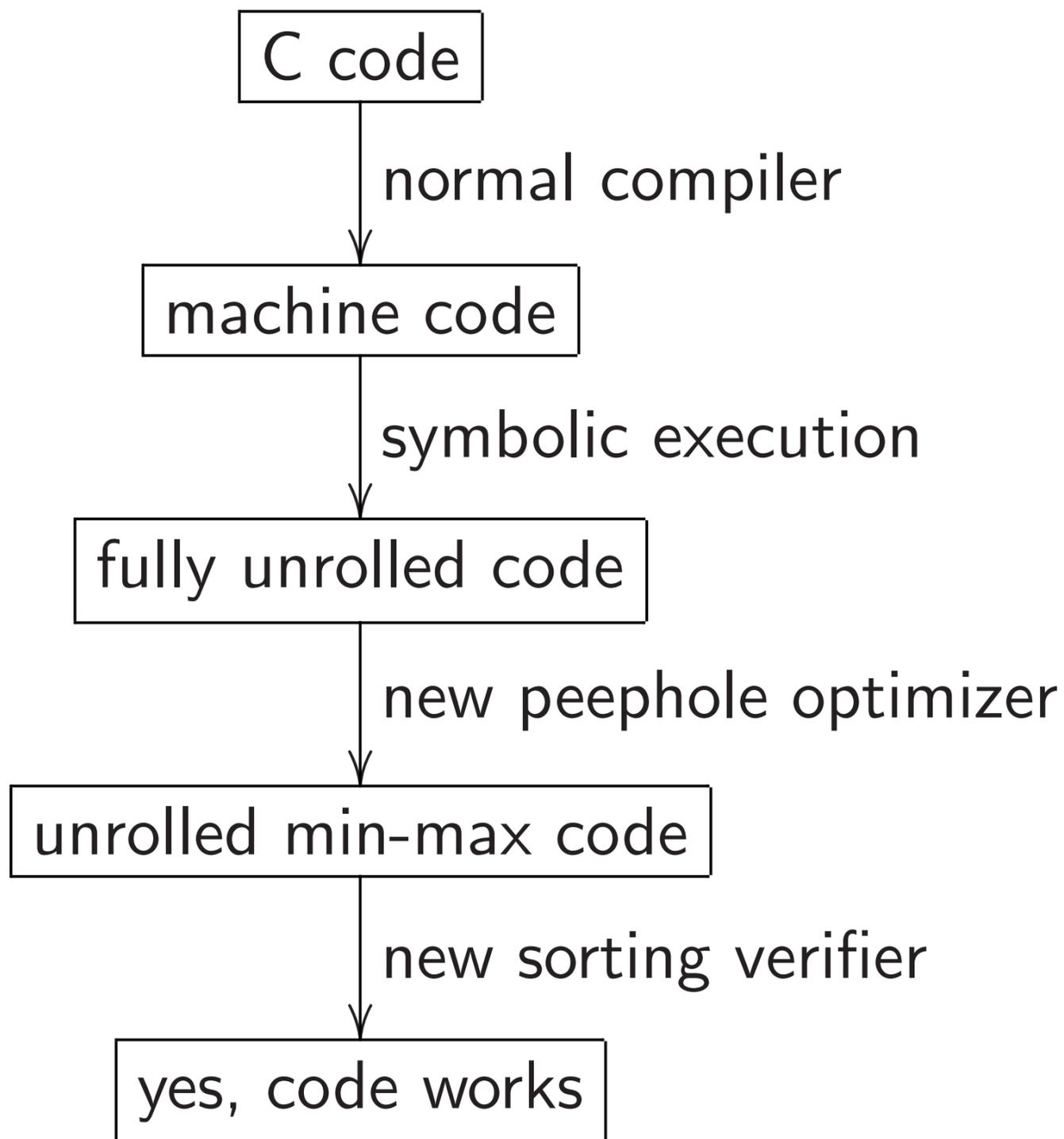
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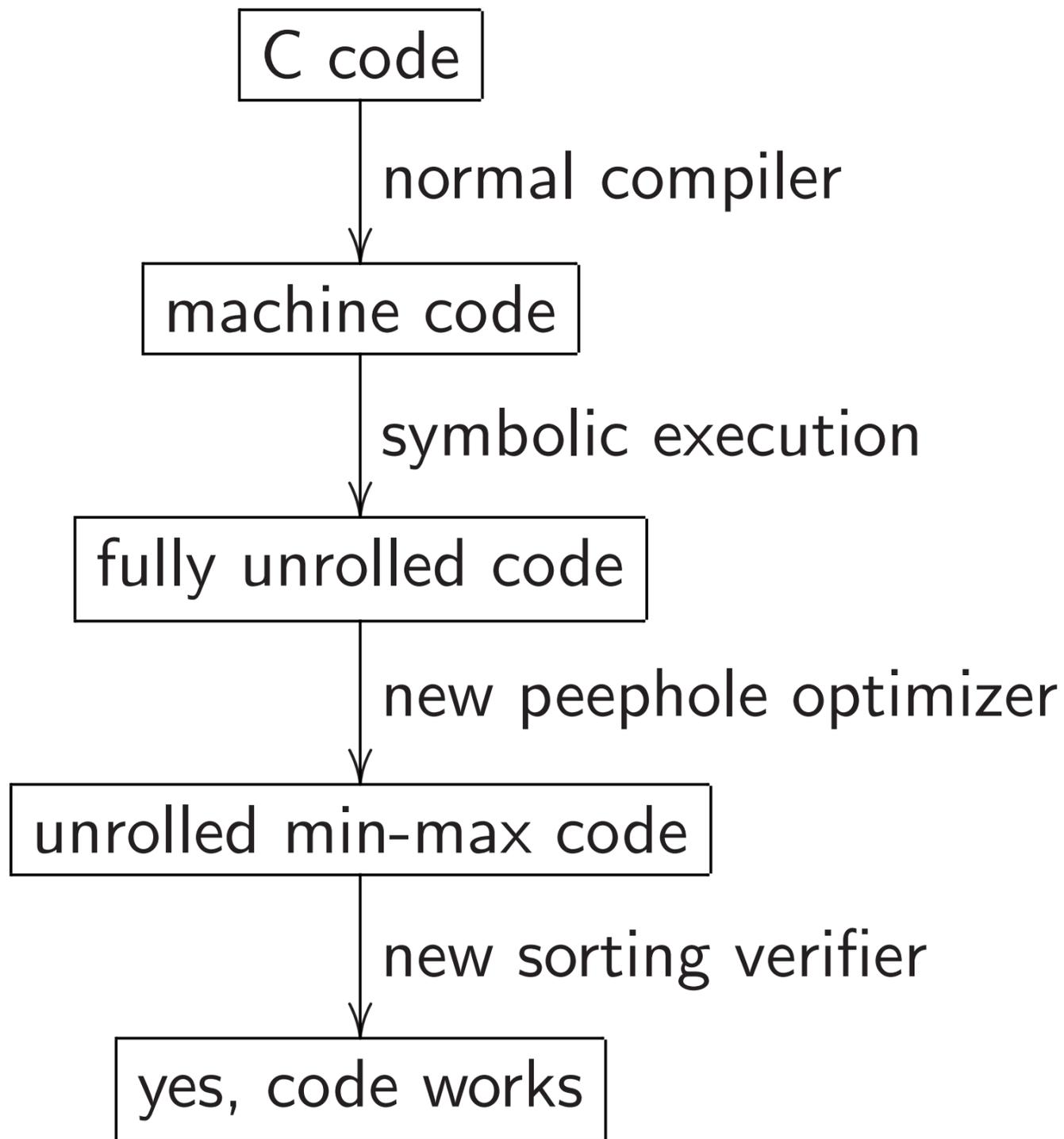
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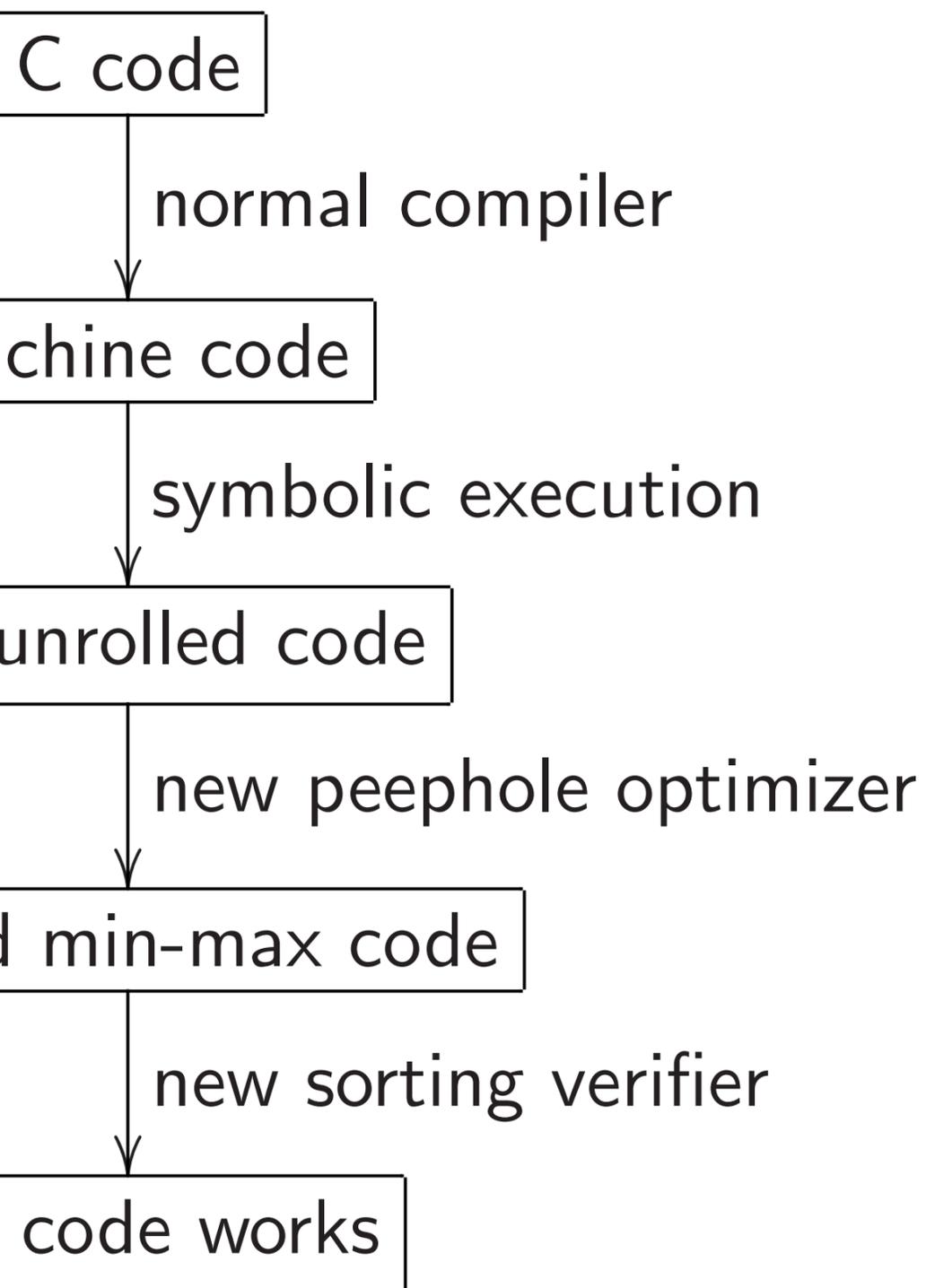
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Verify each merging network using generalization of 2007

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Current djbsort release
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verified portable int32,
fast uint32, fast float32):

sorting.cr.yp.to

Includes the sorting code;
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