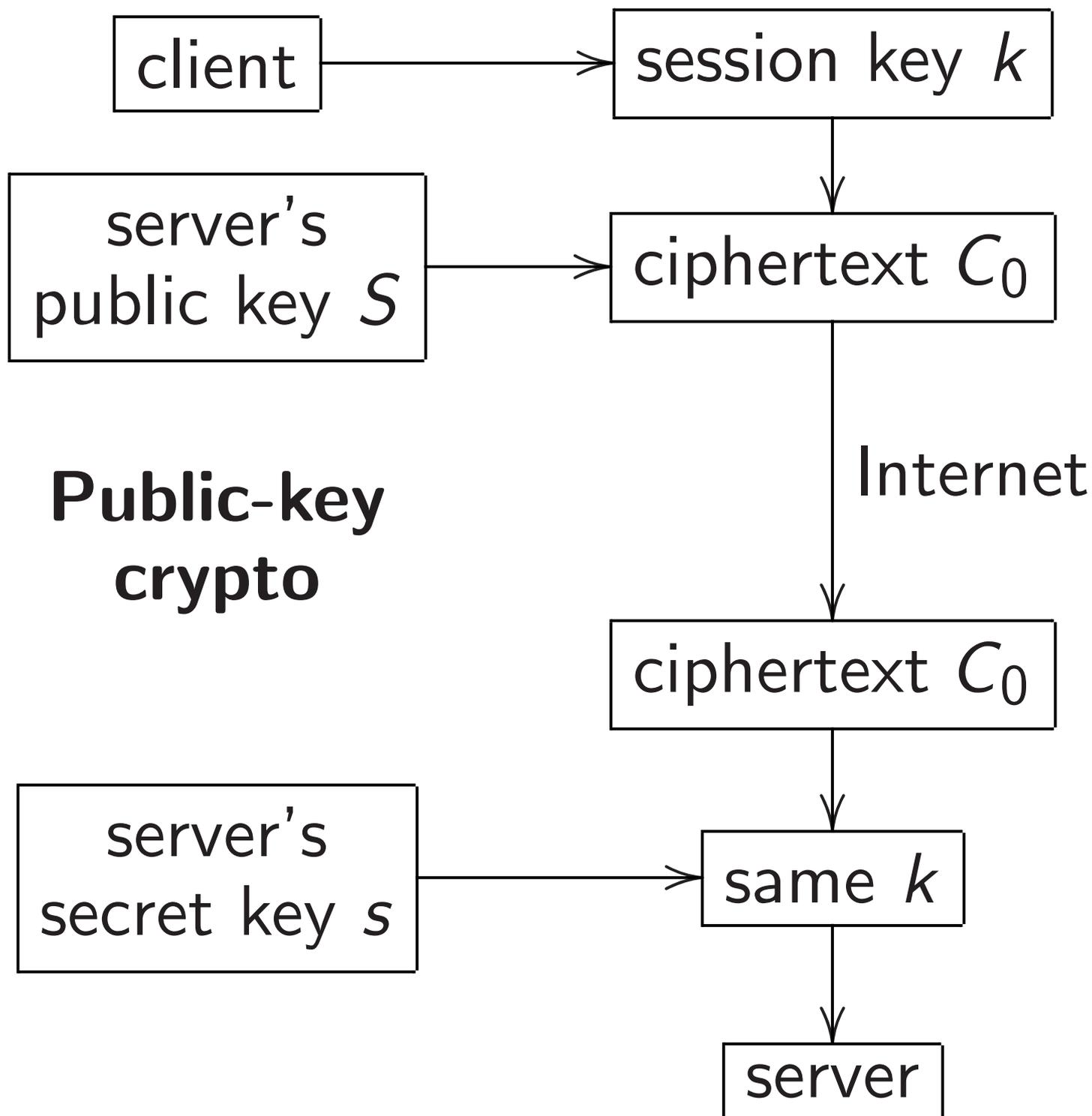
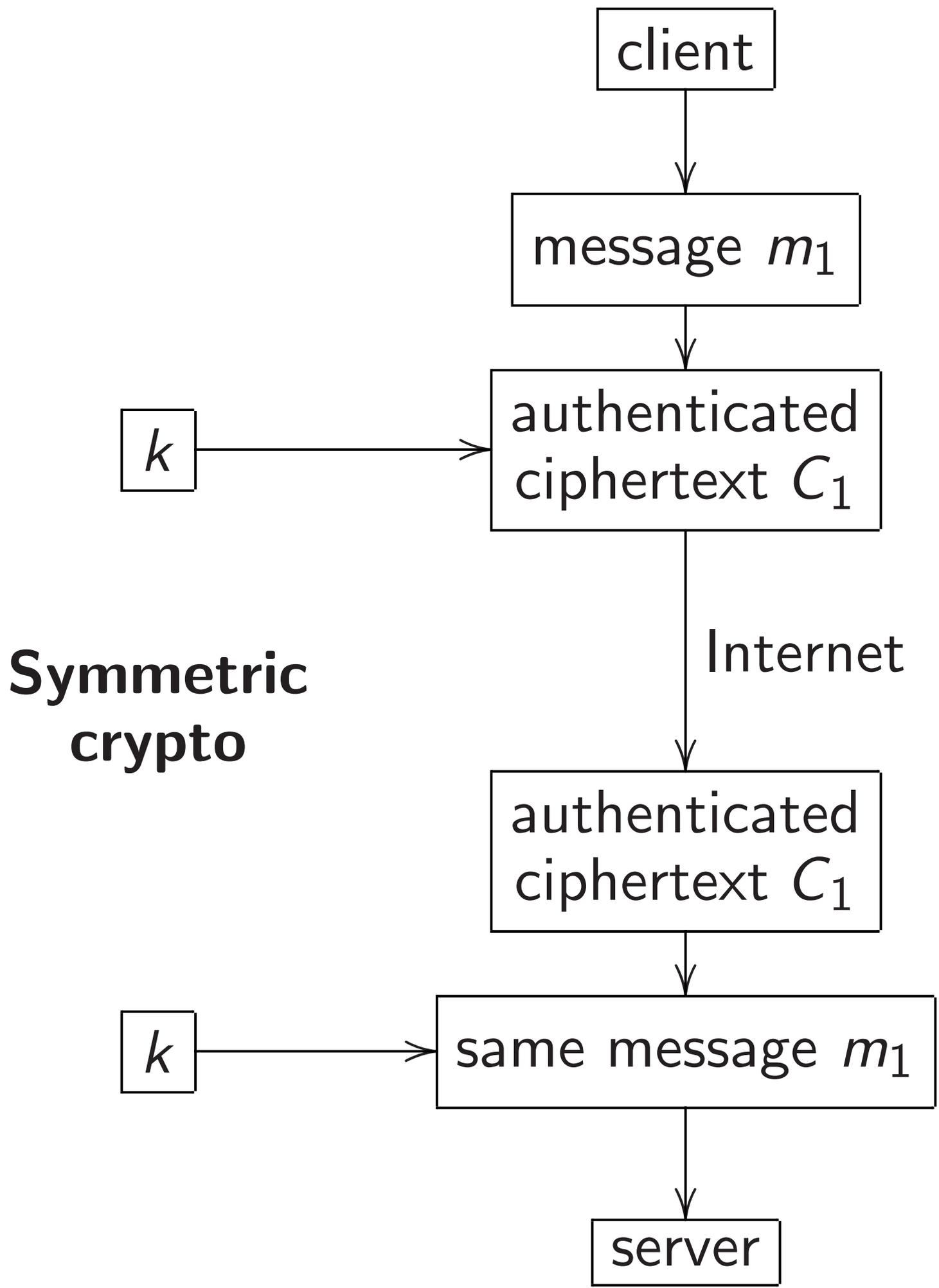
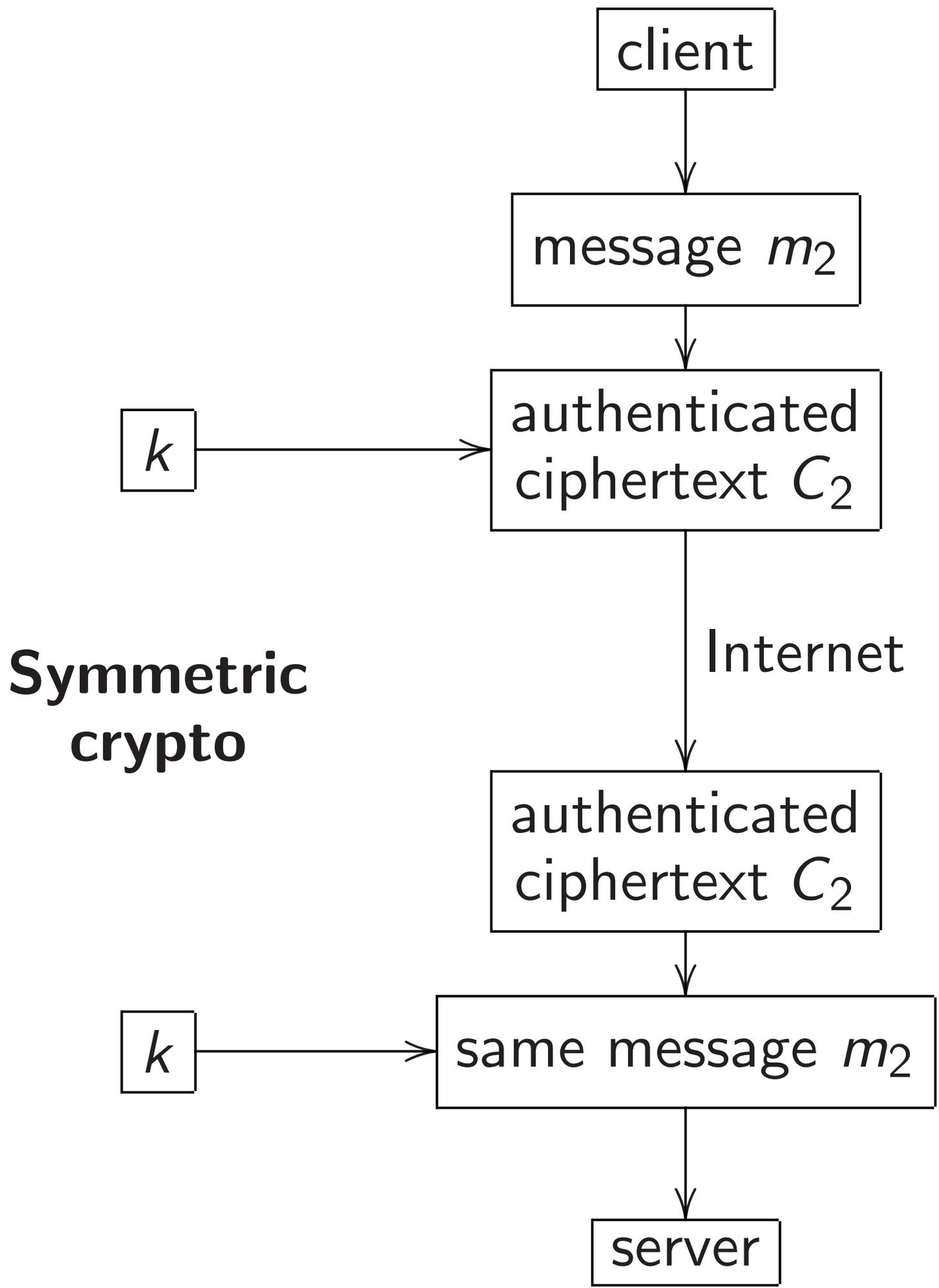


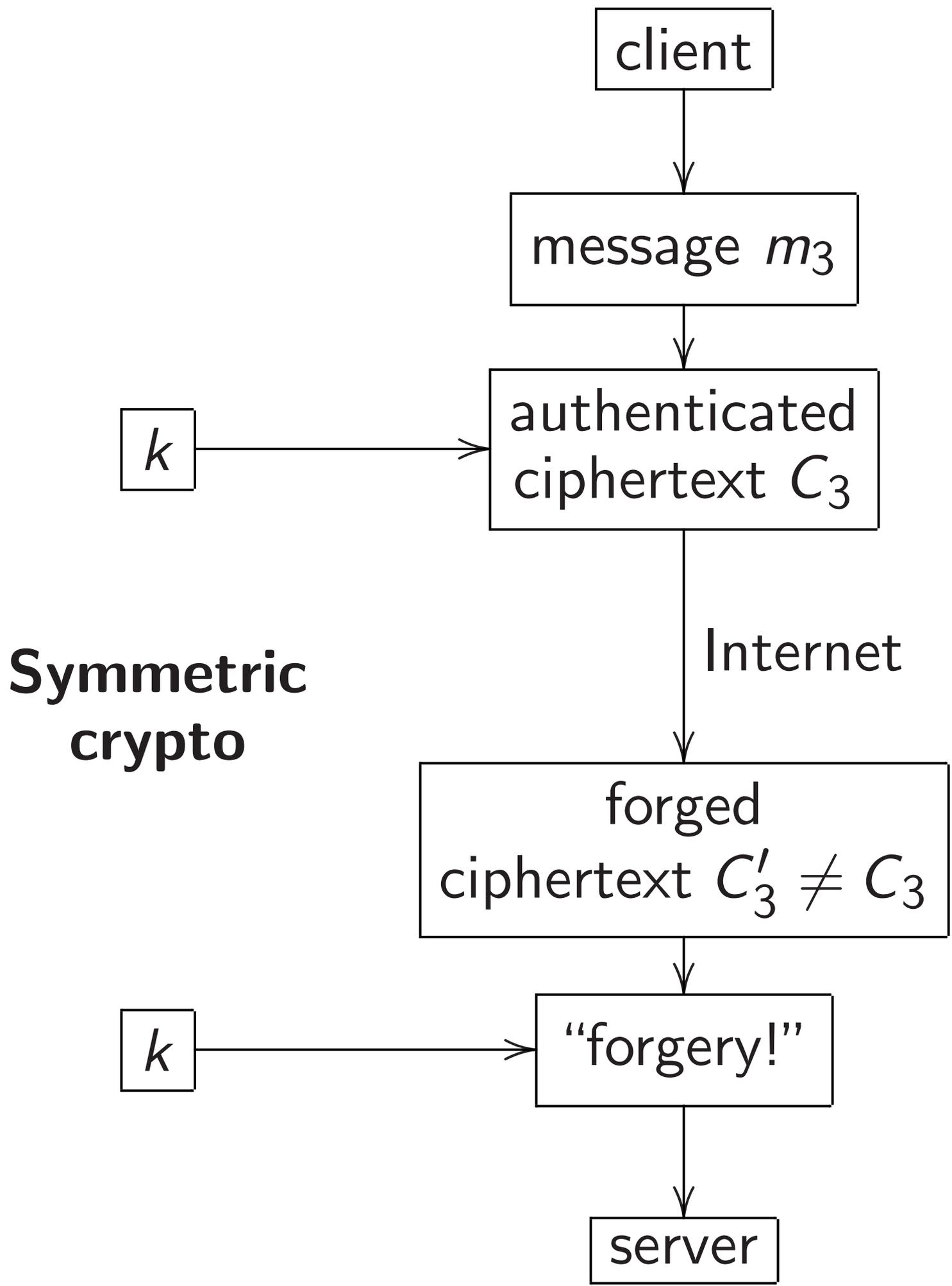
Symmetric crypto, part 2

D. J. Bernstein









Symmetric crypto: main objectives

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Attacker can't forge ciphertexts.

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Confidentiality: Attacker seeing ciphertexts can't figure out message contents. (But can see message number, length, timing.)

Can define further objectives.

Example: If crypto is too slow, attacker can flood server's CPU.

Real client messages are lost.

This damages **availability**.

Easy encryption mechanism:

Assume 30-digit messages.

Assume client, server know
secret 30-digit numbers

t_1 to use for message 1;

t_2 to use for message 2;

t_3 to use for message 3; etc.

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$$C_1 = (m_1 + t_1) \bmod 10^{30};$$

$$C_2 = (m_2 + t_2) \bmod 10^{30};$$

$$C_3 = (m_3 + t_3) \bmod 10^{30}; \text{ etc.}$$

This protects confidentiality.

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This protects confidentiality.

AES-GCM, ChaCha20-Poly1305

work this way, scaled up to

groups larger than $\mathbf{Z}/10^{30}$.

Last time: For each message
compute **authenticator**
using another secret number.

Sender attaches authenticator
to message before sending it.
Receiver checks authenticator.
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Details use multiplications.
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This would be the whole picture
if client, server started with
enough secret random numbers.

AES expands 256-bit secret k
into $F(k, 1), F(k, 2), F(k, 3), \dots$
simulating many independent
secrets r, s_1, t_1, \dots

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Definition of **PRG**

(“pseudorandom generator”):

Attacker can't distinguish

$F(k, 1), F(k, 2), F(k, 3), \dots$

from string of independent uniform random blocks.

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(“pseudorandom generator”):

Attacker can't distinguish

$F(k, 1), F(k, 2), F(k, 3), \dots$

from string of independent uniform random blocks.

Warning: “pseudorandom” has many other meanings.

PRF (“pseudorandom function”):

Attacker can't distinguish

$F(k, 1), F(k, 2), F(k, 3), \dots$ from

independent uniform random

blocks, given access to a server

that returns $F(k, i)$ given i .

Server is called an **oracle**.

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If block size is big then

$\text{PRP} \Rightarrow \text{PRF} \Rightarrow \text{PRG}$.

Small block sizes are dangerous.
PRF property fails, and often
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sweet32.info: Triple-DES
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AES block size: 128 bits.

PRF attack chance $\approx q^2 / 2^{129}$

if AES is used for q blocks.

Is this safe? How big is q ?

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ChaCha20 block size: 512 bits.

Can prove confidentiality and integrity of AES-GCM and ChaCha20-Poly1305 *assuming* AES and ChaCha20 are PRFs.

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Generalization: Prove security of $M(F)$ assuming cipher F is a PRF. M is a **mode of use** of F .

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Good modes: CTR (“counter mode”), CBC, OFB, many more.

Bad modes: ECB, many more.

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Generalization: Prove security of $M(F)$ assuming cipher F is a PRF. M is a **mode of use** of F .

Good modes: CTR (“counter mode”), CBC, OFB, many more.

Bad modes: ECB, many more.

Mode that claimed proof but was recently broken: OCB2.
Have to check proofs carefully!

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“All of these attacks fail and we don't have better attack ideas.”

Remaining slides today:

- Simple example of block cipher. Seems to be a good cipher, except block size is too small.
- Variants of this block cipher that look similar but can be quickly broken.

1994 Wheeler–Needham “TEA,
a tiny encryption algorithm”:

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

`uint32`: 32 bits $(b_0, b_1, \dots, b_{31})$
representing the “unsigned”
integer $b_0 + 2b_1 + \dots + 2^{31}b_{31}$.

`+`: addition mod 2^{32} .

`c += d`: same as `c = c + d`.

`^`: xor; \oplus ; addition of
each bit separately mod 2.

Lower precedence than `+` in C,
so spacing is not misleading.

`<<4`: multiplication by 16, i.e.,
 $(0, 0, 0, 0, b_0, b_1, \dots, b_{27})$.

`>>5`: division by 32, i.e.,
 $(b_5, b_6, \dots, b_{31}, 0, 0, 0, 0, 0)$.

Functionality

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Input: 128-bit key (namely $k[0], k[1], k[2], k[3]$);
64-bit **plaintext** ($b[0], b[1]$).

Output: 64-bit **ciphertext** (final $b[0], b[1]$).

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64-bit **plaintext** ($b[0], b[1]$).

Output: 64-bit **ciphertext** (final $b[0], b[1]$).

Can efficiently **encrypt**:
 $(\text{key}, \text{plaintext}) \mapsto \text{ciphertext}$.

Can efficiently **decrypt**:
 $(\text{key}, \text{ciphertext}) \mapsto \text{plaintext}$.

Wait, how can we decrypt?

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y+c ^ (y<<4)+k[0]
                ^ (y>>5)+k[1];
        y += x+c ^ (x<<4)+k[2]
                ^ (x>>5)+k[3];
    }
    b[0] = x; b[1] = y;
}
```

Answer: Each step is invertible.

```
void decrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 32 * 0x9e3779b9;
    for (r = 0; r < 32; r += 1) {
        y -= x+c ^ (x<<4)+k[2]
                ^ (x>>5)+k[3];
        x -= y+c ^ (y<<4)+k[0]
                ^ (y>>5)+k[1];
        c -= 0x9e3779b9;
    }
    b[0] = x; b[1] = y;
}
```

Generalization, **Feistel network**
(used in, e.g., “Lucifer” from
1973 Feistel–Coppersmith):

```
x += function1(y,k);  
y += function2(x,k);  
x += function3(y,k);  
y += function4(x,k);  
...
```

Decryption, inverting each step:

```
...  
y -= function4(x,k);  
x -= function3(y,k);  
y -= function2(x,k);  
x -= function1(y,k);
```

TEA again for comparison

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

XORTEA: a bad cipher

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x ^= y ^ c ^ (y << 4) ^ k[0]
                ^ (y >> 5) ^ k[1];
        y ^= x ^ c ^ (x << 4) ^ k[2]
                ^ (x >> 5) ^ k[3];
    }
    b[0] = x; b[1] = y;
}
```

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But output bits are linear functions of input bits!

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e.g. First output bit is

$$\begin{aligned}
 &1 \oplus k_0 \oplus k_1 \oplus k_3 \oplus k_{10} \oplus k_{11} \oplus k_{12} \oplus \\
 &k_{20} \oplus k_{21} \oplus k_{30} \oplus k_{32} \oplus k_{33} \oplus k_{35} \oplus \\
 &k_{42} \oplus k_{43} \oplus k_{44} \oplus k_{52} \oplus k_{53} \oplus k_{62} \oplus \\
 &k_{64} \oplus k_{67} \oplus k_{69} \oplus k_{76} \oplus k_{85} \oplus k_{94} \oplus \\
 &k_{96} \oplus k_{99} \oplus k_{101} \oplus k_{108} \oplus k_{117} \oplus k_{126} \oplus \\
 &b_1 \oplus b_3 \oplus b_{10} \oplus b_{12} \oplus b_{21} \oplus b_{30} \oplus b_{32} \oplus \\
 &b_{33} \oplus b_{35} \oplus b_{37} \oplus b_{39} \oplus b_{42} \oplus b_{43} \oplus \\
 &b_{44} \oplus b_{47} \oplus b_{52} \oplus b_{53} \oplus b_{57} \oplus b_{62}.
 \end{aligned}$$

There is a matrix M
with coefficients in \mathbf{F}_2
such that, for all (k, b) ,
 $\text{XORTEA}_k(b) = (1, k, b)M$.

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$$\begin{aligned} \text{XORTEA}_k(b_1) \oplus \text{XORTEA}_k(b_2) \\ = (0, 0, b_1 \oplus b_2)M. \end{aligned}$$

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Very fast attack:

if $b_4 = b_1 \oplus b_2 \oplus b_3$ then

$$\begin{aligned} \text{XORTEA}_k(b_1) \oplus \text{XORTEA}_k(b_2) = \\ \text{XORTEA}_k(b_3) \oplus \text{XORTEA}_k(b_4). \end{aligned}$$

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This breaks PRP (and PRF):

uniform random permutation

(or function) F almost never has

$$F(b_1) \oplus F(b_2) = F(b_3) \oplus F(b_4).$$

TEA again for comparison

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

LEFTEA: another bad cipher

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y+c ^ (y<<4)+k[0]
                ^ (y<<5)+k[1];
        y += x+c ^ (x<<4)+k[2]
                ^ (x<<5)+k[3];
    }
    b[0] = x; b[1] = y;
}
```

Addition is not \mathbf{F}_2 -linear,
but addition mod 2 is \mathbf{F}_2 -linear.

First output bit is

$$1 \oplus k_0 \oplus k_{32} \oplus k_{64} \oplus k_{96} \oplus b_{32}.$$

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How TEA avoids this problem:
>>5 **diffuses** nonlinear changes
from high bits to low bits.

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Higher output bits
are increasingly nonlinear
but they never affect first bit.

How TEA avoids this problem:
>>5 **diffuses** nonlinear changes
from high bits to low bits.

(Diffusion from low bits to high
bits: <<4; carries in addition.)

TEA again for comparison

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

TEA4: another bad cipher

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 4; r += 1) {
        c += 0x9e3779b9;
        x += y+c ^ (y<<4)+k[0]
                ^ (y>>5)+k[1];
        y += x+c ^ (x<<4)+k[2]
                ^ (x>>5)+k[3];
    }
    b[0] = x; b[1] = y;
}
```

Fast attack:

$\text{TEA4}_k(x + 2^{31}, y)$ and

$\text{TEA4}_k(x, y)$ have same first bit.

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$\text{TEA4}_k(x, y)$ have same first bit.

Trace x, y differences

through steps in computation.

$r = 0$: multiples of $2^{31}, 2^{26}$.

$r = 1$: multiples of $2^{21}, 2^{16}$.

$r = 2$: multiples of $2^{11}, 2^6$.

$r = 3$: multiples of $2^1, 2^0$.

Fast attack:

$\text{TEA4}_k(x + 2^{31}, y)$ and

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Trace x, y differences

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$r = 0$: multiples of $2^{31}, 2^{26}$.

$r = 1$: multiples of $2^{21}, 2^{16}$.

$r = 2$: multiples of $2^{11}, 2^6$.

$r = 3$: multiples of $2^1, 2^0$.

Uniform random function F :

$F(x + 2^{31}, y)$ and $F(x, y)$ have

same first bit with probability $1/2$.

Fast attack:

$\text{TEA4}_k(x + 2^{31}, y)$ and

$\text{TEA4}_k(x, y)$ have same first bit.

Trace x, y differences

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$r = 2$: multiples of $2^{11}, 2^6$.

$r = 3$: multiples of $2^1, 2^0$.

Uniform random function F :

$F(x + 2^{31}, y)$ and $F(x, y)$ have

same first bit with probability $1/2$.

PRF advantage $1/2$.

Two pairs (x, y) : advantage $3/4$.

More sophisticated attacks:
trace *probabilities* of differences;
probabilities of linear equations;
probabilities of higher-order
differences $C(x + \delta + \epsilon) -$
 $C(x + \delta) - C(x + \epsilon) + C(x)$; etc.
Use algebra+statistics to exploit
non-randomness in probabilities.

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Attacks get beyond $r = 4$
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Very far from full TEA.

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but rapidly lose effectiveness.

Very far from full TEA.

Hard question in cipher design:
How many “rounds” are
really needed for security?

TEA again for comparison

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

REPTEA: another bad cipher

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0x9e3779b9;
    for (r = 0; r < 1000; r += 1) {
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

$$\text{REPTEA}_k(b) = I_k^{1000}(b)$$

where I_k does $x+=\dots; y+=\dots$

$$\text{REPTEA}_k(b) = I_k^{1000}(b)$$

where I_k does $x += \dots ; y += \dots$

Try list of 2^{32} inputs b .

Collect outputs $\text{REPTEA}_k(b)$.

$$\text{REPTEA}_k(b) = I_k^{1000}(b)$$

where I_k does $x += \dots ; y += \dots$

Try list of 2^{32} inputs b .

Collect outputs $\text{REPTEA}_k(b)$.

Good chance that some b in list also has $a = I_k(b)$ in list. Then

$$\text{REPTEA}_k(a) = I_k(\text{REPTEA}_k(b)).$$

$$\text{REPTEA}_k(b) = I_k^{1000}(b)$$

where I_k does $x += \dots ; y += \dots$

Try list of 2^{32} inputs b .

Collect outputs $\text{REPTEA}_k(b)$.

Good chance that some b in list also has $a = I_k(b)$ in list. Then

$$\text{REPTEA}_k(a) = I_k(\text{REPTEA}_k(b)).$$

For each (b, a) from list:

Try solving equations $a = I_k(b)$,

$$\text{REPTEA}_k(a) = I_k(\text{REPTEA}_k(b))$$

to figure out k . (More equations: try re-encrypting these outputs.)

$$\text{REPTEA}_k(b) = I_k^{1000}(b)$$

where I_k does $x += \dots; y += \dots$

Try list of 2^{32} inputs b .

Collect outputs $\text{REPTEA}_k(b)$.

Good chance that some b in list also has $a = I_k(b)$ in list. Then

$$\text{REPTEA}_k(a) = I_k(\text{REPTEA}_k(b)).$$

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This is a **slide attack**.

TEA avoids this by varying c .

What about original TEA?

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

Related keys: e.g.,

$$\text{TEA}_{k'}(b) = \text{TEA}_k(b)$$

where $(k'[0], k'[1], k'[2], k'[3]) = (k[0] + 2^{31}, k[1] + 2^{31}, k[2], k[3])$.

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Related keys $\Rightarrow g$ succeeds with chance 2^{-126} . Still very small.

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But advertised as “related-key cryptanalysis” and claimed to justify recommendations for designers regarding key scheduling.

Some ways to learn more about cipher attacks, hash-function attacks, etc.:

Take upcoming course “Selected areas in cryptology”. Includes symmetric attacks.

Read attack papers, especially from FSE conference.

Try to break ciphers yourself: e.g., find attacks on FEAL.

Reasonable starting point: 2000 Schneier “Self-study course in block-cipher cryptanalysis”.