Small cryptographic bytecode

D. J. Bernstein

elaborating on an idea from
Adam Langley
“Line search”: trying to find minimum of function $f$ defined on $x$-line.

e.g. “Bisection”, trying to find minimum in interval $[x_0, x_1]$:
Replace interval with either $[x_0, (x_0+x_1)/2]$ or $[(x_0+x_1)/2, x_1]$; try to make sensible choice.
Iterate many times.
“Line search”: trying to find minimum of function \( f \) defined on \( x \)-line.

e.g. “Bisection”, trying to find minimum in interval \([x_0, x_1]\):
Replace interval with either
\([x_0, (x_0+x_1)/2]\) or \([(x_0+x_1)/2, x_1]\);
try to make sensible choice.
Iterate many times.

Can try to reduce \#iterations using smarter models of \( f \):
see, e.g., “secant method”. 
“Line search”: trying to find minimum of function $f$ defined on $x$-line.

e.g. “Bisection”, trying to find minimum in interval $[x_0, x_1]$:
Replace interval with either $[x_0, (x_0 + x_1)/2]$ or $[(x_0 + x_1)/2, x_1]$; try to make sensible choice.
Iterate many times.

Can try to reduce #iterations using smarter models of $f$: see, e.g., “secant method”.

Harder when $f$ varies more.
How to find minimum of function $f$ defined on $(x, y)$-plane?

“Gradient descent”: Starting from $(x_0, y_0)$, try to figure out direction where $f$ decreases fastest.
How to find minimum of function \( f \) defined on \((x, y)\)-plane?

“Gradient descent”: Starting from \((x_0, y_0)\), try to figure out direction where \( f \) decreases fastest.

Could do line search to find minimum in that direction. Then find a new direction.
How to find minimum of function $f$ defined on $(x, y)$-plane?

“Gradient descent”: Starting from $(x_0, y_0)$, try to figure out direction where $f$ decreases fastest.

Could do line search to find minimum in that direction. Then find a new direction.

Better: Step down that direction. Then find a new direction.
How to find minimum of function $f$ defined on $(x, y)$-plane?

“Gradient descent”:  
Starting from $(x_0, y_0)$,  
try to figure out direction where $f$ decreases fastest.

Could do line search to find minimum in that direction.  
Then find a new direction.

Better: Step down that direction.  
Then find a new direction.

Silly: Line search in $x$ direction;  
line search in $y$ direction; repeat.
Keccak optimization

Goal: Fastest C code for Keccak on a Cortex-M4 CPU core.

You start with simple C code implementing Keccak.
Keccak optimization

Goal: Fastest C code for Keccak on a Cortex-M4 CPU core.

You start with simple C code implementing Keccak.

You compile it; see how fast it is; modify it to try to make it faster; repeat; eventually stop trying.
Keccak optimization

Goal: Fastest C code for Keccak on a Cortex-M4 CPU core.

You start with simple C code implementing Keccak.

You compile it; see how fast it is; modify it to try to make it faster; repeat; eventually stop trying.

You publish your fastest code.

Maybe lots of people use it, and care about its speed.
Compiler writer learns about your Keccak Cortex-M4 C code.
Compiler writer learns about your Keccak Cortex-M4 C code.

Compiles it; sees how fast it is. Modifies *compiler* to try to make the compiled code faster. Repeats; eventually stops trying.
Compiler writer learns about your Keccak Cortex-M4 C code.

Compiles it; sees how fast it is. Modifies compiler to try to make the compiled code faster. Repeats; eventually stops trying.

Publishes a new compiler version.
Compiler writer learns about your Keccak Cortex-M4 C code. Compiles it; sees how fast it is. Modifies compiler to try to make the compiled code faster. Repeats; eventually stops trying. Publishes a new compiler version. Later: Maybe you try the new compiler. Whole process repeats.
Compiler writer learns about your Keccak Cortex-M4 C code.

Compiles it; sees how fast it is. Modifies \textit{compiler} to try to make the compiled code faster. Repeats; eventually stops trying.

Publishes a new compiler version.

Later: Maybe you try the new compiler. Whole process repeats.

You treat compiler as constant. Compiler treats code as constant.
Define $f(x, y)$ as time taken by code $x$ with compiler $y$. 
Define $f(x, y)$ as time taken by code $x$ with compiler $y$.

$x_0$: initial code.

$y_0$: initial compiler.
Define $f(x, y)$ as time taken by code $x$ with compiler $y$.

$x_0$: initial code.

$y_0$: initial compiler.

You try to minimize $f(x, y_0)$.

$x_1$: new code from this line search in $x$ direction.
Define $f(x, y)$ as time taken by code $x$ with compiler $y$.

$x_0$: initial code.

$y_0$: initial compiler.

You try to minimize $f(x, y_0)$.

$x_1$: new code from this line search in $x$ direction.

Compiler writer: $f(x_1, y)$.

$y_1$: new compiler from this line search in $y$ direction.
Define $f(x, y)$ as time taken by code $x$ with compiler $y$.

$x_0$: initial code.

$y_0$: initial compiler.

You try to minimize $f(x, y_0)$.

$x_1$: new code from this line search in $x$ direction.

Compiler writer: $f(x_1, y)$.

$y_1$: new compiler from this line search in $y$ direction.

This whole approach is silly.
min\{f(x, y)\} is the time taken by fastest Keccak Cortex-M4 asm.
\( \min\{f(x, y)\} \) is the time taken by fastest Keccak Cortex-M4 asm.

Slowly bouncing between \( x \)-line searches, \( y \)-line searches is a silly way to approach this min.
\( \text{min}\{f(x, y)\} \) is the time taken by fastest Keccak Cortex-M4 asm.

Slowly bouncing between \( x \)-line searches, \( y \)-line searches is a silly way to approach this min.

Clearly min can be achieved by many different pairs \((x, y)\).

Which pair is easiest to find?
min\{f(x, y)\} is the time taken by fastest Keccak Cortex-M4 asm.

Slowly bouncing between \(x\)-line searches, \(y\)-line searches is a silly way to approach this min.

Clearly min can be achieved by many different pairs \((x, y)\). Which pair is easiest to find?

Generalize from C to other languages: which language makes min easiest to find?

Why did goal say “C code”? End user doesn’t need C.
Does end user need Cortex-M4?
Does end user need Cortex-M4?

CPU designer learns about your Keccak Cortex-M4 asm.
Does end user need Cortex-M4?

CPU designer learns about your Keccak Cortex-M4 asm.

Modifies the CPU design to try to make this code faster.
Repeats; eventually stops trying.
Does end user need Cortex-M4?

CPU designer learns about your Keccak Cortex-M4 asm.

Modifies the CPU design to try to make this code faster.
Repeats; eventually stops trying.

Years later, sells a new CPU.
You reoptimize for this CPU.
Does end user need Cortex-M4?

CPU designer learns about your Keccak Cortex-M4 asm.

Modifies the CPU design to try to make this code faster. Repeats; eventually stops trying.

Years later, sells a new CPU. You reoptimize for this CPU.

_Sometimes_ CPUs try extending or replacing instruction set, but this is poorly coordinated with programmers, compiler writers.
Generalize $f(x, y)$ definition: $f(x, y)$ is time taken by code $x$ on platform $y$.

If compiler $y$ on code $x$ produces $\text{asm } y(x)$ for Cortex-M4: $f(x, y) = f(y(x), \text{Cortex-M4})$. 
Generalize $f(x, y)$ definition: $f(x, y)$ is time taken by code $x$ on platform $y$.

If compiler $y$ on code $x$ produces asm $y(x)$ for Cortex-M4: $f(x, y) = f(y(x), \text{Cortex-M4})$.

Without the CPU changing: Minimize $f(a, \text{Cortex-M4})$. Search for $(x, y)$ with $y(x) = a$. 
Generalize \( f(x, y) \) definition:
\( f(x, y) \) is time taken by code \( x \) on platform \( y \).

If compiler \( y \) on code \( x \) produces asm \( y(x) \) for Cortex-M4:
\[ f(x, y) = f(y(x), \text{Cortex-M4}). \]

Without the CPU changing:
Minimize \( f(a, \text{Cortex-M4}) \).
Search for \( (x, y) \) with \( y(x) = a \).

Typical CPU designer:
View \( a \) as a constant;
try to minimize \( f(a, y) \).
Silly optimization approach.
“I know the minimum!
I’ve developed the fastest circuit
that computes Keccak.
This circuit is my CPU.”
“I know the minimum!
I’ve developed the fastest circuit that computes Keccak.
This circuit is my CPU.”

Wait a minute: “CPU” concept is more restrictive than “chip”.

Perspective of CPU designer: This chip can do anything!

People want this chip to support SHA-1, SHA-2, SHA-3, SHAMir; all sorts of block ciphers; public-key cryptosystems; non-cryptographic computations.
Adding fast Keccak circuit ("Keccak coprocessor") to CPU adds area to CPU.

Adding fast coprocessors for desired mix of operations adds even more area to CPU.
Adding fast Keccak circuit ("Keccak coprocessor") to CPU adds area to CPU.

Adding fast coprocessors for desired mix of operations adds even more area to CPU.

For same CPU area, obtain much better throughput by building many copies of original CPU core without these coprocessors.
Adding fast Keccak circuit ("Keccak coprocessor") to CPU adds area to CPU.

Adding fast coprocessors for desired mix of operations adds even more area to CPU.

For same CPU area, obtain much better throughput by building many copies of original CPU core without these coprocessors.

Fast Keccak chip is special case. Doesn’t reflect general case.
CPU designer’s metric:
What is best performance
for a specified mix of operations
within a particular CPU area?
CPU designer’s metric: What is best performance for a specified mix of operations within a particular CPU area?

CPU designer is much more likely to consider incorporating a small Keccak coprocessor.
CPU designer’s metric: What is best performance for a specified mix of operations within a particular CPU area?

CPU designer is much more likely to consider incorporating a **small** Keccak coprocessor.

“So we should design the **smallest** Keccak circuit?”
CPU designer’s metric: What is best performance for a specified mix of operations within a particular CPU area?

CPU designer is much more likely to consider incorporating a small Keccak coprocessor.

“So we should design the smallest Keccak circuit?”

—Maybe, but will this extreme be faster than using existing CPU instructions without coprocessor?
Intel typically designs quite large CPU cores: 32KB L1 data cache, 32KB L1 instruction cache, several fast multipliers, many different instructions, out-of-order unit, etc.

“So it’s small cost for Intel to add instruction-set extension for my favorite crypto!”
Intel typically designs quite large CPU cores: 32KB L1 data cache, 32KB L1 instruction cache, several fast multipliers, many different instructions, out-of-order unit, etc.

“So it’s small cost for Intel to add instruction-set extension for my favorite crypto!”

—Yes, but even smaller benefit for Intel’s mix of operations.
Intel did add instruction for 1 round of AES.

How many parallel S-boxes are in an AES-round coprocessor?

Can be 16: big; fast.
8: smaller but slower.
4: even smaller but slower.
... 1: probably not worthwhile compared to skipping coprocessor and using other CPU instructions.
Intel did add instruction for 1 round of AES.

How many parallel S-boxes are in an AES-round coprocessor?

Can be 16: big; fast.
8: smaller but slower.
4: even smaller but slower.
... 1: probably not worthwhile compared to skipping coprocessor and using other CPU instructions.

An instruction for 4 rounds of SHA-256 is in a few Intel CPUs.
Lightweight crypto

Frequent claim in literature, where $X$ might be

- Keccak;
- any secure hash;
- a secure cipher; . . . :

“Resource-constrained IoT devices need the smallest circuit for $X$.”
Lightweight crypto

Frequent claim in literature, where $X$ might be

- Keccak;
- any secure hash;
- a secure cipher; ...:

“Resource-constrained IoT devices need the smallest circuit for $X$.”

—Even if speed is acceptable, who will use smallest $X$ circuit?
Lightweight crypto

Frequent claim in literature, where $X$ might be
- Keccak;
- any secure hash;
- a secure cipher; . . . :

“Resource-constrained IoT devices need the smallest circuit for $X$.”

—Even if speed is acceptable, who will use smallest $X$ circuit?

Why should minimum area for $X$ give minimum area for IoT+$X$?
An idea from Adam Langley

Consider a device that receives public keys from trusted sources; receives data supposedly signed under these public keys; verifies these signatures.

E.g. an SSL client.

Painful historical event: all clients needed upgrades to support new hash functions since old functions were broken.
A public key is a signature-verification program in a limited language.

Langley’s idea:
Replace this language with a full programming language.
Then can upgrade hash function (or upgrade to post-quantum signatures!) by changing public keys, with no changes to clients.
A public key is a signature-verification program in a limited language.

Langley’s idea: Replace this language with a full programming language. Then can upgrade hash function (or upgrade to post-quantum signatures!) by changing public keys, with no changes to clients.

Same for public-key encryption systems: public key is program.
Say verification device is a chip of area $A$.
How small can public keys be?
Have to consider, e.g., size of a SHA-256 program, size of a Keccak program, etc.
Say verification device is a chip of area $A$.
How small can public keys be?
Have to consider, e.g., size of a SHA-256 program, size of a Keccak program, etc.
Similar question to optimizing total size of a CPU with a SHA-256 instruction, a Keccak instruction, etc.
Say verification device is a chip of area $A$.

How small can public keys be?

Have to consider, e.g., size of a SHA-256 program, size of a Keccak program, etc.

Similar question to optimizing total size of a CPU with a SHA-256 instruction, a Keccak instruction, etc.

Not the usual code-size question.
Change the language!