Sorting integer arrays: security, speed, and verification

D. J. Bernstein
Bob’s laptop screen:

From: Alice

Thank you for your submission. We received many interesting papers, and unfortunately your

Bob assumes this message is something Alice actually sent.

But today’s “security” systems fail to guarantee this property. Attacker could have modified or forged the message.
Trusted computing base (TCB)

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Security policy for this talk:
If message is displayed on Bob’s screen as “From: Alice” then message is from Alice.
If TCB works correctly, then message is guaranteed to be from Alice, no matter what the rest of the system does.
Examples of attack strategies:

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Massive TCB has many bugs, including many security holes. Any hope of fixing this?
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e.g. Bob runs many VMs:

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<th>...</th>
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Browser in VM C isn’t in TCB. Can’t touch data in VM A, if TCB works correctly.

Alice also runs many VMs.
Cryptography

How does Bob’s laptop know that incoming network data is from Alice’s laptop?

Cryptographic solution: Message-authentication codes.

Alice’s message

\[ k \]

\[ \text{authenticated message} \]

untrusted network

\[ \text{authenticated message} \]

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Alice’s message

\[ \rightarrow \]

authenticated message

untrusted network

modified message

“Alert: forgery!”

\[ \rightarrow \]
Important for Alice and Bob to share the same secret $k$.

What if attacker was spying on their communication of $k$?
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What if attacker was spying on their communication of $k$?

Solution 1: Public-key encryption.
Solution 2:
Public-key signatures.
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No more shared secret $k$ but Alice still has secret $a$.

Cryptography requires TCB to protect secrecy of keys, even if user has no other secrets.
Constant-time software

Large portion of CPU hardware: optimizations depending on addresses of memory locations.

Consider data caching, instruction caching, parallel cache banks, store-to-load forwarding, branch prediction, etc.
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Many attacks (e.g. TLBleed from 2018 Gras–Razavi–Bos–Giuffrida) show that this portion of the CPU has trouble keeping secrets.
Typical literature on this topic:
Understand this portion of CPU. But details are often proprietary, not exposed to security review.

Try to push attacks further. This becomes very complicated.

Tweak the attacked software to try to stop the known attacks.
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For researchers: This is great!

For auditors: This is a nightmare. Many years of security failures. No confidence in future security.
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TCB analysis: Need this portion of the CPU to be correct, but don’t need it to keep secrets. Makes auditing much easier.
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TCB analysis: Need this portion of the CPU to be correct, but don’t need it to keep secrets. Makes auditing much easier.

Good match for attitude and experience of CPU designers: e.g., Intel issues errata for correctness bugs, not for information leaks.
Case study: Constant-time sorting

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Subroutine in some submissions: sort array of secret integers. e.g. sort 768 32-bit integers.
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One submission to competition: “Radix sort is used as constant-time sorting algorithm.” Some versions of radix sort avoid secret branches. But data addresses in radix sort still depend on secrets.
Foundation of solution: a **comparator** sorting 2 integers.

\[ \min\{x, y\} \quad \max\{x, y\} \]

Easy constant-time exercise in C. Warning: C standard allows compiler to screw this up.

Even easier exercise in asm.
Combine comparators into a **sorting network** for more inputs.

Example of a sorting network:
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Speed is a serious issue in the post-quantum competition. “Cost” is evaluation criterion; “we’d like to stress this once again on the forum that we’d really like to see more platform-optimized implementations”; etc.
void int32_sort(int32 *x, int64 n) {
    int64 t, p, q, i;

    if (n < 2) return;

    t = 1;
    while (t < n - t) t += t;

    for (p = t; p > 0; p >>= 1) {
        for (i = 0; i < n - p; ++i)
            if (!(i & p))
                minmax(x+i, x+i+p);

        for (q = t; q > p; q >>= 1)
            for (i = 0; i < n - q; ++i)
                if (!(i & p))
                    minmax(x+i+p, x+i+q);
    }
}
Previous slide: C translation of 1973 Knuth “merge exchange”, which is a simplified version of 1968 Batcher “odd-even merge” sorting networks.

\[ \approx n (\log_2 n)^2 / 4 \] comparators.

Much faster than bubble sort.

Warning: many other descriptions of Batcher’s sorting networks require \( n \) to be a power of 2. Also, Wikipedia says “Sorting networks . . . are not capable of handling arbitrarily large inputs.”
This constant-time sorting code

↓

vectorization (for Haswell)

↓

Constant-time sorting code included in 2017 Bernstein–Chuengsatiansup–Lange–van Vredendaal “NTRU Prime” software release

↓

revamped for higher speed

↓

New: “djbsort” constant-time sorting code
The slowdown for constant time

Massive fast-sorting literature.

2015 Gueron–Krasnov: AVX and AVX2 (Haswell) optimization of quicksort. For 32-bit integers:
\[
\approx 45 \text{ cycles/byte for } n \approx 2^{10},
\]
\[
\approx 55 \text{ cycles/byte for } n \approx 2^{20}.
\]

Slower than “the radix sort implemented of IPP, which is the fastest in-memory sort we are aware of”: 32, 40 cycles/byte.

IPP: Intel’s Integrated Performance Primitives library.
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Warning: Comparison for $n \approx 2^{20}$ involves microarchitecture details beyond Haswell core. Should measure all code on same CPU.
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Answer: well-known trends in CPU design, reflecting fundamental hardware costs of various operations.
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Every cycle, Haswell core can do 8 “min” ops on 32-bit integers + 8 “max” ops on 32-bit integers.
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Loading a 32-bit integer from a random address: much slower.

Conditional branch: much slower.
Verification

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Test the sorting software on many random inputs, increasing inputs, decreasing inputs. Seems to work.
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But are there *occasional* inputs where this sorting software fails to sort correctly?

History: Many security problems involve occasional inputs where TCB works incorrectly.
For each used $n$ (e.g., 768):

1. C code
2. normal compiler
3. machine code
4. symbolic execution
5. fully unrolled code
6. new peephole optimizer
7. unrolled min-max code
8. new sorting verifier
9. yes, code works
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First djbsort release, verified int32 on AVX2:

https://sorting.cryp.to

Includes the sorting code; automatic build-time tests; simple benchmarking program; verification tools.

Web site shows how to use the verification tools.

Next release planned: verified ARM NEON code and verified portable code.