Software optimization

Almost all software is much slower than it could be.

Previous part:
• General software engineering.
• Using const-time instructions.
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Is software applied to much data? Usually not. Usually the wasted CPU time is negligible.
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But *crypto software* should be applied to all communication.

Crypto that’s too slow
⇒ fewer users
⇒ fewer cryptanalysts
⇒ less attractive for everybody.
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Typical situation:

$X$ is a cryptographic system.

You have written a (const-time) reference implementation.

You want (const-time) software that computes $X$ as efficiently as possible.

You have chosen a target CPU.
(Can repeat for other CPUs.)

You measure performance of the implementation. Now what?
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A simplified example

Target CPU: TI LM4F120H5QR microcontroller containing one ARM Cortex-M4F core.

Reference implementation:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; ++i)
        result += x[i];
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Counting cycles:

```c
static volatile unsigned int *const DWT_CYCCNT = (void *) 0xE0001004;
...
int beforesum = *DWT_CYCCNT;
int result = sum(x);
int aftersum = *DWT_CYCCNT;
UARTprintf("sum %d %d\n", result, aftersum - beforesum);
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Output shows 8012 cycles.

Change 1000 to 500: 4012.
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Apply random “optimizations” (and tweak compiler options) until you get bored.
Keep the fastest results.
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Try `-Os`: 8012 cycles.
Counting cycles:

static volatile unsigned int
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int beforesum = *DWT_CYCCNT;
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Try -Os: 8012 cycles.
Try -O1: 8012 cycles.
Counting cycles:

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int result = sum(x);
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Try -Os: 8012 cycles.
Try -O1: 8012 cycles.
Try -O2: 8012 cycles.
Counting cycles:

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Try -O1: 8012 cycles.
Try -O2: 8012 cycles.
Try -O3: 8012 cycles.
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Try -0s: 8012 cycles.
Try -01: 8012 cycles.
Try -02: 8012 cycles.
Try -03: 8012 cycles.

Try moving the pointer:

```c
int sum(int *x)
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    int result = 0;
    int i;
    for (i = 0;i < 1000;++i)
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Try counting down:
```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 1000; i > 0; --i)
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Try using an end pointer:

```c
int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    while (x != y)
        result += *x++;
    return result;
}
```

8010 cycles.
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```

Back to original. Try unrolling:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 2) {

        result += x[i];
        result += x[i + 1];
    }
    return result;
}
```
Try counting down:

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    result += x[i];
    return result;
}
```
Try using an end pointer:

```c
int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    while (x != y)
    result += *x++;
    return result;
}
```

8010 cycles.

Back to original. Try unrolling:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0;i < 1000;i += 2) {
        result += x[i];
        result += x[i + 1];
    }
    return result;
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5016 cycles.
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Back to original. Try unrolling:

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int sum(int *x)
{
int result = 0;
int i;
for (i = 0; i < 1000; i += 2) {
    result += x[i];
    result += x[i + 1];
}
return result;
}
```

5016 cycles.
Back to original. Try unrolling:

```c
int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 5)
    {
        result += x[i];
        result += x[i + 1];
        result += x[i + 2];
        result += x[i + 3];
        result += x[i + 4];
    }
    return result;
}
```

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Try using an end pointer:

```c
int sum(int *x)
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    int result = 0;
    int *y = x + 1000;
    while (x != y)
        result += *x++;
    return result;
}
```

8010 cycles.

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int sum(int *x)
{
    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 2) {
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int sum(int *x)
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    int result = 0;
    int i;
    for (i = 0; i < 1000; i += 2) {
        result += x[i];
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5016 cycles.
Back to original. Try unrolling:

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        result += x[i + 4];
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    return result;
}
4016 cycles. “Are we done yet?”
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Figure out lower bound for cycles spent on arithmetic etc.
Understand gap between lower bound and observed time.
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Points to the “ARMv7-M Architecture Reference Manual”, which defines instructions: e.g., “ADD” for 32-bit addition.

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Each element of x array needs to be “loaded” into a register.

Basic load instruction: LDR. Manual says 2 cycles but adds a note about “pipelining”.

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\( n \) consecutive LDRs takes only \( n + 1 \) cycles (“more multiple LDRs can be pipelined together”). Can achieve this speed in other ways (LDRD, LDM) but nothing seems faster.

Lower bound for \( n \) LDR + \( n \) ADD: \( 2n + 1 \) cycles, including \( n \) cycles of arithmetic.

Why observed time is higher: non-consecutive LDRs; costs of manipulating \( i \).

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Lower bound for \( n \) LDR + \( n \) ADD: \( 2n + 1 \) cycles, including \( n \) cycles of arithmetic.

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Inputs and output of ADD are "integer registers". ARMv7-M has 16 integer registers, including special-purpose “stack pointer” and “program counter”.

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Load instruction: LDR. Manual says 2 cycles but adds a note about “pipelining”.

More explanation: if next instruction is also LDR (with address not based on first LDR) then it saves 1 cycle.

n consecutive LDRs takes only n + 1 cycles (“more multiple LDRs can be pipelined together”).

Can achieve this speed in other ways (LDRD, LDM) but nothing seems faster.

Lower bound for n LDR + n ADD: 2n + 1 cycles, including n cycles of arithmetic.

Why observed time is higher: non-consecutive LDRs; costs of manipulating i.

int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    int x0,x1,x2,x3,x4,
        x5,x6,x7,x8,x9;
    while (x != y) {
        x0 = 0[(volatile int *)x];
        x1 = 1[(volatile int *)x];
        x2 = 2[(volatile int *)x];
        x3 = 3[(volatile int *)x];
        x4 = 4[(volatile int *)x];
        x5 = 5[(volatile int *)x];
        x6 = 6[(volatile int *)x];
        x7 = 7[(volatile int *)x];
        x8 = 8[(volatile int *)x];
        x9 = 9[(volatile int *)x];
    }
    return result;
}
Inputs and output of ADD are "integer registers". ARMv7-M has 16 integer registers, including special-purpose "stack pointer" and "program counter".

Each element of array needs to be "loaded" into a register.

Operation: LDR. Basic load instruction: LDR. Manual says 2 cycles but adds a note about "pipelining".

Can achieve this speed in other ways (LDRD, LDM) but nothing seems faster.

Lower bound for $n$ LDR + $n$ ADD:

$$2n + 1 \text{ cycles, including } n \text{ cycles of arithmetic.}$$

Why observed time is higher:

non-consecutive LDRs; costs of manipulating i.

\[ \text{int sum(int *x)} \{ \]
\[ \text{int result = 0;} \]
\[ \text{int *y = x + 1000;} \]
\[ \text{int x0,x1,x2,x3,x4,} \]
\[ \text{x5,x6,x7,x8,x9;} \]
\[ \text{while (x != y)} \{ \]
\[ \text{x0 = 0[(volatile int *)x];} \]
\[ \text{x1 = 1[(volatile int *)x];} \]
\[ \text{x2 = 2[(volatile int *)x];} \]
\[ \text{x3 = 3[(volatile int *)x];} \]
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Inputs and output of ADD are "integer registers". ARMv7-M has 16 integer registers, including special-purpose "stack pointer" and "program counter".

Each element of \( x \) array needs to be "loaded" into a register. Basic load instruction: LDR.

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Why observed time is higher: non-consecutive LDRs; costs of manipulating $i$.

```c
int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    int x0,x1,x2,x3,x4,x5,x6,x7,x8,x9;

    while (x != y) {
        x0 = 0[(volatile int *)x];
        x1 = 1[(volatile int *)x];
        x2 = 2[(volatile int *)x];
        x3 = 3[(volatile int *)x];
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        x5 = 5[(volatile int *)x];
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    }
}
```
$n$ consecutive LDRs takes only $n + 1$ cycles ("more multiple LDRs can be pipelined together").

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int sum(int *x)
{
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    int *y = x + 1000;
    int x0,x1,x2,x3,x4,
        x5,x6,x7,x8,x9;

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        x0 = 0[(volatile int *)x];
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        x6 = 6[(volatile int *)x];
    }

    return result;
}
```
consecutive LDRs takes only \(n + 1\) cycles (multiple LDRs can be pipelined together).

Can achieve this speed in other ways (LDRD, LDM) but nothing seems faster.

Lower bound for \(n\) LDR + \(n\) ADD: \(2n + 1\) cycles, including \(n\) cycles of arithmetic.

Why observed time is higher: non-consecutive LDRs; costs of manipulating \(i\).

int sum(int *x)
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        x0 = 0[(volatile int *)x];
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        x5 = 5[(volatile int *)x];
        x6 = 6[(volatile int *)x];
        x7 = 7[(volatile int *)x];
        x8 = 8[(volatile int *)x];
        x9 = 9[(volatile int *)x];
        result += x0;
        result += x1;
        result += x2;
        result += x3;
        result += x4;
        result += x5;
        result += x6;
        result += x7;
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        result += x9;
        x0 = 10[(volatile int *)x];
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n consecutive LDRs takes only n + 1 cycles ("more multiple LDRs can be pipelined together").

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Lower bound for \( n \) LDR + \( n \) ADD:

\[ 2n + 1 \text{ cycles}, \]

including \( n \) cycles of arithmetic.

Why observed time is higher:

- non-consecutive LDRs;
- costs of manipulating \( i \).

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int sum(int *x)
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        result += x0;
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Lower bound for \( n \) LDR + \( n \) ADD:

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        result += x0;
        result += x1;
        result += x2;
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        result += x9;
        x0 = 10[(volatile int *)x];
        x1 = 11[(volatile int *)x];
    }
}
```c
int sum(int *x)
{
    int result = 0;
    int *y = x + 1000;
    int x0,x1,x2,x3,x4,
        x5,x6,x7,x8,x9;

    while (x != y) {
        x0 = 0[(volatile int *)x];
        x1 = 1[(volatile int *)x];
        x2 = 2[(volatile int *)x];
        x3 = 3[(volatile int *)x];
        x4 = 4[(volatile int *)x];
        x5 = 5[(volatile int *)x];
        x6 = 6[(volatile int *)x];
        x7 = 7[(volatile int *)x];
        x8 = 8[(volatile int *)x];
        x9 = 9[(volatile int *)x];
        result += x0;
        result += x1;
        result += x2;
        result += x3;
        result += x4;
        result += x5;
        result += x6;
        result += x7;
        result += x8;
        result += x9;
        x0 = 10[(volatile int *)x];
        x1 = 11[(volatile int *)x];
        x2 = 12[(volatile int *)x];
        x3 = 13[(volatile int *)x];
        x4 = 14[(volatile int *)x];
        x5 = 15[(volatile int *)x];
        x6 = 16[(volatile int *)x];
        x7 = 17[(volatile int *)x];
        x8 = 18[(volatile int *)x];
        x9 = 19[(volatile int *)x];
        result += x0;
        result += x1;
        result += x2;
        result += x3;
        result += x4;
        result += x5;
        result += x6;
        result += x7;
        result += x8;
        result += x9;
        x0 = 20;
        result += x0;
        x += 20;
    }
    return result;
}
```
int sum(int *x) {
    int result = 0;
    int *y = x + 1000;
    int x0, x1, x2, x3, x4, x5, x6, x7, x8, x9;
    while (x != y) {
        x0 = 0[(volatile int *)x];
        x1 = 1[(volatile int *)x];
        x2 = 2[(volatile int *)x];
        x3 = 3[(volatile int *)x];
        x4 = 4[(volatile int *)x];
        x5 = 5[(volatile int *)x];
        x6 = 6[(volatile int *)x];
        x7 = 7[(volatile int *)x];
        x8 = 8[(volatile int *)x];
        x9 = 9[(volatile int *)x];
        result += x0;
        result += x1;
        result += x2;
        result += x3;
        result += x4;
        result += x5;
        result += x6;
        result += x7;
        result += x8;
        result += x9;
        x0 = 10[(volatile int *)x];
        x1 = 11[(volatile int *)x];
        x2 = 12[(volatile int *)x];
        x3 = 13[(volatile int *)x];
        x4 = 14[(volatile int *)x];
        x5 = 15[(volatile int *)x];
        x6 = 16[(volatile int *)x];
        x7 = 17[(volatile int *)x];
        x8 = 18[(volatile int *)x];
        x9 = 19[(volatile int *)x];
        result += x0;
        result += x1;
        result += x2;
        result += x3;
        result += x4;
        result += x5;
        result += x6;
        result += x7;
        result += x8;
        result += x9;
        x0 = 20;
        x1 = 21;
        x2 = 22;
        x3 = 23;
        x4 = 24;
        x5 = 25;
        x6 = 26;
        x7 = 27;
        x8 = 28;
        x9 = 29;
        result += x0;
        result += x1;
        result += x2;
        result += x3;
        result += x4;
        result += x5;
        result += x6;
        result += x7;
        result += x8;
        result += x9;
        x0 = 30;
        x1 = 31;
        x2 = 32;
        x3 = 33;
        x4 = 34;
        x5 = 35;
        x6 = 36;
        x7 = 37;
        x8 = 38;
        x9 = 39;
        result += x0;
        result += x1;
        result += x2;
        result += x3;
        result += x4;
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        x6 = 6[(volatile int *)x];
        x7 = 7[(volatile int *)x];
        x8 = 8[(volatile int *)x];
        x9 = 9[(volatile int *)x];
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        result += x1;
        result += x2;
        result += x3;
        result += x4;
        result += x5;
        result += x6;
        result += x7;
        result += x8;
        result += x9;
        x0 = 10[(volatile int *)x];
        x1 = 11[(volatile int *)x];
        x2 = 12[(volatile int *)x];
        x3 = 13[(volatile int *)x];
        x4 = 14[(volatile int *)x];
        x5 = 15[(volatile int *)x];
        x6 = 16[(volatile int *)x];
        x7 = 17[(volatile int *)x];
        x8 = 18[(volatile int *)x];
        x9 = 19[(volatile int *)x];
        x += 20;
        result += x0;
        result += x1;
        result += x2;
        result += x3;
        result += x4;
        result += x5;
    }
}
x7 = 7[(volatile int *)x];
x8 = 8[(volatile int *)x];
x9 = 9[(volatile int *)x];
result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
result += x5;
result += x6;
result += x7;
result += x8;
x0 = 10[(volatile int *)x];
x1 = 11[(volatile int *)x];
x2 = 12[(volatile int *)x];
x3 = 13[(volatile int *)x];
x4 = 14[(volatile int *)x];
x5 = 15[(volatile int *)x];
x6 = 16[(volatile int *)x];
x7 = 17[(volatile int *)x];
x8 = 18[(volatile int *)x];
x9 = 19[(volatile int *)x];
x += 20;
x += 20;
result += x0;
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result += x2;
result += x3;
result += x4;
result += x5;
x7 = 7[(volatile int *)x];
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result += x0;
result += x1;
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result += x1;
result += x2;
result += x3;
result += x4;
result += x5;
result += x6;
result += x7;
result += x8;
result += x9;
return result;
tile int *)x];
x7 = 7[(volatile int *)x];
x8 = 8[(volatile int *)x];
x9 = 9[(volatile int *)x];
result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
result += x5;
result += x6;
result += x7;
result += x8;
result += x9;
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x2 = 12[(volatile int *)x];
x3 = 13[(volatile int *)x];
x4 = 14[(volatile int *)x];
x5 = 15[(volatile int *)x];
x6 = 16[(volatile int *)x];
x7 = 17[(volatile int *)x];
x8 = 18[(volatile int *)x];
x9 = 19[(volatile int *)x];
x += 20;
result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
result += x5;
result += x6;
result += x7;
result += x8;
result += x9;
}
return result;
x2 = 12[(volatile int *)x];
x3 = 13[(volatile int *)x];
x4 = 14[(volatile int *)x];
x5 = 15[(volatile int *)x];
x6 = 16[(volatile int *)x];
x7 = 17[(volatile int *)x];
x8 = 18[(volatile int *)x];
x9 = 19[(volatile int *)x];
x += 20;
result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
result += x5;
result += x6;
result += x7;
result += x8;
result += x9;
} return result;
x2 = 12[(volatile int *)x];
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x += 20;
result += x0;
result += x1;
result += x2;
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result += x4;
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result += x6;
result += x7;
result += x8;
result += x9;
}
return result;
}
x2 = 12[(volatile int *)x];
x3 = 13[(volatile int *)x];
x4 = 14[(volatile int *)x];
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x6 = 16[(volatile int *)x];
x7 = 17[(volatile int *)x];
x8 = 18[(volatile int *)x];
x9 = 19[(volatile int *)x];
x += 20;
result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
result += x5;  
result += x6;
result += x7;
result += x8;
result += x9;
}
return result;
}

2526 cycles. Even better in asm.
x2 = 12[(volatile int *)x];
x3 = 13[(volatile int *)x];
x4 = 14[(volatile int *)x];
x5 = 15[(volatile int *)x];
x6 = 16[(volatile int *)x];
x7 = 17[(volatile int *)x];
x8 = 18[(volatile int *)x];
x9 = 19[(volatile int *)x];
x += 20;
result += x0;
result += x1;
result += x2;
result += x3;
result += x4;
result += x5;
x6 = 12[(volatile int *)x];
x7 = 13[(volatile int *)x];
x8 = 14[(volatile int *)x];
x9 = 15[(volatile int *)x];
x10 = 16[(volatile int *)x];
x11 = 17[(volatile int *)x];
x12 = 18[(volatile int *)x];
x13 = 19[(volatile int *)x];
x14 += 20;
result += x6;
result += x7;
result += x8;
result += x9;
result += x10;
result += x11;
result += x12;
result += x13;
return result;
}

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Wikipedia: “By the late 1990s for even performance sensitive code, optimizing compilers exceeded the performance of human experts.”
\[ x2 = 12[(volatile \text{ int } *)x]; \]
\[ x3 = 13[(volatile \text{ int } *)x]; \]
\[ x4 = 14[(volatile \text{ int } *)x]; \]
\[ x5 = 15[(volatile \text{ int } *)x]; \]
\[ x6 = 16[(volatile \text{ int } *)x]; \]
\[ x7 = 17[(volatile \text{ int } *)x]; \]
\[ x8 = 18[(volatile \text{ int } *)x]; \]
\[ x9 = 19[(volatile \text{ int } *)x]; \]
\[ x += 20; \]
\[ \text{result } += x0; \]
\[ \text{result } += x1; \]
\[ \text{result } += x2; \]
\[ \text{result } += x3; \]
\[ \text{result } += x4; \]
\[ \text{result } += x5; \]
\[ \text{result } += x6; \]
\[ \text{result } += x7; \]
\[ \text{result } += x8; \]
\[ \text{result } += x9; \]
\]
\[ \text{return result; } \]

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— [citation needed]
result += x6;
result += x7;
result += x8;
result += x9;
}
return result;

2526 cycles. Even better in asm.

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— [citation needed]
```c
volatile int x;
result += x;
result += x2;
result += x3;
result += x4;
result += x5;
return result;
```

2526 cycles. Even better in asm.

Wikipedia: “By the late 1990s for even performance sensitive code, optimizing compilers exceeded the performance of human experts.” — [citation needed]
result += x6;
result += x7;
result += x8;
result += x9;
}
return result;
}

2526 cycles. Even better in asm.

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— [citation needed]

A real example

Salsa20 reference software: 30.25 cycles/byte on this CPU.

Lower bound for arithmetic: 64 bytes require $21 \cdot 16$ 1-cycle ADDs, $20 \cdot 16$ 1-cycle XORs, so at least 10.25 cycles/byte.

Also many rotations, but ARMv7-M instruction set includes free rotation as part of XOR instruction. (Compiler knows this.)
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result += x6;
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Detailed benchmarks show several cycles/byte spent on load_littleendian and store_littleendian.
Can replace with LDR and STR.
(Compiler doesn't see this.)

Then observe 23 cycles/byte:
18 cycles/byte for rounds, plus 5 cycles/byte overhead.
Still far above 10.25 cycles/byte.
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Detailed benchmarks show
several cycles/byte spent on
load\textunderscore littleendian and
store\textunderscore littleendian.

Can replace with LDR and SDR.
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Gap is mostly loads, stores.
Minimize load/store cost by
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20·16 1-cycle ADDs,
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Which of the 16 Salsa20 words
should be in registers?
Don’t trust compiler to
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64 bytes require
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On bigger CPUs, selecting vector instructions is critical for performance.
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http://bench.cr.yp.to includes 2392 implementations of 614 cryptographic primitives.
> 20 implementations of Salsa20.

Haswell: Reasonably simple ref implementation compiled with gcc -O3 -fomit-frame-pointer
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Fast random permutations
Goal: Put list \((x_1, \ldots, x_n)\) into a random order.
Which of the 16 Salsa20 words should be in registers?

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merged implementation with “machine-independent” optimizations and best of 121 compiler options: 4.52× slower.

Fast random permutations

Goal: Put list \((x_1, \ldots, x_n)\) into a random order.
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Simulate uniform random \(r_i\) using RNG: e.g., stream cipher.
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Restart on collision?
Uniform distribution; some cost.
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Restart on collision?
Uniform distribution; some cost.

Example: \(n = 6960\) bits;
weight 119; 31-bit \(r_i\); no restart.

Any output is produced in \(\leq 119!(n - 119)!\left(\frac{2^{31} + n - 1}{n}\right)\) ways;
i.e., < \(1.02 \cdot \frac{2^{31n}}{119}\) ways.
Factor < 1.02 increase in attacker’s chance of winning.
Fast random permutations

Goal: Put list \((x_1, \ldots, x_n)\) into a random order.

One textbook strategy:
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Any output is produced in \(\leq 119!(n - 119)!(2^{31} + n - 1)\) ways;
i.e., \(< 1.02 \cdot 2^{31n}/(\binom{n}{119})\) ways.

Factor \(< 1.02\) increase in attacker’s chance of winning.

Which sorting algorithm?
Reference bubblesort code does \(n(n - 1)\) = \(2\) minmax operations.
Fast random permutations

Goal: Put list \((x_1; \ldots; x_n)\) into a random order.

One textbook strategy:
Sort \((x_1; \ldots; M x_n)\) for random \((r_1; \ldots; r_n)\), suitable \(M\).

McEliece encryption example:
Randomly order 6960 bits \((1; \ldots; 1; 0; \ldots; 0)\), weight 119.

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Randomly order 761 trits \((-1; \ldots; -1; 0; \ldots; 0)\), wt 286.

Simulate uniform random \(r_i\) using RNG: e.g., stream cipher.

How many bits in \(r_i\)? Negligible collisions? Occasional collisions?
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Uniform distribution; some cost.

Example: \( n = 6960 \) bits; weight 119; 31-bit \( r_i \); no restart.
Any output is produced in
\[
\leq 119!(n - 119)! \left( \binom{2^{31}}{n} + n^{-1} \right) \text{ ways};
\]
i.e., \(< 1.02 \cdot 2^{31n}/\binom{n}{119} \) ways.

Factor \(< 1.02 \) increase in attacker’s chance of winning.
Simulate uniform random $r_i$ using RNG: e.g., stream cipher.

How many bits in $r_i$? Negligible collisions? Occasional collisions?

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Uniform distribution; some cost.

Example: $n = 6960$ bits;
weight 119; 31-bit $r_i$; no restart.

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$\leq 119!(n - 119)!(\binom{2^{31} + n - 1}{n})$ ways;
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$$\leq 119!(n - 119)! \left(\frac{2^{31} + n - 1}{n}\right)$$

ways; i.e., $< 1.02 \cdot 2^{31n}/\binom{n}{119}$ ways.

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Which sorting algorithm?

Reference bubblesort code does

$$n(n - 1)/2$$

minmax operations.

Many standard algorithms use fewer operations: mergesort, quicksort, heapsort, radixsort, etc.

But these algorithms rely on secret branches and secret indices.
Simulate uniform random $r_i$ using RNG: e.g., stream cipher.

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Restart on collision?

Uniform distribution; some cost.

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Factor $< 1.02$ increase in attacker’s chance of winning.

Which sorting algorithm?

Reference bubblesort code does $n(n - 1)/2$ minmax operations.

Many standard algorithms use fewer operations: mergesort, quicksort, heapsort, radixsort, etc.

But these algorithms rely on secret branches and secret indices.

Exercise: convert mergesort into constant-time mergesort using $\Theta(n^2)$ operations.
Simulate uniform random \( r_i \) using \( \text{RNG} \): e.g., stream cipher.

How many bits in \( r_i \)? Negligible collisions? Occasional collisions? Restart on collision?

Uniform distribution; some cost.

Example:
\[ n = 6960 \text{ bits; weight } 119; 31\text{-bit } r_i; \text{ no restart.} \]

Output is produced in \( \leq \frac{2^{31} + n - 1}{n} \) ways; \( \leq 1.02 \cdot 2^{31n}/(119) \) ways.

\(<1.02 \text{ increase in } \text{attacker’s chance of winning.}\)

Which sorting algorithm?
Reference bubblesort code does \( n(n - 1)/2 \) \( \text{minmax} \) operations.
Many standard algorithms use fewer operations: \( \text{mergesort, quicksort, heapsort, radixsort, etc.} \)

But these algorithms rely on secret branches and secret indices.

Exercise: convert \( \text{mergesort} \) into constant-time \( \text{mergesort} \) using \( \Theta(n^2) \) operations.
Simulate uniform random $r_i$ using RNG: e.g., stream cipher.

How many bits in $r_i$? Negligible collisions? Non-negligible?

Restart on collision? Uniform distribution; some cost.

Example: $n = 6960$ bits; weight $119$; $31$-bit $r_i$; no restart.

Any output is produced in

\[ \leq \binom{n}{119} \cdot 2^{31+n-1}\]

\[= \binom{n}{119} \cdot 2^{31+n-1} \]

ways; i.e.,

\[ < 1 : 02 \cdot 2^{31}n = \binom{n}{119} \]

ways.

Factor $< 1 : 02$ increase in attacker's chance of winning.

Which sorting algorithm?

Reference bubblesort code does

\[ n(n-1)/2 \]

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Many standard algorithms use fewer operations: mergesort, quicksort, heapsort, radixsort, etc.

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Exercise: convert mergesort into constant-time mergesort using $\Theta(n^2)$ operations.

Converting bubblesort into constant-time bubblesort loses only a constant factor in cost of constant-time.
Simulate uniform random $r_i$ using RNG: e.g., stream cipher.

How many bits in $r_i$? Negligible collisions? Occasional collisions?

Restart on collision? Uniform distribution; some cost.

Example: $n = 6960$ bits; weight $119$; $31$-bit $r_i$; no restart.

Any output is produced in $\leq n!(n-119)! \cdot 2^{31-n}$ ways; i.e., $< 1 : 0.2 \cdot 2^{31-n}$ ways.

Factor $< 1 : 0.2$ increase in attacker's chance of winning.

Which sorting algorithm?

Reference bubblesort code does $n(n-1)/2$ minmax operations.

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Exercise: convert mergesort into constant-time mergesort using $\Theta(n^2)$ operations.

Converting bubblesort into constant-time bubblesort loses only a constant factor: cost of constant-time minmax.
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“Sorting network”: sorting algorithm built as constant sequence of minmax operations (“comparators”).
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“Sorting network”: sorting algorithm built as constant sequence of minmax operations (“comparators”).

Sorting network on next slide: Batcher’s merge-exchange sort. \( \Theta(n(\log n)^2) \) minmax operations; \( (1/4)(e^2 - e + 4)n - 1 \) for \( n = 2^e \).
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Sorting network on next slide: Batcher’s merge-exchange sort.

$\Theta(n(\log n)^2)$ minmax operations;

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void sort(int32 *x,long long n)
{ long long t,p,q,i;
  t = 1; if (n < 2) return;
  while (t < n-t) t += t;
  for (p = t;p > 0;p >>= 1) {
    for (i = 0;i < n-p;++i)
      if (!(i & p))
        minmax(x+i,x+i+p);
    for (q = t;q > p;q >>= 1)
      for (i = 0;i < n-q;++i)
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Which sorting algorithm?

Reference bubblesort code does \( (n - 1) = 2 \) minmax operations.

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Sorting network on next slide: Batcher's merge-exchange sort. \( \Theta(n (\log n)^2) \) minmax operations; \((1 = 4)(e^2 - e + 4)n - 1\) for \(n = 2^e\).

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void sort(int32 *x, long long n) {
    long long t, p, q, i;
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        for (i = 0; i < n - p; ++i)
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}
```

How many cycles on, e.g., Intel Haswell CPU core?

Every cycle: a vector of 8 32-bit "min" operations and a vector of 8 32-bit "max" operations.
Converting bubblesort into constant-time bubblesort loses only a constant factor: cost of constant-time minmax.

"Sorting network": sorting algorithm built as constant sequence of minmax operations ("comparators").

Sorting network on next slide: Batcher's merge-exchange sort.

Θ(\(n (\log n)^2\)) minmax operations; (1 = 4)(\(e^2 - e + 4\))\(n - 1\) for \(n = 2^e\).

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How many cycles on, e.g., Intel Haswell CPU core?
Every cycle: a vector of 8 32-bit “min” operations and a vector of 8 32-bit “max” operations.
≥3008 cycles for \( n = 1024 \).
Current software: 7328 cycles.
void sort(int32 *x, long long n)
{
    long long t, p, q, i;
    t = 1; if (n < 2) return;
    while (t < n-t) t += t;
    for (p = t; p > 0; p >>= 1) {
        for (i = 0; i < n-p; ++i)
            if (!(i & p))
                minmax(x+i, x+i+p);
        for (q = t; q > p; q >>= 1)
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The function `sort` takes an integer array `x` and a long long `n` as arguments and sorts the array.

1. Set initial size `t = 1`.
2. If `n` is less than 2, return.
3. While `t` is less than `n-t`, increment `t` by `t`.
4. For each power of 2 `p`, perform `minmax` operations on elements at even indices.
5. For each power of 2 `q`, perform `minmax` operations on elements at odd indices.

The `minmax` function compares two elements and updates the minimum and maximum values accordingly.
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People optimize algorithms for a naive model of CPUs:
• Branches are fast.
• Random access is fast.
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Fundamental hardware costs of constant-time arithmetic are much lower than random access.
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Basic ECC operations: add, sub, mul of, e.g., integers mod \( 2^{255} - 19 \).

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Typical “big-integer library”:

A variable-length uint32 string $(f_0, f_1, \ldots, f_{\ell-1})$ represents the nonnegative integer $f_0 + 2^{32}f_1 + \cdots + 2^{32(\ell-1)}f_{\ell-1}$.

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Library provides functions acting on this representation: (1) $f \mapsto fg$; (2) $f \mapsto f \mod g$; etc.
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ECC implementor using library:
multiply $f, g$ mod $2^{255} - 19$
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ECC implementor using library:
multiply $f, g \mod 2^{255} - 19$ by (1) multiplying $f$ by $g$;
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But these functions take variable time to ensure uniqueness!
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ECC implementor using library: multiply $f, g \mod 2^{255} - 19$ by (1) multiplying $f$ by $g$; (2) reducing mod $2^{255} - 19$.

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Need a different representation for constant-time arithmetic. Can also gain speed this way.
Modular arithmetic

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Library provides functions acting on this representation: (1) $f, g \mapsto fg$; (2) $f, g \mapsto f \bmod g$; etc.

ECC implementor using library:
multiply $f, g \bmod 2^{255} - 19$
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Need a different representation for constant-time arithmetic.
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Constant-time bigint library:
a constant-length uint32 string $(f_0, f_1, \ldots, f_{\ell-1})$ represents the nonnegative integer $f_0 + 2^{32}f_1 + \cdots + 2^{32(\ell-1)}f_{\ell-1}$.

Adding two $\ell$-limb integers:
always allocate $\ell + 1$ limbs.
Don’t remove top zero limb.
Modular arithmetic

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Uniqueness: \( f_0 \neq 0 \) or \( f_{\ell-1} \neq 0 \).

Library provides functions acting on this representation: (1) \( f, g \mapsto fg \); (2) \( f, g \mapsto f \mod g \); etc.

ECC implementor using library: multiply \( f, g \mod 2^{255} - 19 \) by (1) multiplying \( f \) by \( g \);
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multiply $f, g \mod 2^{255} - 19$
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Can also track bounds more refined than $2^0, 2^{32}, 2^{64}, 2^{96}, \ldots$;
but no limbs $\mapsto$ bounds data flow.
Library provides functions acting on this representation: (1) $f, g \mapsto fg$; (2) $f, g \mapsto f \mod g$; etc.

ECC implementor using library: multiply $f, g \mod 2^{255} - 19$ by (1) multiplying $f$ by $g$; (2) reducing mod $2^{255} - 19$.

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$f \mod p$ is as short as $p$. 
Library provides functions acting on this representation: (1) \( f, g \mapsto f \cdot g \); (2) \( f, g \mapsto f \mod g \); etc.

ECC implementor using library:
- (1) multiplying \( f \) by \( g \);
- (2) reducing mod \( 2^{255} - 19 \).

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\( f \mod p \) is as short as \( p \).

Usually faster representation:
\texttt{uint32} string \((f_0; f_1; \ldots; f_9)\) represents \( f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9 \).

Constant bound on each \( f_i \).

More limbs than before, but save time by avoiding overflows and delaying carries.

After multiplication, replace \( 2^{255} \) with \( 19 \).

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ECC implementor using library:

- multiply \( f, g \mod 2^{255} - 19 \) by \( f \) by \( g \);
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\(f \mod p\) is as short as \(p\).

Usually faster representation: uint32 string \((f_0, f_1, \ldots, f_9)\) represents 
\[ f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9. \]

Constant bound on each \(f_i\).

More limbs than before, but save time by avoiding overflows and delaying carries.

After multiplication, replace \(2^{255}\) with 19.
Constant-time bigint library: a constant-length `uint32` string
\((f_0, f_1, \ldots, f_{\ell-1})\) represents
the nonnegative integer
\(f_0 + 2^{32} f_1 + \cdots + 2^{32}(\ell-1) f_{\ell-1}\).

Adding two \(\ell\)-limb integers:
always allocate \(\ell + 1\) limbs.
Don’t remove top zero limb.

Can also track bounds more
refined than \(2^0, 2^{32}, 2^{64}, 2^{96}, \ldots\);
but no limbs \(\rightarrow\) bounds data flow.

\(f \mod p\) is as short as \(p\).

Usually faster representation:
`uint32` string \((f_0, f_1, \ldots, f_9)\)
represents
\(f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9\).

Constant bound on each \(f_i\).

More limbs than before,
but save time by avoiding
overflows and delaying carries.

After multiplication,
replace \(2^{255}\) with 19.

Slightly faster on some CPUs:
`int32` string \((f_0, f_1, \ldots, f_9)\).

Usually faster representation:

uint32 string \((f_0, f_1, \ldots, f_9)\)
represents 
\[ f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9. \]

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Constant-time bigint library:
a constant-length uint32 string
( f_0 ; f_1 ; : : : ; f_{\ell-1} ) represents
the nonnegative integer
f_0 + 2^{32(\ell-1)} f_{\ell-1}.

- integers:
  + 1 limbs.
- zero limb.

- bounds more
  2^{32}, 2^{64}, 2^{96}, \ldots;
- bounds data flow.
- short as p.

Usually faster representation:
uint32 string ( f_0, f_1, \ldots, f_9 )
represents f_0 + 2^{26} f_1 + 2^{51} f_2 +
2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 +
2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9.

Constant bound on each f_i.

More limbs than before,
but save time by avoiding
overflows and delaying carries.

After multiplication,
replace 2^{255} with 19.

Slightly faster on some CPUs:
int32 string ( f_0, f_1, \ldots, f_9 ).

int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
\ldots
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
f7_2 \times (int64) g7_19; \ldots
int64 h4 = f0g4 + f2g2 + f4g0 + f6g8_19 + f8g6_19;
\ldots
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 \times 2^{26};
Constant-time bigint library:

A constant-length \( \text{uint32} \) string \((f_0, f_1, \ldots, f_{n-1})\) represents
the nonnegative integer
\[ f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9. \]

Adding two \( \text{-limb} \) integers:
Always allocate \( +1 \) limbs.
Don't remove top zero limb.
Can also track bounds more refined than \( 2^0; 2^{32}; 2^{64}; 2^{96}; \ldots \)
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\( f \mod p \) is as short as \( p \).

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\( \text{uint32} \) string \((f_0, f_1, \ldots, f_9)\)
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Constant bound on each \( f_i \).

More limbs than before, but save time by avoiding overflows and delaying carries.

After multiplication, replace \( 2^{255} \) with 19.

Slightly faster on some CPUs:

\( \text{int32} \) string \((f_0, f_1, \ldots, f_9)\).

\[ \begin{align*}
\text{int32 } f7_2 &= 2 \times f7; \\
\text{int32 } g7_19 &= 19 \times g7; \\
\vdots \\
\text{int64 } f0g4 &= f0 \times (\text{int64}) g4; \\
\text{int64 } f7g7_38 &= f7_2 \times (\text{int64}) g7_19; \\
\vdots \\
\text{int64 } h4 &= f0g4 + f1g3_2 + f2g2 + f3g1_2 + f4g0 + f5g9_38 + f6g8_19 + f7g7_38 + f8g6_19 + f9g5; \\
\vdots \\
c4 &= (h4 + (\text{int64})(1<<25)) >> 26; \\
h5 &= h5 + c4; h4 &= h4 - c4 << 26; \\
\end{align*} \]
Usually faster representation:
\[ \text{uint32 string } (f_0, f_1, \ldots, f_9) \]
represents \[ f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{77} f_3 + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{179} f_7 + 2^{204} f_8 + 2^{230} f_9. \]

Constant bound on each \( f_i \).

More limbs than before, but save time by avoiding overflows and delaying carries.

After multiplication, replace \( 2^{255} \) with 19.

Slightly faster on some CPUs:
\[ \text{int32 string } (f_0, f_1, \ldots, f_9). \]

```c
int32 f7_2 = 2 * f7;
int32 g7_19 = 19 * g7;
...
int64 f0g4 = f0 * (int64) g4;
int64 f7g7_38 =
    f7_2 * (int64) g7_19;
...
int64 h4 = f0g4 + f1g3_2 
    + f2g2 + f3g1_2
    + f4g0 + f5g9_38
    + f6g8_19 + f7g7_38
    + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```
Usually faster representation:

\[ \text{uint32 string} \left( f_0, f_1, \ldots, f_9 \right) \]

represents

\[ f_0 + 2^{26} f_1 + 2^{51} f_2 + \ldots + 2^{102} f_4 + 2^{128} f_5 + 2^{153} f_6 + 2^{204} f_8 + 2^{230} f_9. \]

Constant bound on each \( f_i \).

More limbs than before, but save time by avoiding overflows and delaying carries.

After multiplication, replace \( 2^{255} \) with 19.

Slightly faster on some CPUs:

\[ \text{int32 string} \left( f_0, f_1, \ldots, f_9 \right). \]

Initial computation of \( h_0, \ldots, h_9 \) is polynomial multiplication modulo \( x^{10} - 19 \).

Exercise: Which polynomials are being multiplied?

\[
\begin{align*}
\text{int32 } f_7_2 &= 2 \times f_7; \\
\text{int32 } g_7_19 &= 19 \times g_7; \\
&\quad \ldots \\
\text{int64 } f_0g_4 &= f_0 \times (\text{int64}) g_4; \\
\text{int64 } f_7g_7_38 &= f_7_2 \times (\text{int64}) g_7_19; \\
&\quad \ldots \\
\text{int64 } h_4 &= f_0g_4 + f_1g_3_2 \\
&\quad + f_2g_2 + f_3g_1_2 \\
&\quad + f_4g_0 + f_5g_9_38 \\
&\quad + f_6g_8_19 + f_7g_7_38 \\
&\quad + f_8g_6_19 + f_9g_5_38; \\
&\quad \ldots \\
c_4 &= (h_4 + (\text{int64})(1\ll25)) >> 26; \\
h_5 &= c_4; h_4 &= c_4 \ll 26;
\end{align*}
\]
Usually faster representation:
\[ \text{uint32 string } (f_0; f_1; \ldots; f_9) \]
represents \( f_0 + 2^{26} f_1 + 2^{51} f_2 + 2^{128} f_3 + 2^{153} f_6 + 2^{230} f_9. \)

This involves multiplying each \( f_i \).

More limbs than before, but save time by avoiding overflows and delaying carries.

After multiplication, replace \( 2^{255} \) with 19.

Slightly faster on some CPUs:
\[ \text{int32 string } (f_0; f_1; \ldots; f_9). \]

---

Initial computation of \( h_0, \ldots, h_9 \) is polynomial multiplication modulo \( x^{10} - 19. \).
Exercise: Which polynomials are being multiplied?

```c
int32 f7_2 = 2 * f7;
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int64 f0g4 = f0 * (int64) g4;
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int64 h4 = f0g4 + f1g3_2
    + f2g2 + f3g1_2
    + f4g0 + f5g9_38
    + f6g8_19 + f7g7_38
    + f8g6_19 + f9g5_38;
...
c4 = (h4 + (int64)(1<<25)) >> 26;
h5 += c4; h4 -= c4 << 26;
```
34
\[
\begin{align*}
\text{int32 } f_7_2 &= 2 \times f_7; \\
\text{int32 } g_{7_19} &= 19 \times g_7; \\
\text{...}
\end{align*}
\]

35
\[
\begin{align*}
\text{int64 } f_{0g4} &= f_0 \times (\text{int64}) \times g_4; \\
\text{int64 } f_{7g7_38} &= f_7_2 \times (\text{int64}) \times g_{7_19}; \\
\text{...}
\end{align*}
\]

36
\[
\begin{align*}
\text{int64 } h_4 &= f_{0g4} + f_{1g3_2} \\
&\quad + f_{2g2} + f_{3g1_2} \\
&\quad + f_{4g0} + f_{5g9_38} \\
&\quad + f_{6g8_19} + f_{7g7_38} \\
&\quad + f_{8g6_19} + f_{9g5_38}; \\
\text{...}
\end{align*}
\]

34
Initial computation of $h_0, \ldots, h_9$ is polynomial multiplication modulo $x^{10} - 19$.

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Exercise: Which polynomials are being multiplied?

Reduction modulo $x^{10} - 19$ and carries such as $h_4 \rightarrow h_5$ squeeze the product into limited-size representation suitable for next multiplication.
Initial computation of $h_0, \ldots, h_9$ is polynomial multiplication modulo $x^{10} - 19$.

Exercise: Which polynomials are being multiplied?

Reduction modulo $x^{10} - 19$ and carries such as $h_4 \rightarrow h_5$ ***squeeze*** the product into limited-size representation suitable for next multiplication.

At end of computation: ***freeze*** representation into unique representation suitable for network transmission.
\begin{verbatim}
35

f7_2 = 2 * f7;
g7_19 = 19 * g7;

f0g4 = f0 * (int64) g4;
f7g7_38 = f7_2 * (int64) g7_19;

f4 = f0g4 + f1g3_2 + f2g2 + f3g1_2 + f4g0 + f5g9_38
+ f6g8_19 + f7g7_38 + f8g6_19 + f9g5_38;

h4 = f0g4 + f1g3_2 + f2g2 + f3g1_2 + f4g0 + f5g9_38
+ f6g8_19 + f7g7_38 + f8g6_19 + f9g5_38;

h4 += c4; h4 -= c4 << 26;
\end{verbatim}

Initial computation of $h_0, \ldots, h_9$ is polynomial multiplication modulo $x^{10} - 19$.

Exercise: Which polynomials are being multiplied?

Reduction modulo $x^{10} - 19$ and carries such as $h_4 \rightarrow h_5$ squeeze the product into limited-size representation suitable for next multiplication.

At end of computation: freeze representation into unique representation suitable for network transmission.

Much more about ECC speed: see, e.g., 2015 Chou.
Initial computation of $h_0, \ldots, h_9$ is polynomial multiplication modulo $x^{10} - 19$.

Exercise: Which polynomials are being multiplied?

Reduction modulo $x^{10} - 19$ and carries such as $h_4 \rightarrow h_5$ 

**squeeze** the product into limited-size representation suitable for next multiplication.

At end of computation:

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into limited-size representation suitable for next multiplication.

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Verifying constant time: increasingly automated.
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Testing can miss rare bugs that attacker might trigger. Fix: prove that software matches mathematical spec; have computer check proofs.
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Progress in deploying proven fast software: see, e.g., 2015 Bernstein–Schwabe “gfverif”; 2017 HACL* X25519 in Firefox.
Initial computation of $h_0, \ldots, h_9$ is polynomial multiplication modulo $x^{10} - 19$.

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Progress in deploying proven fast software: see, e.g., 2015 Bernstein–Schwabe “gfverif”; 2017 HACL* X25519 in Firefox.

gfverif has verified ref10 implementation of X25519, plus occasional annotations, against the following specification:

$p = 2^{255}-19$

$A = 486662$

$x_2, z_2, x_3, z_3 = 1, 0, x_1, 1$

for $i$ in reversed(range(255)):

$\ni = \text{bit}(n, i)$

$x_2, x_3 = \text{cswap}(x_2, x_3, \ni)$

$z_2, z_3 = \text{cswap}(z_2, z_3, \ni)$

$x_3, z_3 = (4*(x_2*x_3-z_2*z_3)^2, 4*x_1*(x_2*z_3-z_2*x_3)^2)$

$x_2, z_2 = ((x_2^2-z_2^2)^2, 4*x_2*z_2*(x_2^2+A*x_2*z_2+z_2^2))$
The initial computation of $h_0, \ldots, h_9$ is polynomial multiplication modulo $x^{10} - 19$.

Exercise: Which polynomials are being multiplied?

Reduction modulo $x^{10} - 19$ and carries such as $h_4 \rightarrow h_5$ squeeze the product into limited-size representation suitable for next multiplication.

At the end of computation: freeze representation into unique representation suitable for network transmission.

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\[
p = 2^{255} - 19
\]

\[
A = 486662
\]

\[
x_2, z_2, x_3, z_3 = 1, 0, x_1, 1
\]

\[
\text{for } i \text{ in reversed(range(255))}
\]

\[
i = \text{bit}(n, i)
\]

\[
x_2, x_3 = \text{cswap}(x_2, x_3, i)
\]

\[
z_2, z_3 = \text{cswap}(z_2, z_3, i)
\]

\[
x_3, z_3 = (4*(x_2*x_3-z_2*z_3)^2, 4*x_1*(x_2*z_3-z_2*x_3)^2)
\]

\[
x_2, z_2 = ((x_2^2-z_2^2)^2, 4*x_2*z_2*(x_2^2+A*x_2*z_2+z_2^2))
\]
Initial computation of \( h_0, \ldots, h_9 \) is polynomial multiplication modulo \( x^{10} - 19 \).

Exercise: Which polynomials are being multiplied?

Reduction modulo \( x^{10} - 19 \) and carries such as \( h_4 \rightarrow h_5 \) squeeze the product into limited-size representation suitable for next multiplication.

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---

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\[
\begin{align*}
p &= 2^{255} - 19 \\
A &= 486662 \\
x_2, z_2, x_3, z_3 &= 1, 0, x_1, 1 \\
\text{for } i \text{ in reversed(range(255))}:
\end{align*}
\]

\[
\begin{align*}
ni &= \text{bit}(n, i) \\
x_2, x_3 &= \text{cswap}(x_2, x_3, ni) \\
z_2, z_3 &= \text{cswap}(z_2, z_3, ni) \\
x_3, z_3 &= (4*(x_2*x_3-z_2*z_3)^2, 4*x_1*(x_2*z_3-z_2*x_3)^2) \\
x_2, z_2 &= ((x_2**2-z_2**2)**2, 4*x_2*z_2*(x_2**2+A*x_2*z_2+z_2**2))
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\[
p = 2^{255}-19 \\
A = 486662 \\
x_2, z_2, x_3, z_3 = 1, 0, x_1, 1 \\
\text{for } i \text{ in reversed(range(255))}:
\begin{align*}
  & n_i = \text{bit}(n, i) \\
  & x_2, x_3 = \text{cswap}(x_2, x_3, n_i) \\
  & z_2, z_3 = \text{cswap}(z_2, z_3, n_i) \\
  & x_3, z_3 = (4*(x_2*x_3-z_2*z_3)**2, \\
               4*x_1*(x_2*z_3-z_2*x_3)**2) \\
  & x_2, z_2 = ((x_2**2-z_2**2)**2, \\
               4*x_2*z_2*(x_2**2+A*x_2*z_2+z_2**2))
\end{align*}
\]
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for \( i \) in reversed(range(255)):

\[ n_i = \text{bit}(n, i) \]
\[ x_2, x_3 = \text{cswap}(x_2, x_3, n_i) \]
\[ z_2, z_3 = \text{cswap}(z_2, z_3, n_i) \]
\[ x_3, z_3 = (4*(x_2*x_3-z_2*z_3)^2, 4*x_1*(x_2*z_3-z_2*x_3)^2) \]
\[ x_2, z_2 = ((x_2**2-z_2**2)^2, 4*x_2*z_2*(x_2**2+A*x_2*z_2+z_2**2)) \]

What’s verified: output of ref10 is the same as spec mod \( p \), and is between 0 and \( p-1 \).

x3, z3 = (x3%p, z3%p)
x2, z2 = (x2%p, z2%p)
cut(x2)
cut(x3)
cut(z2)
cut(z3)
x2, x3 = cswap(x2, x3, ni)
z2, z3 = cswap(z2, z3, ni)
x2, z2 = ((x2**2-z2**2)**2, 4*x2*z2*(x2**2+A*x2*z2+z2**2))
Much more about ECC speed:
see, e.g., 2015 Chou.

Verifying constant time:
increasingly automated.

Testing can miss rare bugs
that attacker might trigger.

Fix: prove that software
matches mathematical spec;
have computer check proofs.

Progress in deploying proven
fast software: see, e.g., 2015
Bernstein–Schwabe “gfverif”;
2017 HACL* X25519 in Firefox.

gfverif has verified ref10
implementation of X25519,
plus occasional annotations,
against the following specification:

\[ p = 2^{255}-19 \]
\[ A = 486662 \]
\[ x_2, z_2, x_3, z_3 = 1, 0, x_1, 1 \]

for \( i \) in reversed(range(255)):
    \[ n_i = \text{bit}(n, i) \]
    \[ x_2, x_3 = \text{cswap}(x_2, x_3, n_i) \]
    \[ z_2, z_3 = \text{cswap}(z_2, z_3, n_i) \]
    \[ x_3, z_3 = (4^*(x_2*x_3-z_2*z_3)^2, 4^*x_1^*(x_2*z_3-z_2*x_3)^2) \]
    \[ x_2, z_2 = ((x_2^2-z_2^2)^2, 4^*x_2*z_2*(x_2^2+A*x_2*z_2+z_2^2)^2) \]

\[ x_3, z_3 = (x_3^p)^2 \]
\[ x_2, z_2 = (x_2^p)^2 \]
\[ \text{cut}(x_2) \]
\[ \text{cut}(x_3) \]
\[ \text{cut}(z_2) \]
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Testing can miss rare bugs that attacker might trigger.

Fix: prove that software matches mathematical spec;

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Progress in deploying proven fast software:

see, e.g., 2015 Bernstein–Schwabe “gfverif”;

2017 HACL* X25519 in Firefox.

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gfverif has verified ref10 implementation of X25519,

plus occasional annotations,

against the following specification:

\[
p = 2^{255} - 19
\]

\[
A = 486662
\]

\[
x_2, z_2, x_3, z_3 = 1, 0, 1, 1
\]

for \(i\) in reversed(range(255)):

\[
i = \text{bit}(n, i)
\]

\[
x_2, x_3 = \text{cswap}(x_2, x_3, i)
\]

\[
z_2, z_3 = \text{cswap}(z_2, z_3, i)
\]

\[
x_3, z_3 = \left(4 \times (x_2 \times x_3 - z_2 \times z_3)^2, 4 \times x_1 \times (x_2 \times z_3 - z_2 \times x_3)^2\right)
\]

\[
x_2, z_2 = \left((x_2 \times 2 - z_2 \times 2)^2, 4 \times x_2 \times z_2 \times (x_2 \times 2 + A \times x_2 \times z_2 + z_2 \times 2)^2\right)
\]

\[
x_3, z_3 = \left(x_3 \times p, z_3 \times p\right)
\]

\[
x_2, z_2 = \left(x_2 \times p, z_2 \times p\right)
\]

\[
cut(x_2)
\]

\[
cut(x_3)
\]

\[
cut(z_2)
\]

\[
cut(z_3)
\]

\[
x_2, x_3 = \text{cswap}(x_2, x_3, i)
\]

\[
z_2, z_3 = \text{cswap}(z_2, z_3, i)
\]

\[
cut(x_2)
\]

\[
cut(z_2)
\]

\[
return x_2 \times \text{pow}(z_2, p-2, p)
\]

What’s verified: output of ref10 is the same as spec mod \(p\),

and is between 0 and \(p - 1\).
gfverif has verified ref10 implementation of X25519, plus occasional annotations, against the following specification:

\[ p = 2^{255}-19 \]
\[ A = 486662 \]
\[ x_2, z_2, x_3, z_3 = 1, 0, x_1, 1 \]

for \( i \) in \text{reversed}(\text{range}(255)):

\[ \text{ni} = \text{bit}(n, i) \]
\[ x_2, x_3 = \text{cswap}(x_2, x_3, \text{ni}) \]
\[ z_2, z_3 = \text{cswap}(z_2, z_3, \text{ni}) \]
\[ x_3, z_3 = (4*(x_2*x_3-z_2*z_3)**2, 4*x_1*(x_2*z_3-z_2*x_3)**2) \]
\[ x_2, z_2 = ((x_2**2-z_2**2)**2, 4*x_2*z_2*(x_2**2+A*x_2*z_2+z_2**2)) \]

\[ x_3, z_3 = (x_3%p, z_3%p) \]
\[ x_2, z_2 = (x_2%p, z_2%p) \]
\[ \text{cut}(x_2) \]
\[ \text{cut}(x_3) \]
\[ \text{cut}(z_2) \]
\[ \text{cut}(z_3) \]
\[ x_2, x_3 = \text{cswap}(x_2, x_3, \text{ni}) \]
\[ z_2, z_3 = \text{cswap}(z_2, z_3, \text{ni}) \]
\[ \text{cut}(x_2) \]
\[ \text{cut}(z_2) \]
return \( x_2*\text{pow}(z_2, p-2, p) \)

What's verified: output of ref10 is the same as spec mod \( p \), and is between 0 and \( p-1 \).
has verified ref10 implementation of X25519, plus occasional annotations, against the following specification:

\[
p = 2^{255} - 19
\]

\[
A = 486662
\]

\[
x_2, z_2, x_3, z_3 = 1, 0, x_1, 1
\]

for \( i \) in reversed(range(255)):

\[
i = \text{bit}(n, i)
\]

\[
x_2, x_3 = \text{cswap}(x_2, x_3, i)
\]

\[
z_2, z_3 = \text{cswap}(z_2, z_3, i)
\]

\[
x_3, z_3 = (4(x_2 x_3 - z_2 z_3)^2, 4x_1(x_2 z_3 - z_2 x_3)^2)
\]

\[
x_2, z_2 = ((x_2^2 - z_2^2)^2, 4x_2 z_2(x_2^2 + A x_2 z_2 + z_2^2))
\]

\[
x_3, z_3 = (x_3 \mod p, z_3 \mod p)
\]

\[
x_2, z_2 = (x_2 \mod p, z_2 \mod p)
\]

What's verified: output of ref10 is the same as spec mod \( p \), and is between 0 and \( p - 1 \).

“What a difference a prime makes”

NIST P-256 prime \( p \) is

\[
2^{256} - 2^{224} + 2^{192} + 2^{96} - 1.
\]

ECDSA standard specifies reduction procedure given an integer \( A \) less than \( p^2 \):

Write \( A \) as \((A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)\), meaning

\[
P_i A_i 2^{32i}.
\]

Define \( T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4 \) as
ref10

NIST P-256 prime

P = 2^{256} − 2^{224} + 2^{192} − 2^{96} + 1.

ECDSA standard specifies reduction procedure given an integer “A less than p^2”:

Write A as

\( A_{15}; A_{14}; A_{13}; A_{12}; A_{11}; A_{10}; A_9; \ldots; A_1; A_0 \),

meaning \( \sum_i A_i 2^{32i} \).

Define

\( T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4 \)
as

\[
\begin{align*}
  x_3, z_3 &= (x_3 \% p, z_3 \% p) \\
  x_2, z_2 &= (x_2 \% p, z_2 \% p) \\
  \text{cut}(x_2) \\
  \text{cut}(x_3) \\
  \text{cut}(z_2) \\
  \text{cut}(z_3) \\
  x_2, x_3 &= \text{cswap}(x_2, x_3, ni) \\
  z_2, z_3 &= \text{cswap}(z_2, z_3, ni) \\
  \text{cut}(x_2) \\
  \text{cut}(z_2) \\
  \text{return } x_2 * \text{pow}(z_2, p-2, p)
\end{align*}
\]

What’s verified: output of ref10 is the same as spec mod \( p \), and is between 0 and \( p−1 \).
gfverif has verified ref10 implementation of X25519, plus occasional annotations, against the following specification:

\[ p = 2^{255}-19 \]
\[ A = 486662 \]
\[ x2, z2, x3, z3 = 1, 0, x1, 1 \]

For \( i \) in reversed(range(255)):

- \( n_i = \text{bit}(n, i) \)
- \( x2, x3 = \text{cswap}(x2, x3, n_i) \)
- \( z2, z3 = \text{cswap}(z2, z3, n_i) \)
- \( x3, z3 = (4*(x2*x3 - z2*z3)^2, 4*x1*(x2*z3 - z2*x3)^2) \)
- \( x2, z2 = ((x2^2 - z2^2)^2, 4*x2*z2*(x2^2 + A*x2*z2 + z2^2)) \)

\( x3, z3 = (x3 \mod p, z3 \mod p) \)
\( x2, z2 = (x2 \mod p, z2 \mod p) \)
\( \text{cut}(x2) \)
\( \text{cut}(x3) \)
\( \text{cut}(z2) \)
\( \text{cut}(z3) \)
\( x2, x3 = \text{cswap}(x2, x3, n_i) \)
\( z2, z3 = \text{cswap}(z2, z3, n_i) \)
\( \text{cut}(x2) \)
\( \text{cut}(z2) \)
\( \text{return } x2*\text{pow}(z2, p-2, p) \)

What's verified: output of ref10 is the same as spec mod \( p \), and is between 0 and \( p - 1 \).

“What a difference a prime makes”

NIST P-256 prime \( p \) is \( 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1 \).

ECDSA standard specifies reduction procedure given an integer “\( A \) less than \( p^2 \)”:

Write \( A \) as:

\( (A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0) \)

meaning \( \sum_i A_i 2^{32i} \).

Define \( T; S_1; S_2; S_3; S_4; D_1; D_2; D_3 \) as
x3, z3 = (x3%p, z3%p)
x2, z2 = (x2%p, z2%p)
cut(x2)
cut(x3)
cut(z2)
cut(z3)
x2, x3 = cswap(x2, x3, ni)
z2, z3 = cswap(z2, z3, ni)
cut(x2)
cut(z2)
return x2*power(z2, p-2, p)

What's verified: output of ref10 is the same as spec mod p, and is between 0 and p – 1.

“What a difference a prime makes”

NIST P-256 prime p is $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

ECDSA standard specifies reduction procedure given an integer “A less than $p^2$”:

Write A as

$A = (A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$,

meaning $\sum_i A_i 2^{32i}$.

Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as
(x3%p, z3%p)
(0, A15, A14, A13, A12, A11, A10, A9, A8, A7, A6, A5, A4, A3, A2, A1, A0)
(A11, A9, A10, A15, A14, A13, A12, A11, A10, A9, A8, A7, A6, A5, A4, A3, A2, A1, A0)
(A13, 0, A12, A11, A10, A9, A8, A7, A6, A5, A4, A3, A2, A1, A0)

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.
Reduce modulo $p$ "by adding or subtracting a few copies" of $p$. 

"What a difference a prime makes"

NIST P-256 prime $p$ is
$2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

ECDSA standard specifies reduction procedure given an integer "A less than $p^2$":
Write $A$ as
$(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$,
meaning $\sum_i A_i 2^{32i}$.

Define
$T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as

verified: output of ref10 same as spec mod $p$, between 0 and $p - 1$. 

"What a difference a prime makes"
$x_3, z_3 = (x_3 \mod p, z_3 \mod p)$
$x_2, z_2 = (x_2 \mod p, z_2 \mod p)$
cut(x_2)
cut(x_3)
cut(z_2)
cut(z_3)
$x_2, x_3 = \text{cswap}(x_2, x_3, n_i)$
$z_2, z_3 = \text{cswap}(z_2, z_3, n_i)$
cut(x_2)
cut(z_2)
return $x_2 \cdot (z_2 \mod p - 2, p)$

What's verified: output of ref10 is the same as spec mod $p$, and is between 0 and $p - 1$.

“What a difference a prime makes”

NIST P-256 prime $p$ is $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.
ECDSA standard specifies reduction procedure given an integer “A less than $p^2$”:
Write $A$ as $(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$, meaning $\sum_i A_i 2^{32i}$.
Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as

\[(A_7, A_6, A_5, A_4, A_3; A_{15}, A_{14}, A_{13}, A_{12}; 0, A_1, A_0; 0, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0; 0, A_{15}, A_{14}, 0, 0, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0; A_{13}, 0, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0;)
\]
Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.
Reduce modulo $p$ “by adding or subtracting a few copies” of $p$.\]
“What a difference a prime makes”

NIST P-256 prime $p$ is $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

ECDSA standard specifies reduction procedure given an integer “$A$ less than $p^2$”:

Write $A$ as $(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$, meaning $\sum A_i 2^{32i}$.

Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 − D_1 − D_2 − D_3 − D_4$.

Reduce modulo $p$ “by adding or subtracting a few copies” of $p$
“What a difference a prime makes”

NIST P-256 prime $p$ is
\[ 2^{256} - 2^{224} + 2^{192} + 2^{96} - 1. \]

ECDSA standard specifies reduction procedure given an integer “$A$ less than $p^2$”:

Write $A$ as
\[ (A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, A_{10}, A_9, A_8, A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0), \]
meaning $\sum_i A_i 2^{32i}$.

Define
\[ T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4 \]
as
\[ (A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0); (A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0); (0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0); (A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8); (A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9); (A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11}); (A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12}); (A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13}); (A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14}). \]

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 − D_1 − D_2 − D_3 − D_4$.

Reduce modulo $p$ “by adding or subtracting a few copies” of $p$. 
“What a difference a prime makes”

NIST P-256 prime $p$ is $2^{256} - 2^{224} + 2^{192} + 2^{96} - 1$.

ECDSA standard specifies

standard procedure given an integer “$A$ less than $p^2$”:

Write $A$ as $(A_{15}; A_{14}; A_{13}; A_{12}; A_{11}; A_{10}; A_9; A_8; A_7; A_6; A_5; A_4; A_3; A_2; A_1; A_0)$,

meaning $P_i A_i 2^{32i} \sum_i A_i 2^{32i}$.

Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo $p$ “by adding or subtracting a few copies” of $p$.

What is “a few copies”?
Variable-time loop is unsafe.
What a difference a prime makes

The NIST P-256 prime $p$ is

$$2^{256} - 2^{224} + 2^{192} + 2^{96} - 1.$$ 

ECDSA standard specifies reduction procedure given an integer "less than $p^2$": 

Write $A$ as $(A_{15}; A_{14}; A_{13}; A_{12}; A_{11}; A_{10}; A_9; A_8; A_7; A_6; A_5; A_4; A_3; A_2; A_1; A_0)$, meaning $P_i A_i 2^{32i}$.

Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as

- $(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$
- $(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$
- $(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$
- $(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$
- $(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$
- $(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$
- $(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$
- $(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$
- $(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo $p$ "by adding or subtracting a few copies" of $p$.

What is "a few copies"?

Variable-time loop is unsafe.
What a difference a prime makes

NIST P-256 prime $p$ is

$$2^{256} - 2^{224} + 2^{192} + 2^{96} - 1.$$ 

ECDSA standard specifies reduction procedure given an integer “$A$ less than $p$”:

Write $A$ as $(A_{15}; A_{14}; A_{13}; A_{12}; A_{11}; 0; 0; 0)$,

meaning $P_i A_i 2^{32i}$.

Define $T; S_1; S_2; S_3; S_4; D_1; D_2; D_3; D_4$ as

- $(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0)$,
- $(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0)$,
- $(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0)$,
- $(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8)$,
- $(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9)$,
- $(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11})$,
- $(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12})$
- $(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13})$
- $(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14})$.

Compute $T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4$.

Reduce modulo $p$ “by adding or subtracting a few copies” of $p$.
(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0);
(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0);
(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0);
(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8);
(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9);
(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11});
(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12});
(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13});
(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14}).

Compute \(T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4\).

Reduce modulo \(p\) “by adding or subtracting a few copies” of \(p\).
Compute \( T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4 \).

Reduce modulo \( p \) “by adding or subtracting a few copies” of \( p \).

What is “a few copies”? Variable-time loop is unsafe.

Correct but quite slow: conditionally add \( 4p \), conditionally add \( 2p \), conditionally add \( p \), conditionally sub \( 4p \), conditionally sub \( 2p \), conditionally sub \( p \).
\[(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0);
(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0);
(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0);
(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8);
(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9);
(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11});
(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12});
(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13});
(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14}).\]

Compute \( T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4. \)

Reduce modulo \( p \) “by adding or subtracting a few copies” of \( p \).

What is “a few copies”?
Variable-time loop is unsafe.

Correct but quite slow:
conditionally add \(4p\),
conditionally add \(2p\),
conditionally add \(p\),
conditionally sub \(4p\),
conditionally sub \(2p\),
conditionally sub \(p\).

Delay until end of computation?
Trouble: “A less than \(p^2\).”
(A_7, A_6, A_5, A_4, A_3, A_2, A_1, A_0);
(A_{15}, A_{14}, A_{13}, A_{12}, A_{11}, 0, 0, 0);
(0, A_{15}, A_{14}, A_{13}, A_{12}, 0, 0, 0);
(A_{15}, A_{14}, 0, 0, 0, A_{10}, A_9, A_8);
(A_8, A_{13}, A_{15}, A_{14}, A_{13}, A_{11}, A_{10}, A_9);
(A_{10}, A_8, 0, 0, 0, A_{13}, A_{12}, A_{11});
(A_{11}, A_9, 0, 0, A_{15}, A_{14}, A_{13}, A_{12});
(A_{12}, 0, A_{10}, A_9, A_8, A_{15}, A_{14}, A_{13});
(A_{13}, 0, A_{11}, A_{10}, A_9, 0, A_{15}, A_{14}).

Compute \( T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4 \).

Reduce modulo \( p \) “by adding or subtracting a few copies” of \( p \).

What is “a few copies”? Variable-time loop is unsafe.

Correct but quite slow:
conditionally add \( 4p \),
conditionally add \( 2p \),
conditionally add \( p \),
conditionally sub \( 4p \),
conditionally sub \( 2p \),
conditionally sub \( p \).

Delay until end of computation?
Trouble: “\( A \) less than \( p^2 \)”.

Even worse: what about platforms where \( 2^{32} \) isn’t best radix?
What is “a few copies”?

Variable-time loop is unsafe.

Correct but quite slow:
- conditionally add $4p$,
- conditionally add $2p$,
- conditionally add $p$,
- conditionally sub $4p$,
- conditionally sub $2p$,
- conditionally sub $p$.

Delay until end of computation?

Trouble: “A less than $p^2$”.

Even worse: what about platforms where $2^{32}$ isn’t best radix?
(A_7; A_6; A_5; A_4; A_3; A_2; A_1; A_0);
(A_15; A_14; A_13; A_12; A_11; 0; 0; 0);
(0; A_15; A_14; A_13; A_12; A_11);
(A_10; A_9; A_8);
(A_13; A_11; A_10; A_9);
(A_12; A_11;
A_13; A_12, A_11);
(A_14, A_13, A_12);
8, A_15, A_14, A_13);
(A_9, 0, A_15, A_14).

What is “a few copies”? 
Variable-time loop is unsafe.
Correct but quite slow:
conditionally add 4p,
conditionally add 2p,
conditionally add p,
conditionally sub 4p,
conditionally sub 2p,
conditionally sub p.
Delay until end of computation?
Trouble: “A less than p^2”.
Even worse: what about platforms 
where 2^{32} isn’t best radix?

There are many more ways that 
cryptographic design choices 
affect difficulty of building 
correct constant-time 
software. e.g. ECDSA needs 
divisions of scalars. EdDSA uses complete formulas. 
e.g. ECDSA splits elliptic-curve 
additions into several cases.
EdDSA uses complete formulas.
Compute \( T + 2S_1 + 2S_2 + S_3 + S_4 - D_1 - D_2 - D_3 - D_4 \).

Reduce modulo \( p \) "by adding or subtracting a few copies" of \( p \).

What is "a few copies"?

Variable-time loop is unsafe.

Correct but quite slow:
- conditionally add \( 4p \),
- conditionally add \( 2p \),
- conditionally add \( p \),
- conditionally sub \( 4p \),
- conditionally sub \( 2p \),
- conditionally sub \( p \).

Delay until end of computation?

Trouble: "A less than \( p^2 \)."

Even worse: what about platforms where \( 2^{32} \) isn't best radix?

There are many more ways that cryptographic design choices affect difficulty of building fast correct constant-time software.

E.g. ECDSA needs divisions of scalars. EdDSA doesn't.

E.g. ECDSA splits elliptic-curve additions into several cases. EdDSA uses complete formulas.
What is “a few copies”? Variable-time loop is unsafe.

Correct but quite slow: conditionally add 4\(p\), conditionally add 2\(p\), conditionally add \(p\), conditionally sub 4\(p\), conditionally sub 2\(p\), conditionally sub \(p\).

Delay until end of computation? Trouble: “A less than \(p^2\”).

Even worse: what about platforms where \(2^{32}\) isn’t best radix?

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What’s better use of time: implementing ECDSA, or upgrading protocol to EdDSA?