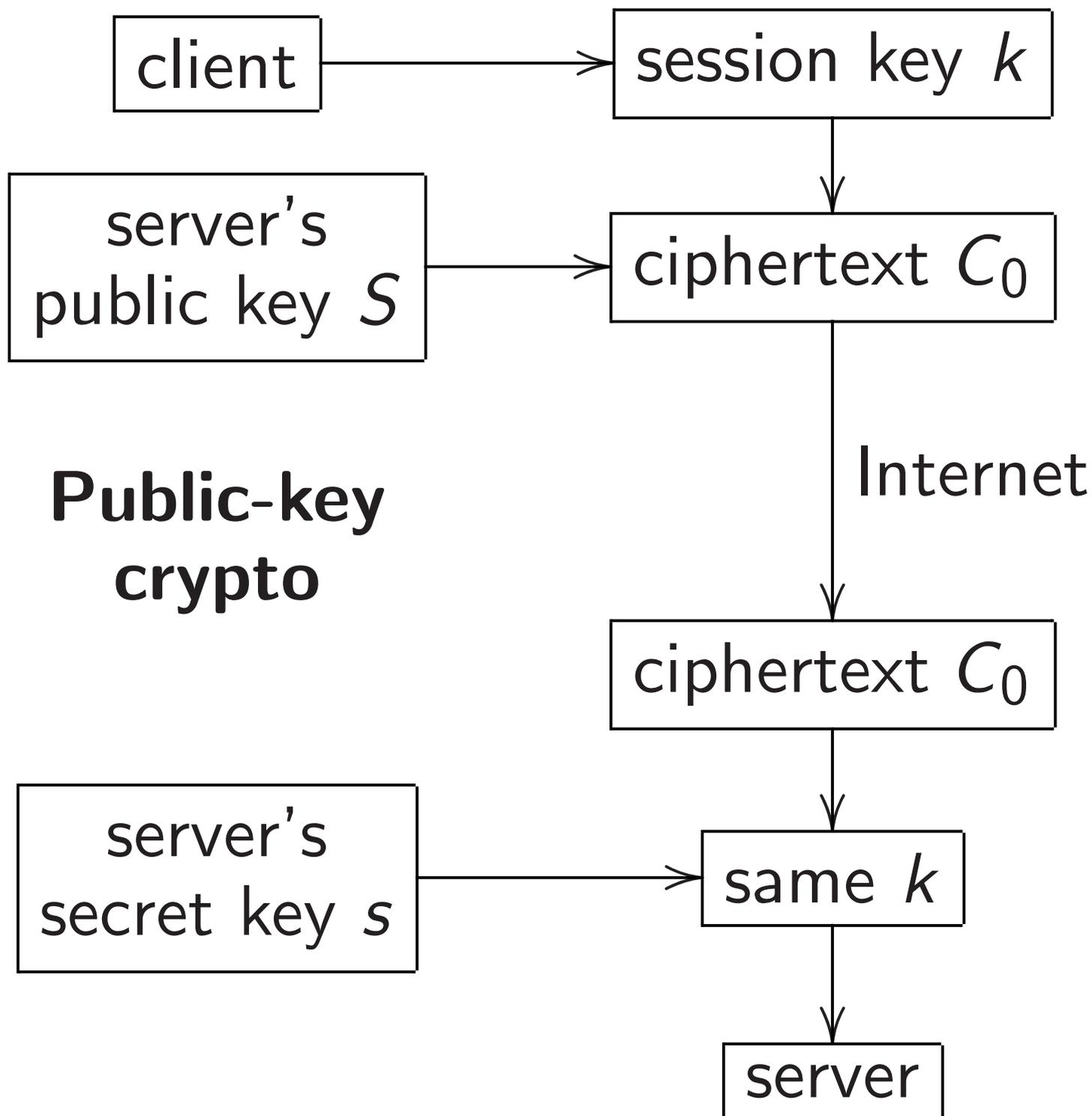
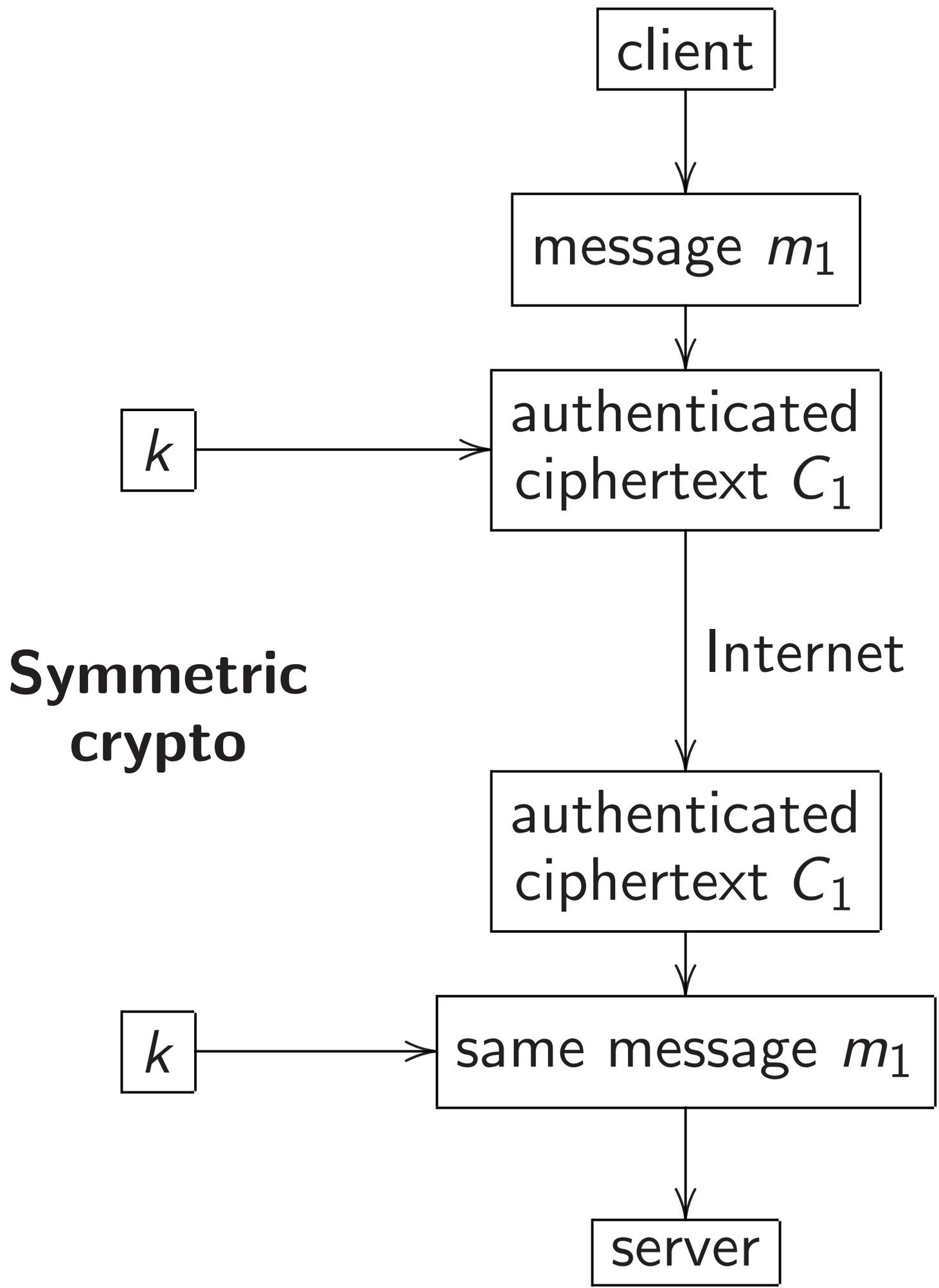


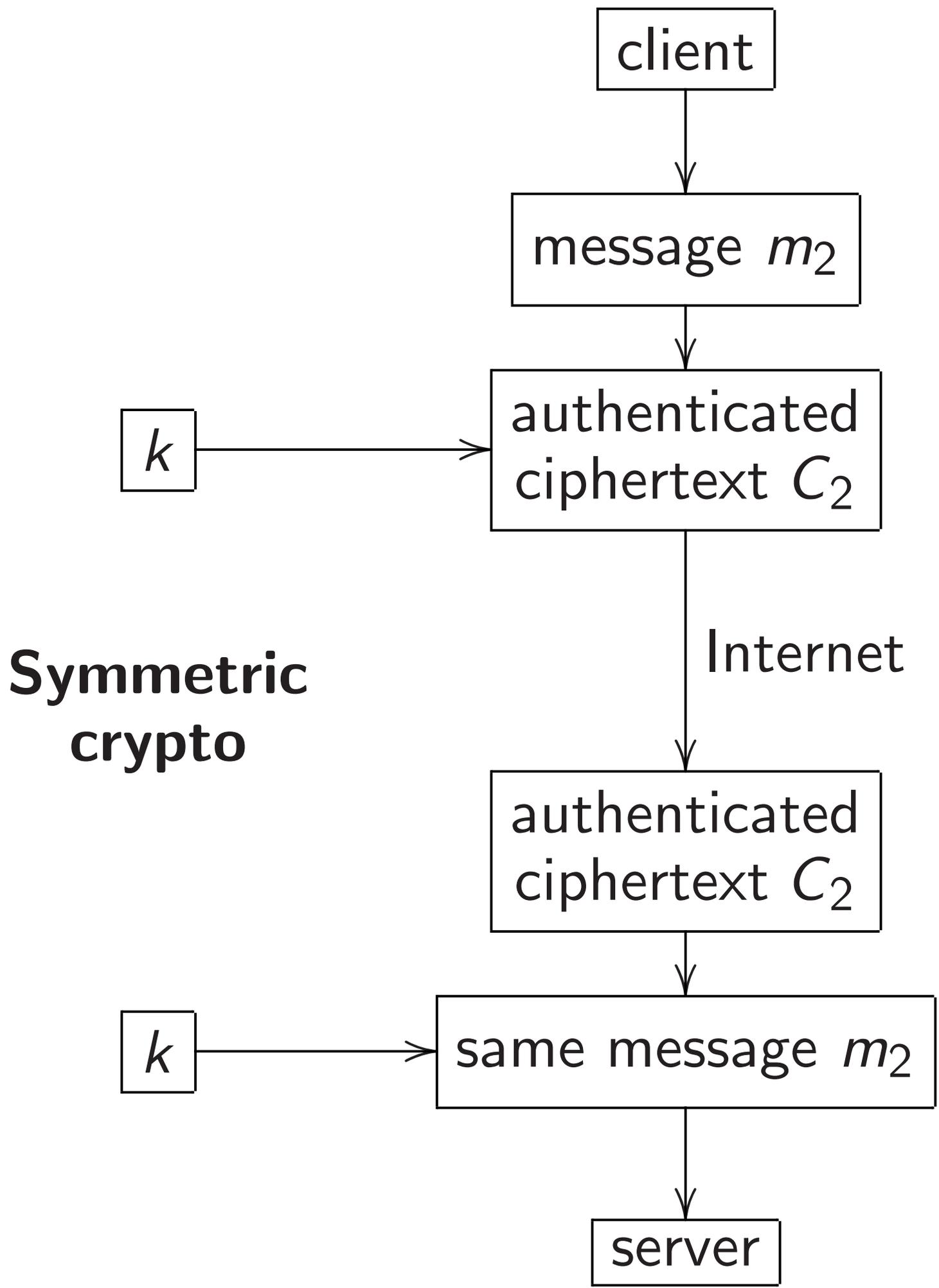
# Symmetric crypto, part 2

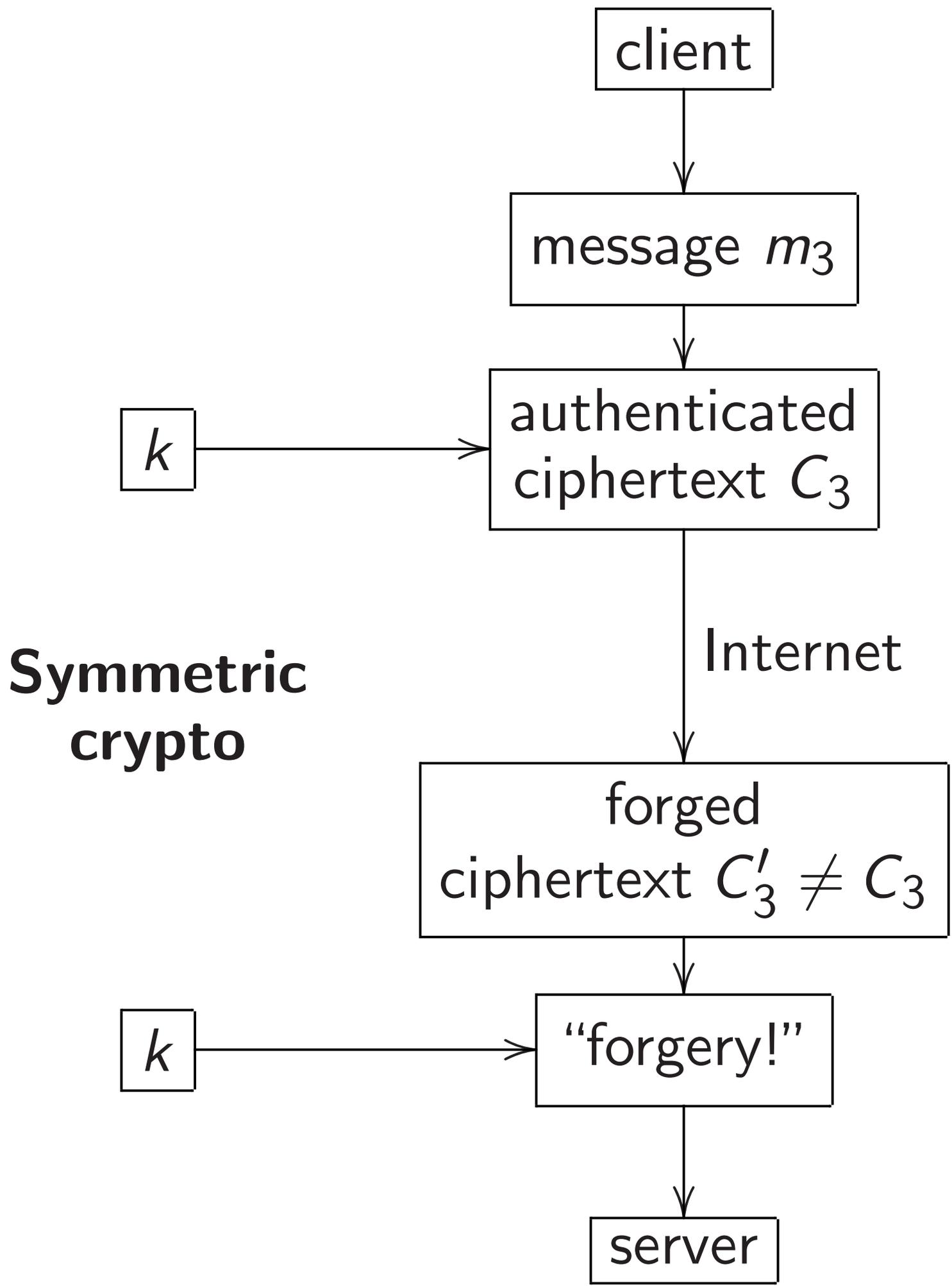
D. J. Bernstein

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# Symmetric crypto: main objectives

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Attacker can't forge ciphertexts.

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Can define further objectives.

Example: If crypto is too slow, attacker can flood server's CPU.

Real client messages are lost.

This damages **availability**.

Easy encryption mechanism:

Assume 30-digit messages.

Assume client, server know  
secret 30-digit numbers

$t_1$  to use for message 1;

$t_2$  to use for message 2;

$t_3$  to use for message 3; etc.

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$$C_1 = (m_1 + t_1) \bmod 10^{30};$$

$$C_2 = (m_2 + t_2) \bmod 10^{30};$$

$$C_3 = (m_3 + t_3) \bmod 10^{30}; \text{ etc.}$$

This protects confidentiality.

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This protects confidentiality.

AES-GCM, ChaCha20-Poly1305

work this way, scaled up to

groups larger than  $\mathbf{Z}/10^{30}$ .

Last time: For each message  
compute **authenticator**  
using another secret number.

Sender attaches authenticator  
to message before sending it.  
Receiver checks authenticator.  
This protects integrity.

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AES-GCM, ChaCha20-Poly1305  
work this way, again scaled up.

This would be the whole picture  
*if* client, server started with  
enough secret random numbers.

AES expands 256-bit secret  $k$   
into  $F(k, 1), F(k, 2), F(k, 3), \dots$   
simulating many independent  
secrets  $r, s_1, t_1, \dots$

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## Definition of **PRG**

(“pseudorandom generator”):

Attacker can't distinguish

$F(k, 1), F(k, 2), F(k, 3), \dots$

from string of independent uniform random blocks.

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Attacker can't distinguish

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from string of independent uniform random blocks.

Warning: “pseudorandom” has many other meanings.

**PRF** (“pseudorandom function”):

Attacker can't distinguish

$F(k, 1), F(k, 2), F(k, 3), \dots$  from

independent uniform random

blocks, given access to a server

that returns  $F(k, i)$  given  $i$ .

Server is called an **oracle**.

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Attacker can't distinguish

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from independent uniform random

**distinct** blocks, given oracle.

If block size is big then

$\text{PRP} \Rightarrow \text{PRF} \Rightarrow \text{PRG}$ .

Small block sizes are dangerous.  
PRF property fails, and often  
application security fails.

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e.g. 2016 Bhargavan–Leurent  
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AES block size: 128 bits.

PRF attack chance  $\approx q^2 / 2^{129}$

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ChaCha20 block size: 512 bits.

Can prove confidentiality and integrity of AES-GCM and ChaCha20-Poly1305 *assuming* AES and ChaCha20 are PRFs.

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Generalization: Prove security of  $M(F)$  assuming cipher  $F$  is a PRF.  $M$  is a **mode of use** of  $F$ .

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Good modes: CTR (“counter mode”), CBC, OFB, many more.

Bad modes: ECB, many more.

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Generalization: Prove security of  $M(F)$  assuming cipher  $F$  is a PRF.  $M$  is a **mode of use** of  $F$ .

Good modes: CTR (“counter mode”), CBC, OFB, many more.

Bad modes: ECB, many more.

Mode that claimed proof but was recently broken: OCB2.  
Have to check proofs carefully!

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Remaining slides today:

- Simple example of block cipher. Seems to be a good cipher, except block size is too small.
- Variants of this block cipher that look similar but can be quickly broken.

1994 Wheeler–Needham “TEA,  
a tiny encryption algorithm”:

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

`uint32`: 32 bits  $(b_0, b_1, \dots, b_{31})$   
representing the “unsigned”  
integer  $b_0 + 2b_1 + \dots + 2^{31}b_{31}$ .

`+`: addition mod  $2^{32}$ .

`c += d`: same as `c = c + d`.

`^`: xor;  $\oplus$ ; addition of  
each bit separately mod 2.

Lower precedence than `+` in C,  
so spacing is not misleading.

`<<4`: multiplication by 16, i.e.,  
 $(0, 0, 0, 0, b_0, b_1, \dots, b_{27})$ .

`>>5`: division by 32, i.e.,  
 $(b_5, b_6, \dots, b_{31}, 0, 0, 0, 0, 0)$ .

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Input: 128-bit key (namely  $k[0], k[1], k[2], k[3]$ );  
64-bit **plaintext** ( $b[0], b[1]$ ).

Output: 64-bit **ciphertext** (final  $b[0], b[1]$ ).

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64-bit **plaintext** ( $b[0], b[1]$ ).

Output: 64-bit **ciphertext**  
(final  $b[0], b[1]$ ).

Can efficiently **encrypt**:  
 $(\text{key}, \text{plaintext}) \mapsto \text{ciphertext}$ .

Can efficiently **decrypt**:  
 $(\text{key}, \text{ciphertext}) \mapsto \text{plaintext}$ .

Wait, how can we decrypt?

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y+c ^ (y<<4)+k[0]
                ^ (y>>5)+k[1];
        y += x+c ^ (x<<4)+k[2]
                ^ (x>>5)+k[3];
    }
    b[0] = x; b[1] = y;
}
```

Answer: Each step is invertible.

```
void decrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 32 * 0x9e3779b9;
    for (r = 0; r < 32; r += 1) {
        y -= x+c ^ (x<<4)+k[2]
                ^ (x>>5)+k[3];
        x -= y+c ^ (y<<4)+k[0]
                ^ (y>>5)+k[1];
        c -= 0x9e3779b9;
    }
    b[0] = x; b[1] = y;
}
```

Generalization, **Feistel network**  
(used in, e.g., “Lucifer” from  
1973 Feistel–Coppersmith):

```
x += function1(y,k);  
y += function2(x,k);  
x += function3(y,k);  
y += function4(x,k);  
...
```

Decryption, inverting each step:

```
...  
y -= function4(x,k);  
x -= function3(y,k);  
y -= function2(x,k);  
x -= function1(y,k);
```

## TEA again for comparison

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
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    }
    b[0] = x; b[1] = y;
}
```

## XORTEA: a bad cipher

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x ^= y ^ c ^ (y << 4) ^ k[0]
                ^ (y >> 5) ^ k[1];
        y ^= x ^ c ^ (x << 4) ^ k[2]
                ^ (x >> 5) ^ k[3];
    }
    b[0] = x; b[1] = y;
}
```

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But output bits are linear functions of input bits!

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But output bits are linear functions of input bits!

e.g. First output bit is

$$\begin{aligned}
 &1 \oplus k_0 \oplus k_1 \oplus k_3 \oplus k_{10} \oplus k_{11} \oplus k_{12} \oplus \\
 &k_{20} \oplus k_{21} \oplus k_{30} \oplus k_{32} \oplus k_{33} \oplus k_{35} \oplus \\
 &k_{42} \oplus k_{43} \oplus k_{44} \oplus k_{52} \oplus k_{53} \oplus k_{62} \oplus \\
 &k_{64} \oplus k_{67} \oplus k_{69} \oplus k_{76} \oplus k_{85} \oplus k_{94} \oplus \\
 &k_{96} \oplus k_{99} \oplus k_{101} \oplus k_{108} \oplus k_{117} \oplus k_{126} \oplus \\
 &b_1 \oplus b_3 \oplus b_{10} \oplus b_{12} \oplus b_{21} \oplus b_{30} \oplus b_{32} \oplus \\
 &b_{33} \oplus b_{35} \oplus b_{37} \oplus b_{39} \oplus b_{42} \oplus b_{43} \oplus \\
 &b_{44} \oplus b_{47} \oplus b_{52} \oplus b_{53} \oplus b_{57} \oplus b_{62}.
 \end{aligned}$$

There is a matrix  $M$   
with coefficients in  $\mathbf{F}_2$   
such that, for all  $(k, b)$ ,  
 $\text{XORTEA}_k(b) = (1, k, b)M$ .

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$$\begin{aligned} \text{XORTEA}_k(b_1) \oplus \text{XORTEA}_k(b_2) \\ = (0, 0, b_1 \oplus b_2)M. \end{aligned}$$

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$$\text{XORTEA}_k(b_1) \oplus \text{XORTEA}_k(b_2) \\ = (0, 0, b_1 \oplus b_2)M.$$

Very fast attack:

if  $b_4 = b_1 \oplus b_2 \oplus b_3$  then

$$\text{XORTEA}_k(b_1) \oplus \text{XORTEA}_k(b_2) = \\ \text{XORTEA}_k(b_3) \oplus \text{XORTEA}_k(b_4).$$

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This breaks PRP (and PRF):

uniform random permutation

(or function)  $F$  almost never has

$$F(b_1) \oplus F(b_2) = F(b_3) \oplus F(b_4).$$

## TEA again for comparison

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

## LEFTEA: another bad cipher

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
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                ^ (y<<5)+k[1];
        y += x+c ^ (x<<4)+k[2]
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    }
    b[0] = x; b[1] = y;
}
```

Addition is not  $\mathbf{F}_2$ -linear,  
but addition mod 2 is  $\mathbf{F}_2$ -linear.

First output bit is

$$1 \oplus k_0 \oplus k_{32} \oplus k_{64} \oplus k_{96} \oplus b_{32}.$$

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How TEA avoids this problem:

$\gg 5$  **diffuses** nonlinear changes  
from high bits to low bits.

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are increasingly nonlinear  
but they never affect first bit.

How TEA avoids this problem:  
>>5 **diffuses** nonlinear changes  
from high bits to low bits.

(Diffusion from low bits to high  
bits: <<4; carries in addition.)

## TEA again for comparison

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

## TEA4: another bad cipher

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 4; r += 1) {
        c += 0x9e3779b9;
        x += y+c ^ (y<<4)+k[0]
                ^ (y>>5)+k[1];
        y += x+c ^ (x<<4)+k[2]
                ^ (x>>5)+k[3];
    }
    b[0] = x; b[1] = y;
}
```

Fast attack:

$\text{TEA4}_k(x + 2^{31}, y)$  and

$\text{TEA4}_k(x, y)$  have same first bit.

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$\text{TEA4}_k(x, y)$  have same first bit.

Trace  $x, y$  differences

through steps in computation.

$r = 0$ : multiples of  $2^{31}, 2^{26}$ .

$r = 1$ : multiples of  $2^{21}, 2^{16}$ .

$r = 2$ : multiples of  $2^{11}, 2^6$ .

$r = 3$ : multiples of  $2^1, 2^0$ .

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Uniform random function  $F$ :

$F(x + 2^{31}, y)$  and  $F(x, y)$  have

same first bit with probability  $1/2$ .

PRF advantage  $1/2$ .

Two pairs  $(x, y)$ : advantage  $3/4$ .

More sophisticated attacks:  
trace *probabilities* of differences;  
probabilities of linear equations;  
probabilities of higher-order  
differences  $C(x + \delta + \epsilon) -$   
 $C(x + \delta) - C(x + \epsilon) + C(x)$ ; etc.  
Use algebra+statistics to exploit  
non-randomness in probabilities.

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Attacks get beyond  $r = 4$   
but rapidly lose effectiveness.

Very far from full TEA.

Hard question in cipher design:  
How many “rounds” are  
really needed for security?

## TEA again for comparison

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y+c ^ (y<<4)+k[0]
                ^ (y>>5)+k[1];
        y += x+c ^ (x<<4)+k[2]
                ^ (x>>5)+k[3];
    }
    b[0] = x; b[1] = y;
}
```

## REPTEA: another bad cipher

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0x9e3779b9;
    for (r = 0; r < 1000; r += 1) {
        x += y+c ^ (y<<4)+k[0]
                ^ (y>>5)+k[1];
        y += x+c ^ (x<<4)+k[2]
                ^ (x>>5)+k[3];
    }
    b[0] = x; b[1] = y;
}
```

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where  $I_k$  does  $x+=\dots; y+=\dots$

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Good chance that some  $b$  in list also has  $a = I_k(b)$  in list. Then

$$\text{REPTEA}_k(a) = I_k(\text{REPTEA}_k(b)).$$

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This is a **slide attack**.

TEA avoids this by varying  $c$ .

## What about original TEA?

```
void encrypt(uint32 *b, uint32 *k)
{
    uint32 x = b[0], y = b[1];
    uint32 r, c = 0;
    for (r = 0; r < 32; r += 1) {
        c += 0x9e3779b9;
        x += y + c ^ (y << 4) + k[0]
                ^ (y >> 5) + k[1];
        y += x + c ^ (x << 4) + k[2]
                ^ (x >> 5) + k[3];
    }
    b[0] = x; b[1] = y;
}
```

Related keys: e.g.,

$$\text{TEA}_{k'}(b) = \text{TEA}_k(b)$$

where  $(k'[0], k'[1], k'[2], k'[3]) = (k[0] + 2^{31}, k[1] + 2^{31}, k[2], k[3])$ .

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Related keys  $\Rightarrow g$  succeeds with chance  $2^{-126}$ . Still very small.

1997 Kelsey–Schneier–Wagner:

Fancier relationship between  $k$ ,  $k'$   
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But advertised as “related-key cryptanalysis” and claimed to justify recommendations for designers regarding key scheduling.

Some ways to learn more about cipher attacks, hash-function attacks, etc.:

Take upcoming course “Selected areas in cryptology”. Includes symmetric attacks.

Read attack papers, especially from FSE conference.

Try to break ciphers yourself: e.g., find attacks on FEAL.

Reasonable starting point: 2000 Schneier “Self-study course in block-cipher cryptanalysis”.