Comparing proofs of security for lattice-based encryption

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Primary objective of this paper: Make a **complete plan** for **thorough security reviews** of 36 target KEMs.

Much harder: Do the reviews! Complete plan is framework to evaluate which pieces are done, and to coordinate further efforts. KEMs vary in what’s needed.
The target KEMs (all proposed for wide deployment, IND-CCA2):

- frodo 640, 976, 1344.
- kyber 512, 768, 1024.
- lac 128, 192, 256.
- newhope 512, 1024.
- ntru hps2048509, hps2048677, hps4096821, hrss701.
- ntrulpr 653, 761, 857.
- round5n1 1, 3, 5.
- round5nd 1.0d, 3.0d, 5.0d, 1.5d, 3.5d, 5.5d.
- saber light, main, fire.
- sntrup 653, 761, 857.
- threebears baby, mama, papa.
One categorization of the KEMs:

- frodo: Product NTRU.
- kyber: Product NTRU.
- lac: Product NTRU.
- newhope: Product NTRU.
- ntru: Quotient NTRU.
- ntrulpr: Product NTRU.
- round5n1: Product NTRU.
- round5nd: Product NTRU.
- saber: Product NTRU.
- sntrup: Quotient NTRU.
- threebears: Product NTRU.
An oversimplified plan

Plan: Verify the security proofs—make sure there are no mistakes.
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I did some sanity checks (tiny part of full verification!) and found unproven theorems claimed by frodo, round5n1, round5nd, saber; also wrong hypotheses for newhope theorem.
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Backup strategies: Clean up proofs. Check proofs by hand. Track bug categories, as in code.
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Revised plan:
1. Verify the “security proofs”.
2. Verify the cryptanalysis of the risks left by the proofs.

Again clean up; check by hand; track failure categories.
Are attack-cost analyses correct? How thorough is exploration of space of optimizations? How thorough is the study of claimed barriers to speedups that work for similar problems? Do the cryptanalytic targets match the proof risks? etc.

Long history of failures: e.g., NSA overstated DES attack cost; $L(1/2)$ optimality conjecture for factorization was wrong; TLS Triple-DES-CBC was broken without Triple-DES attack; etc.
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This paper’s verification plan skips proofs that clearly fail to reduce cost of cryptanalysis: e.g., frodo seed “reduction”.
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Risk #4: Proof is incorrect.
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Attack OW-Passive ("OW-CPA") security of the 36 core PKEs.
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For all targets: KEM proofs allow non-ROM attacks.
The core PKEs ("P")

Key generation:

- Table 8.6: Public multiplier $G$.
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More for Product NTRU:
- Table 8.9: Public $C \approx Ab + M$.
- Table 8.10: Secret $M$. 
OW-Passive vs. IND-CPA ("dist")

Quotient NTRU (ntru, sntrup) asks for OW-Passive cryptanalysis. 2003 Naor: this is "falsifiable".
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Compare 2006 Goldreich: “What concerns us about” DDH is that “DDH is less simple than DH” making it “harder to evaluate.”
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`frodo` seed “reduction”: Useless. Still need non-ROM cryptanalysis.
More hashing (“ROM”)

Want the target KEMs to provide IND-CCA2 security.

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Decryption failures ("fail"/"conj")

2017 Hofheinz–Hövelmanns–Kiltz proofs do not rule out ROM IND-CCA2 attacks with probability \( Q\delta \), even if the PKEs are secure.

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frodo640, kyber512 prove $\delta \leq 2^{-128}$ with security goal $2^{128}$.

frodo976 proves $\delta \leq 2^{-192}$. 
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- 14 KEMs do not claim that $\delta$ is small enough.
- 15 KEMs conjecture $\delta \leq \cdots$ without claiming proof.
- 5 KEMs have proofs but do not clearly use correct $\delta$ definition. (LEDAD uses wrong definition.)
What about quantum attacks?

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(3) Non-QROM attacks.

Sometimes proofs eliminate #1. Ongoing efforts to extend proofs to similarly eliminate #2. Most QROM proofs are loose, but see 2019 Bindel–Hamburg–Hülsing–Persichetti.
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As far as I can tell, none of the target KEMs claim higher $U$-user security.