# Standardization for the black hat

Daniel J. Bernstein

University of Illinois at Chicago &
Technische Universiteit Eindhoven

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(1) bada55.cr.yp.to "BADA55 Crypto" including "How to manipulate curve standards: a white paper for the black hat."

(2) projectbullrun.org including "Dual EC: a standardized back door."



Includes joint work with (in alphabetical order):

Tung Chou (1)
Chitchanok Chuengsatiansup (1)
Andreas Hülsing (1)
Eran Lambooij (1)
Tanja Lange (1) (2)
Ruben Niederhagen (1) (2)
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"is the only DRBG mechanism in this Recommendation whose security is related to a hard problem in number theory."



## Denial of service via hoops

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2006 Schoenmakers–Sidorenko:
Dual EC flunks well-established definition of PRNG security.

Are *all* applications broken?
Obviously not! Standardize!

## Flooding via dishonesty

If we were honest then we would tell Alice+Bob to reuse ciphers/hashes as PRNGs.

But why should we be honest? Let's build PRNGs from scratch!

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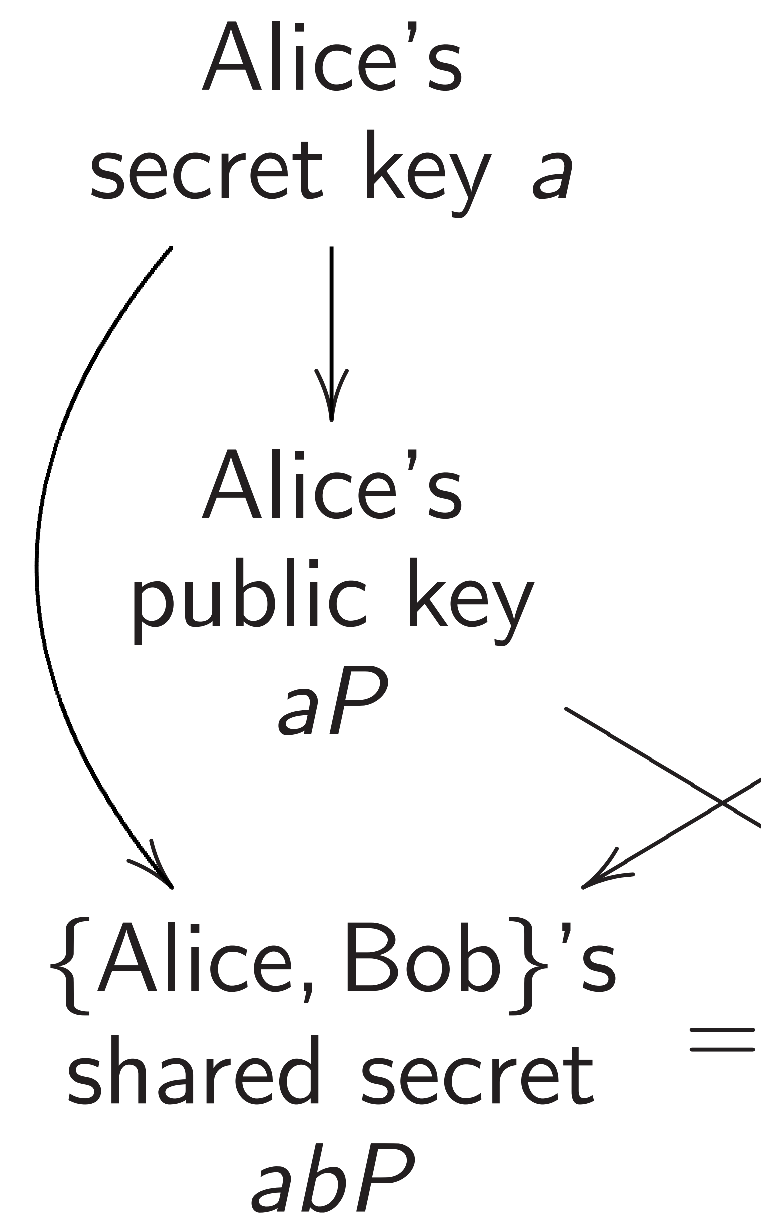

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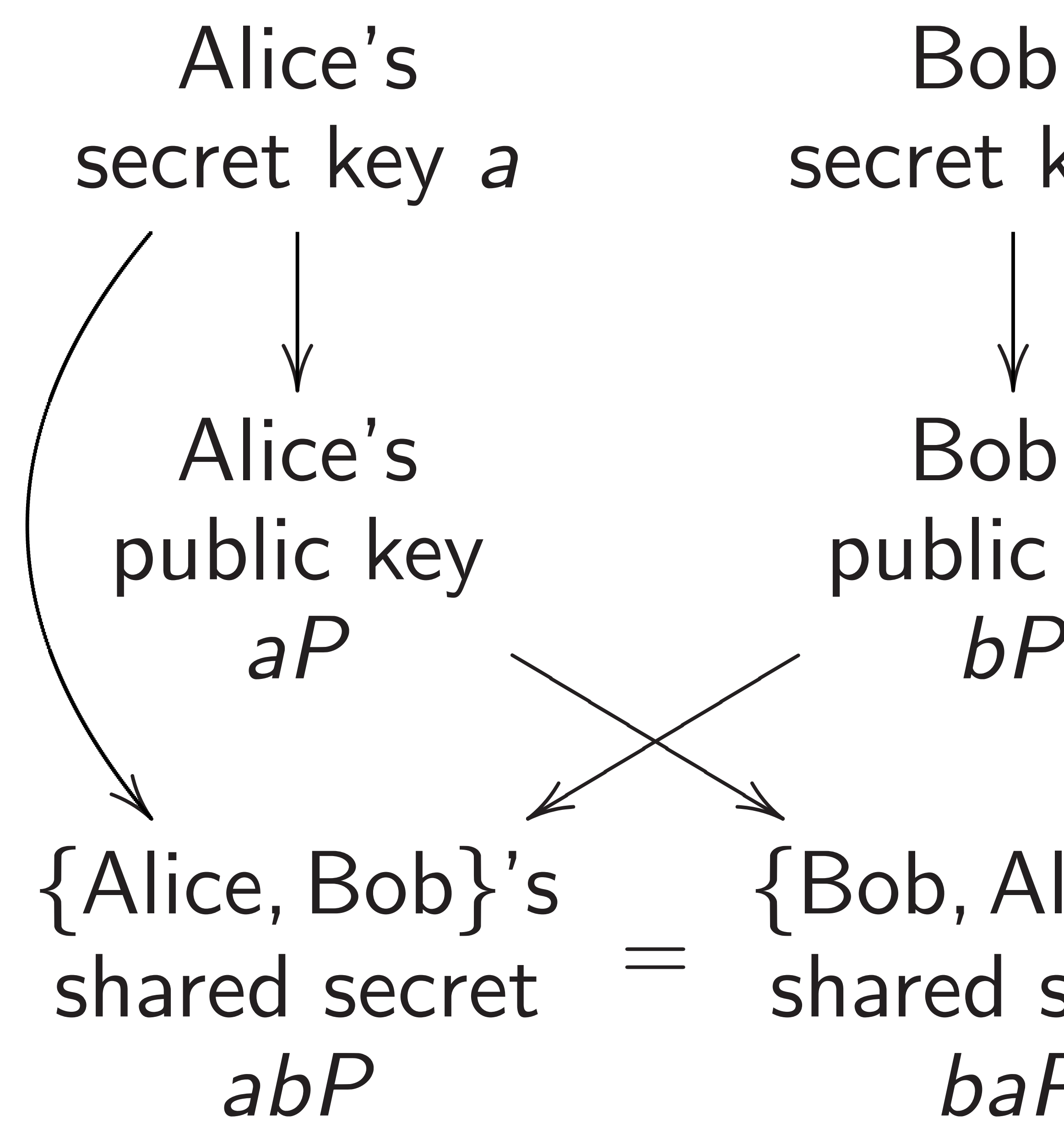
Textbook key exchange using standard point $P$ on a standard elliptic curve



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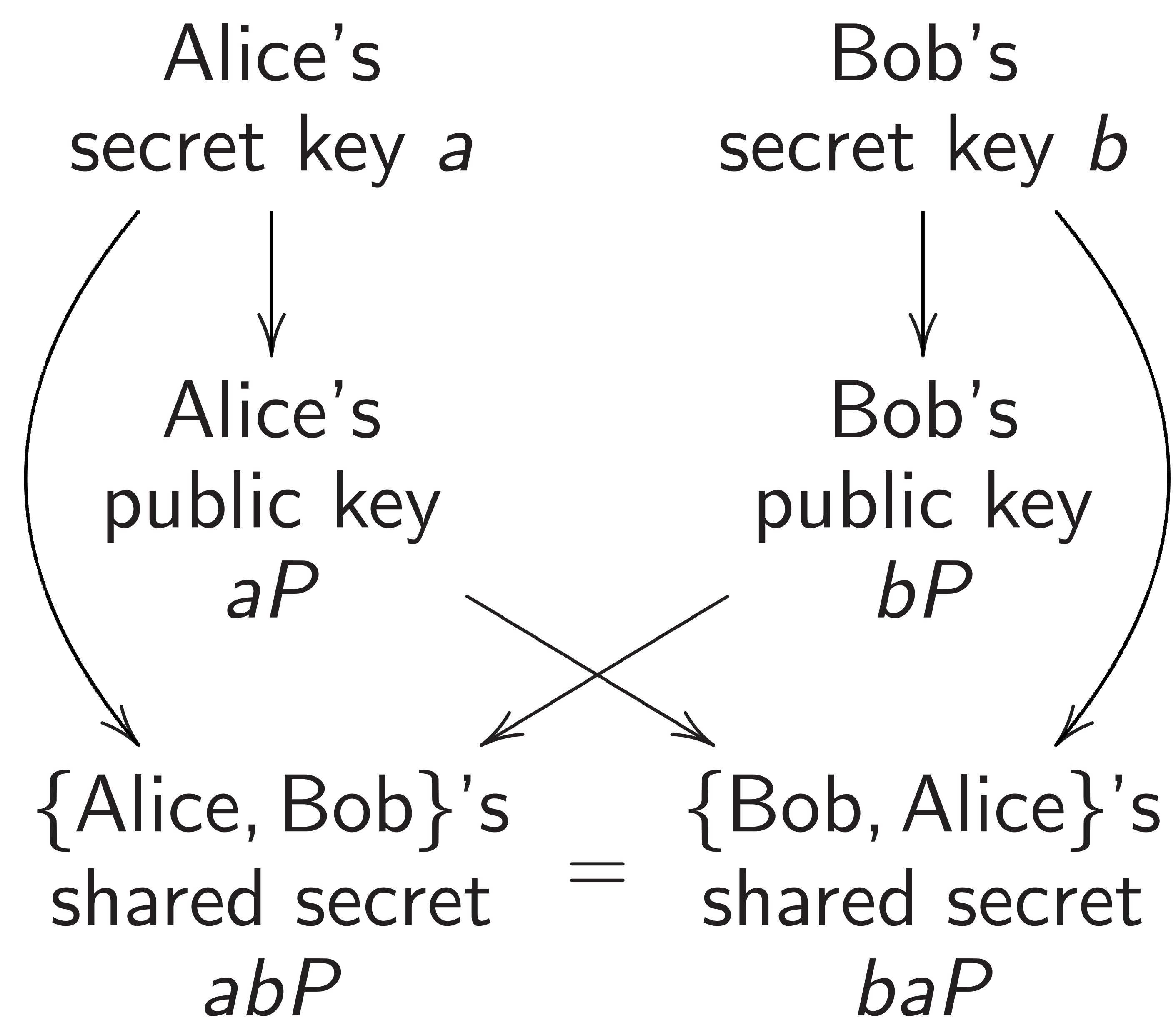

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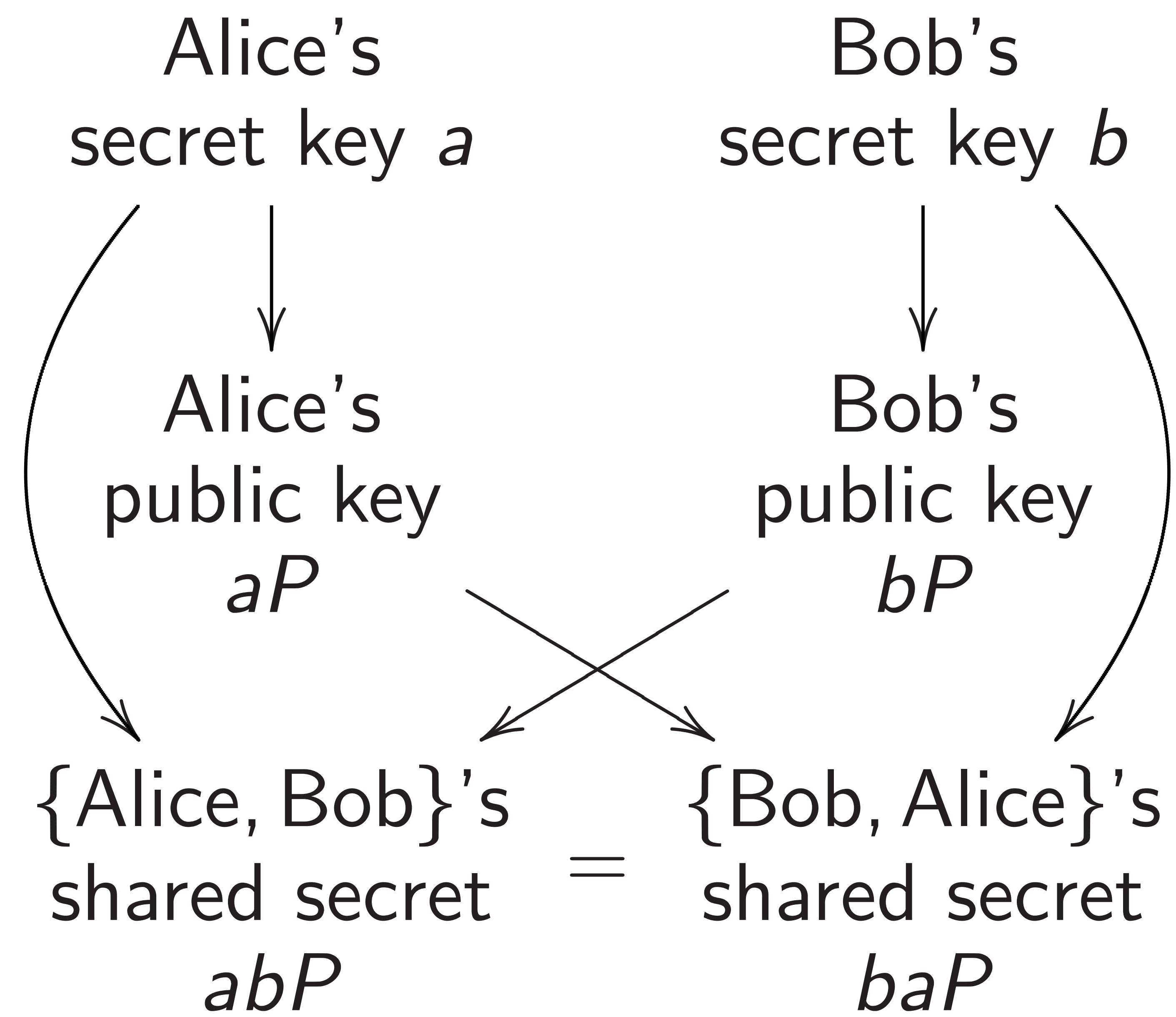


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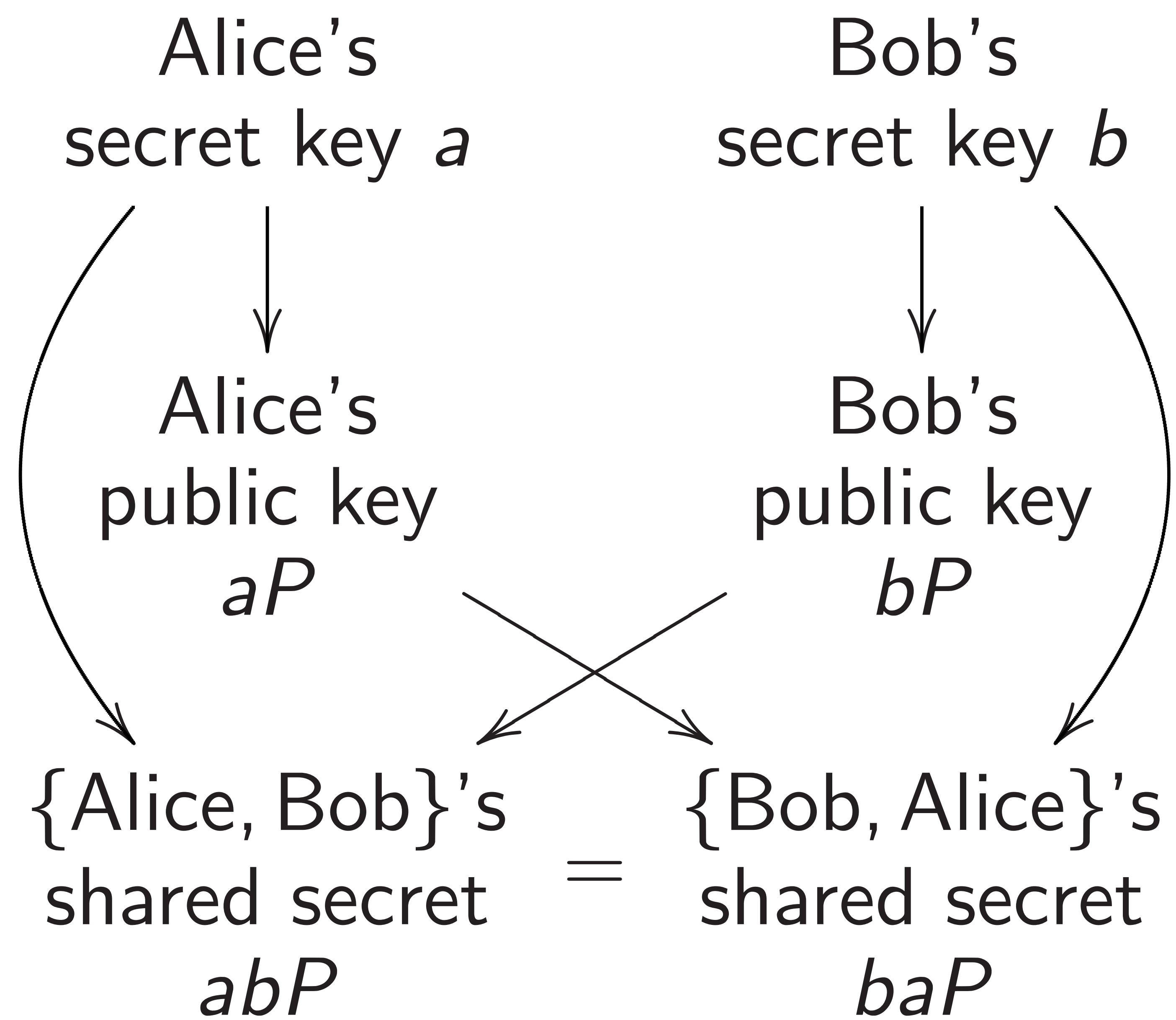


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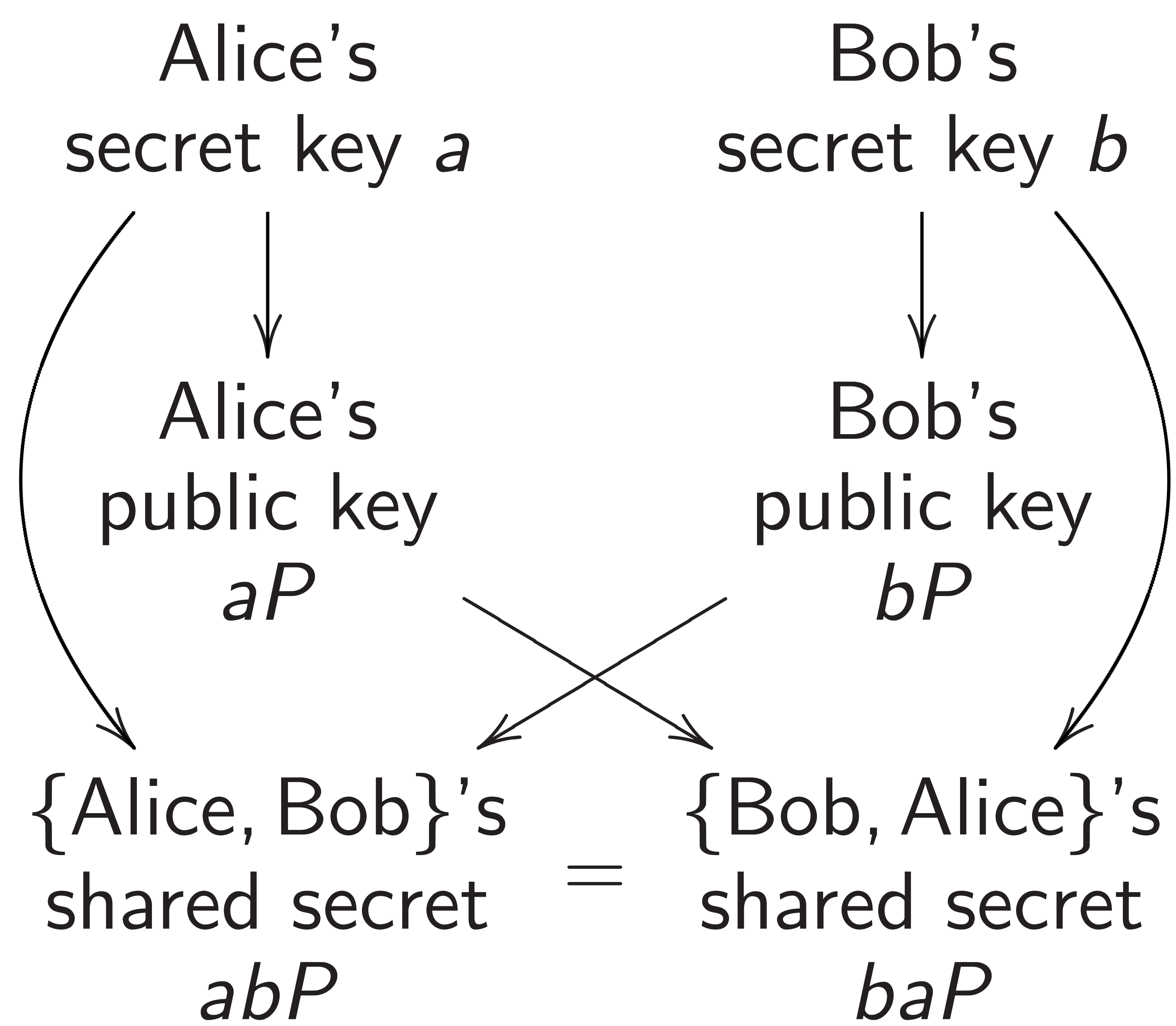


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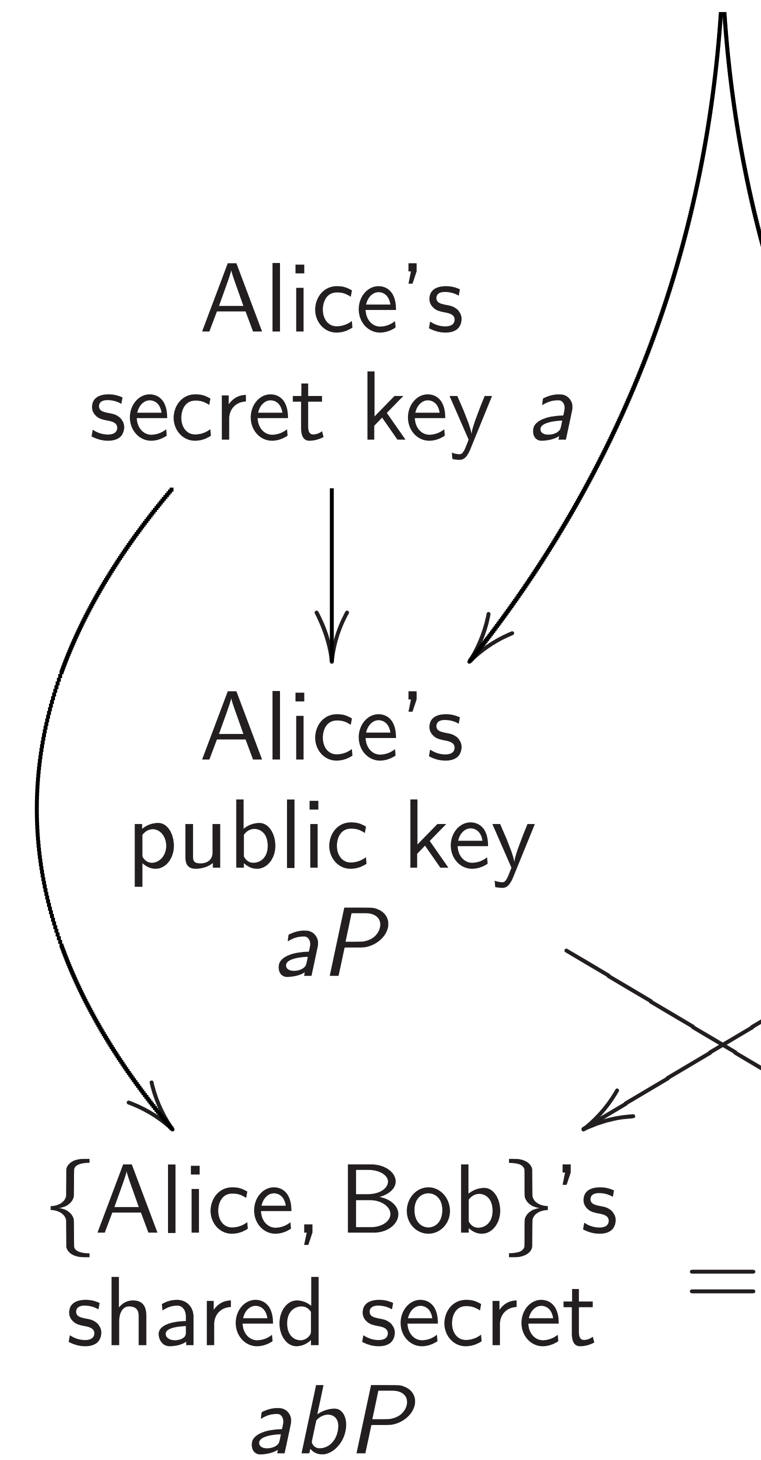


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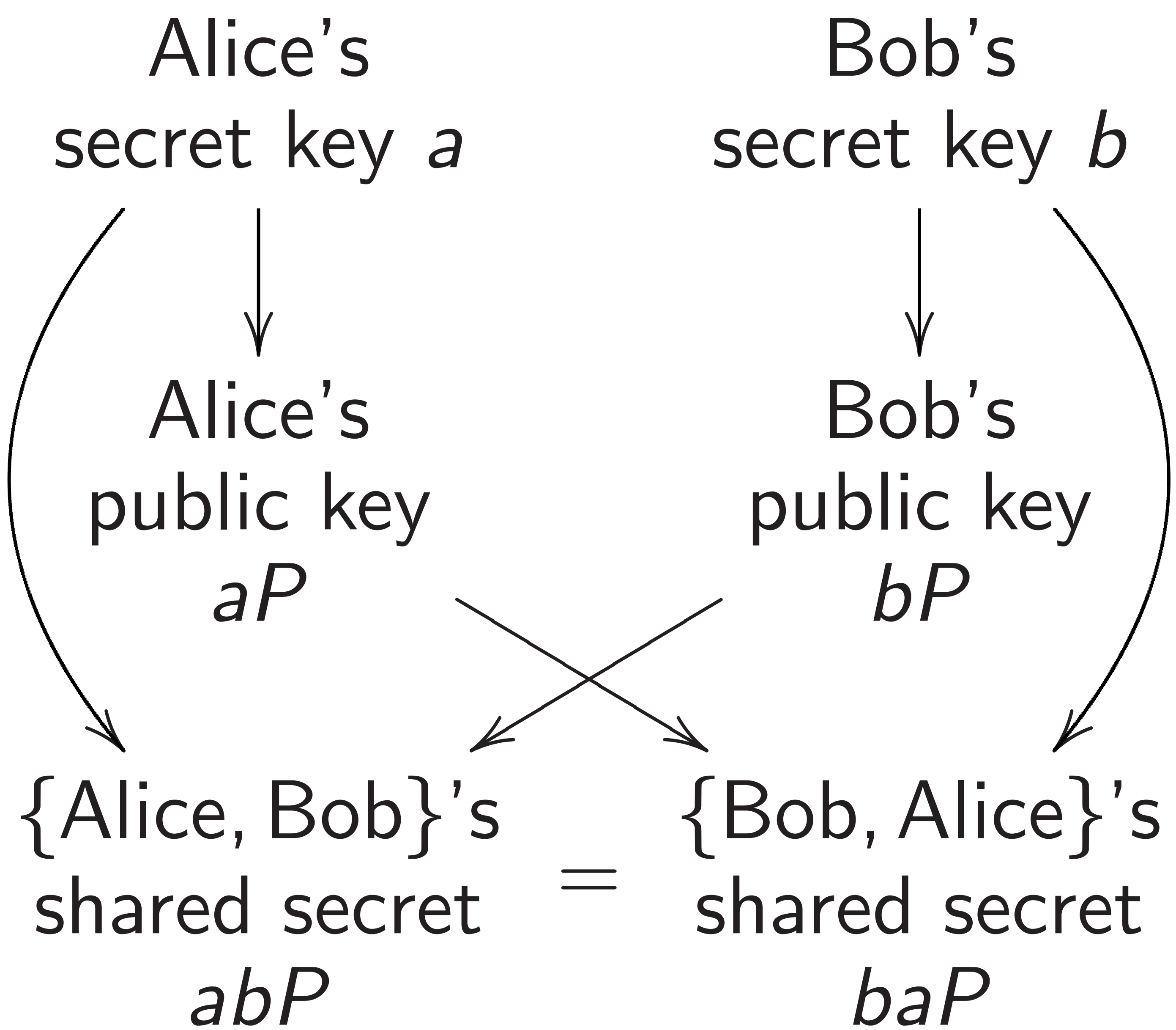


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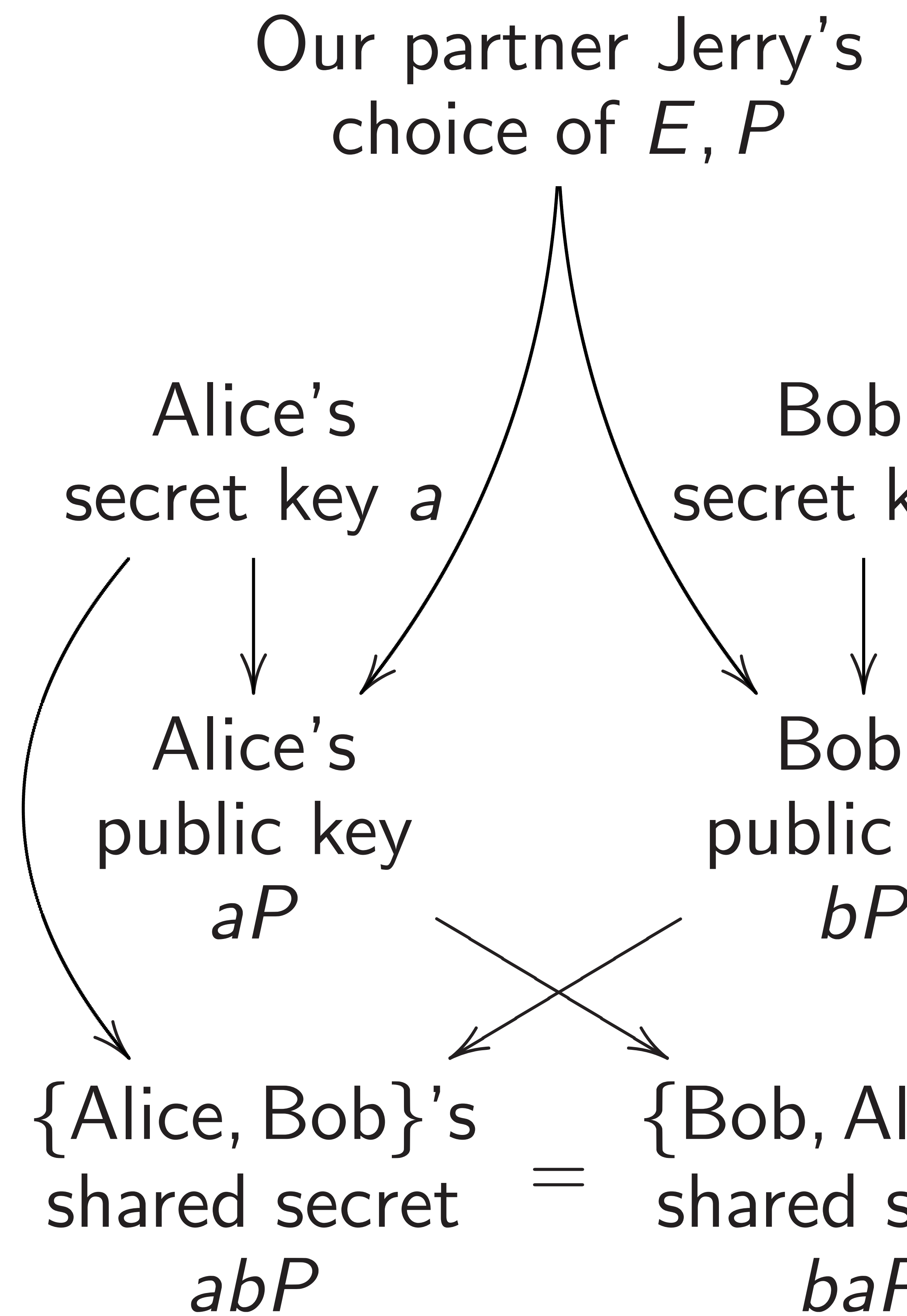


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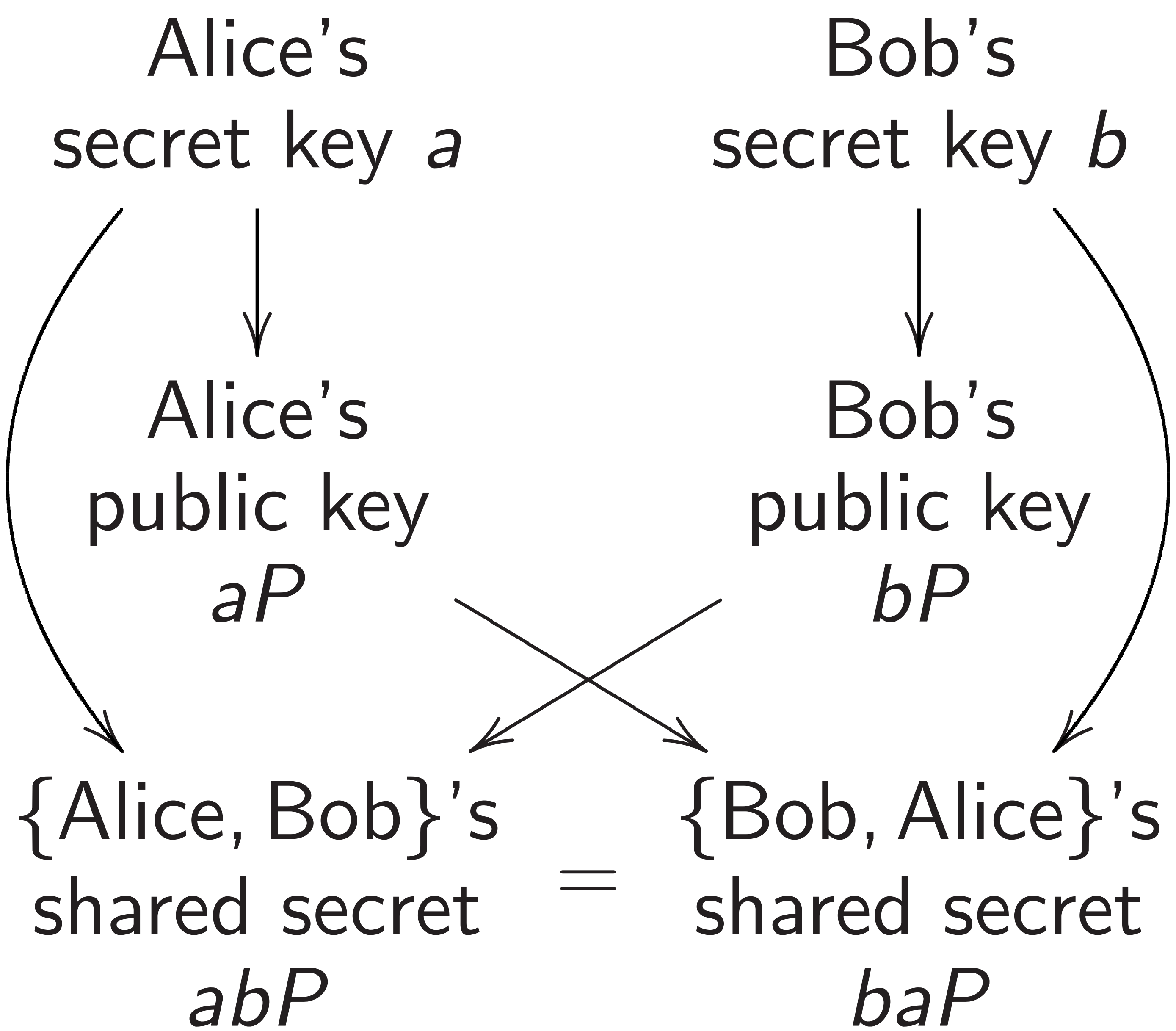


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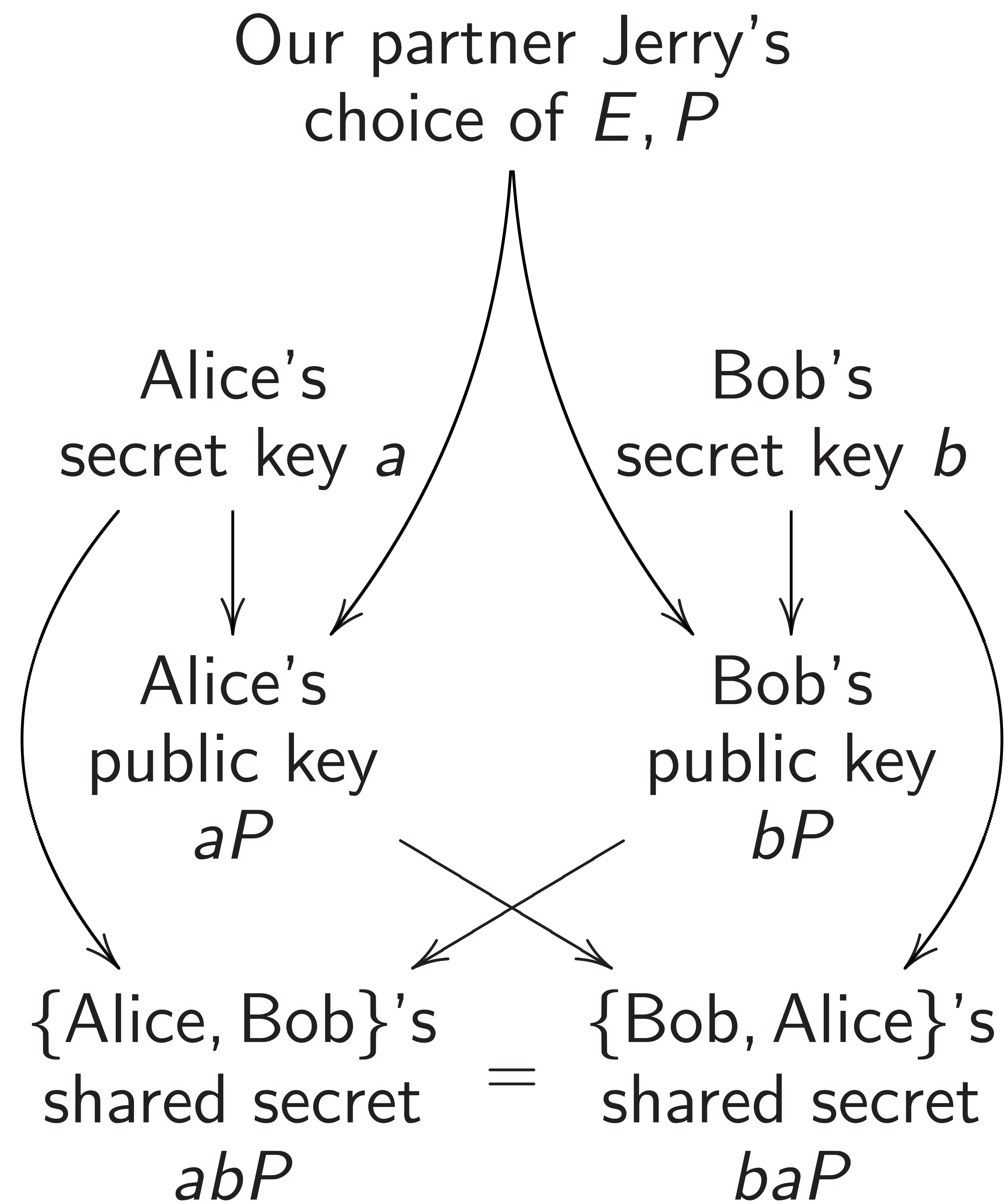


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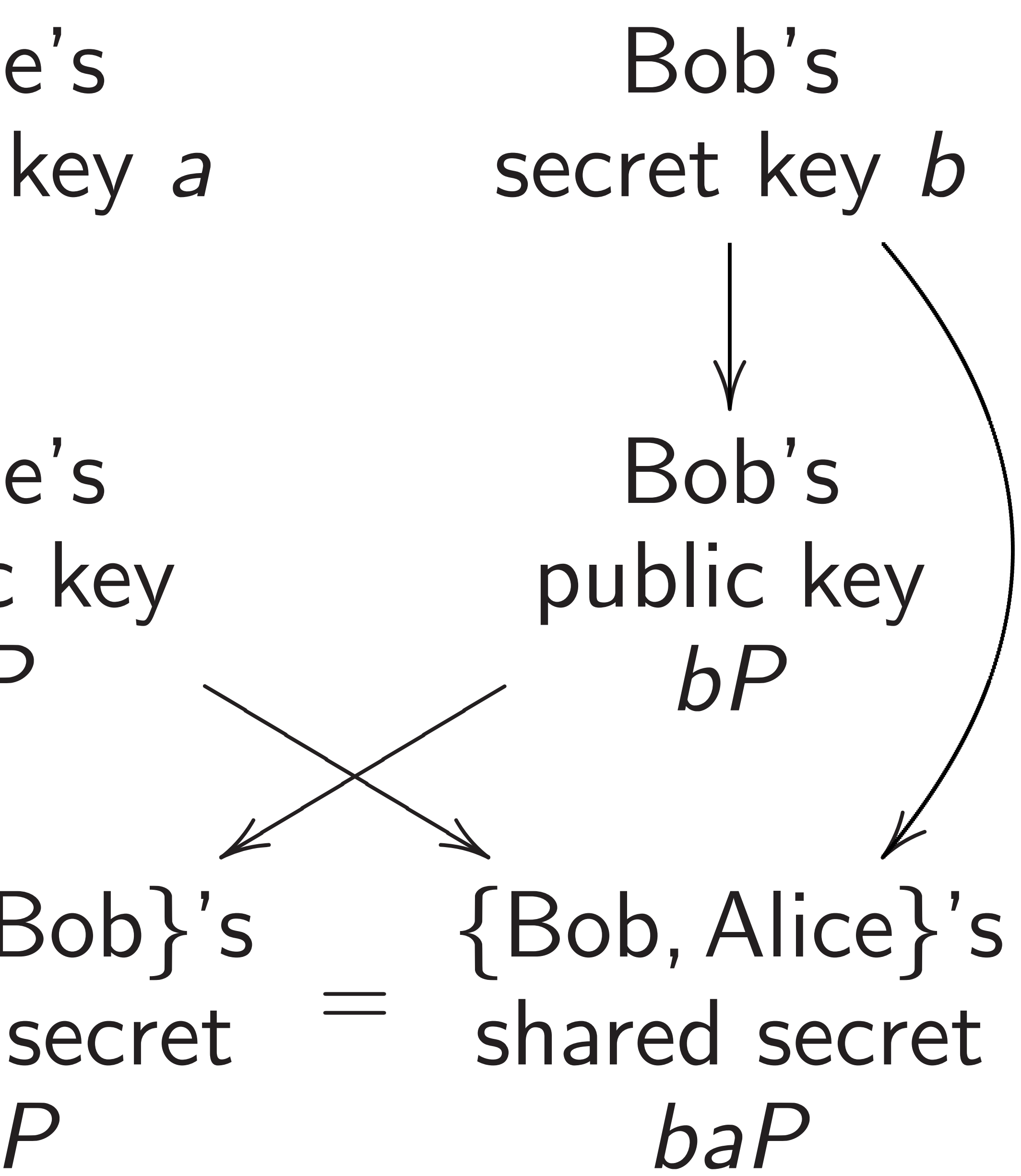


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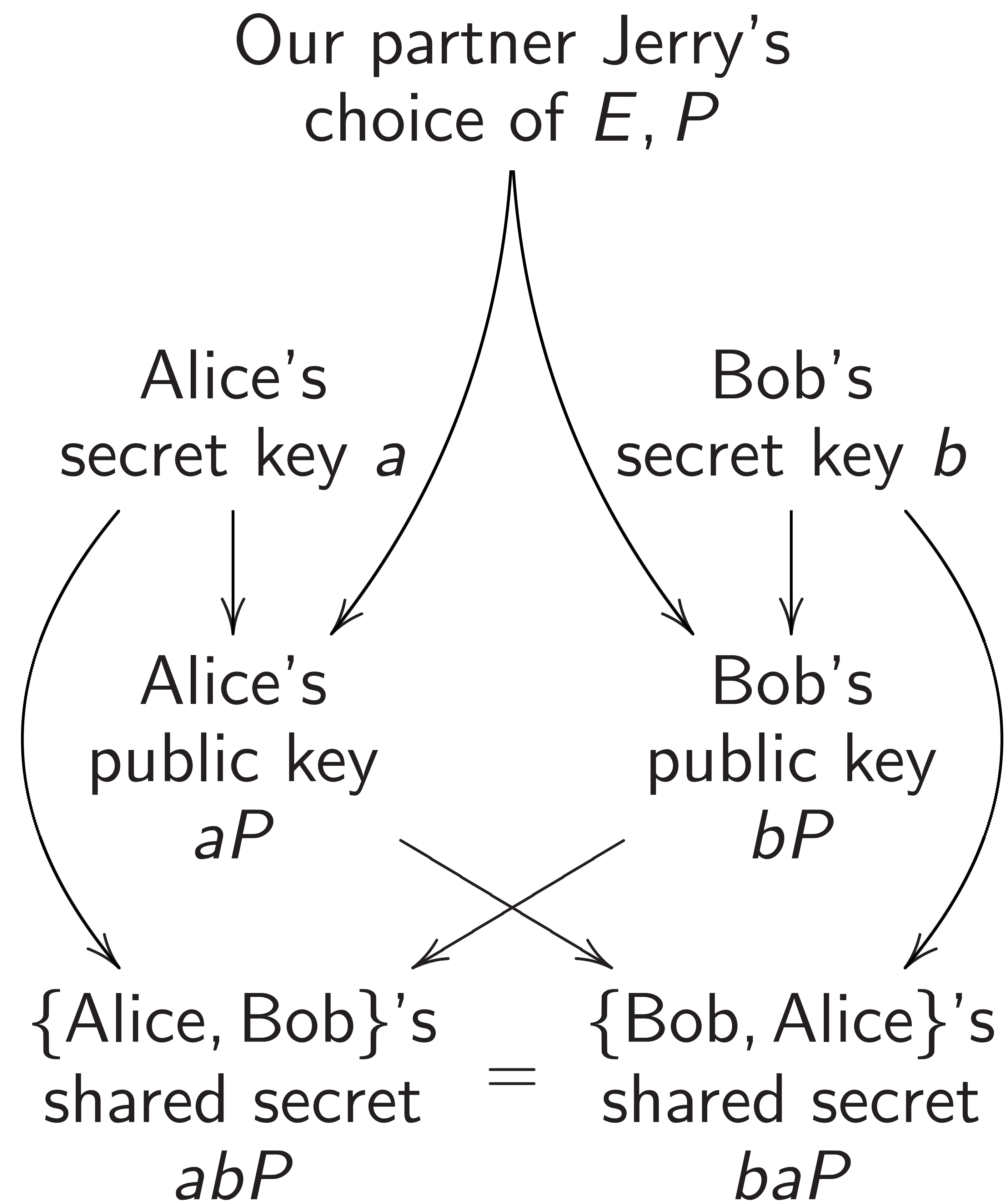


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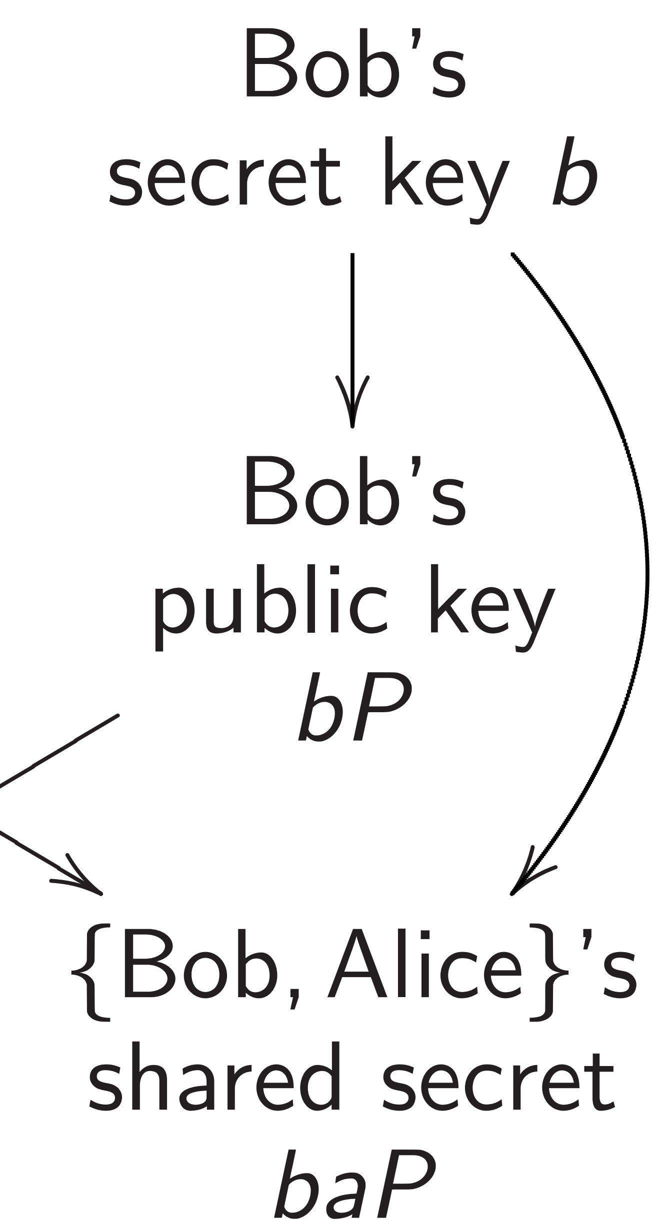


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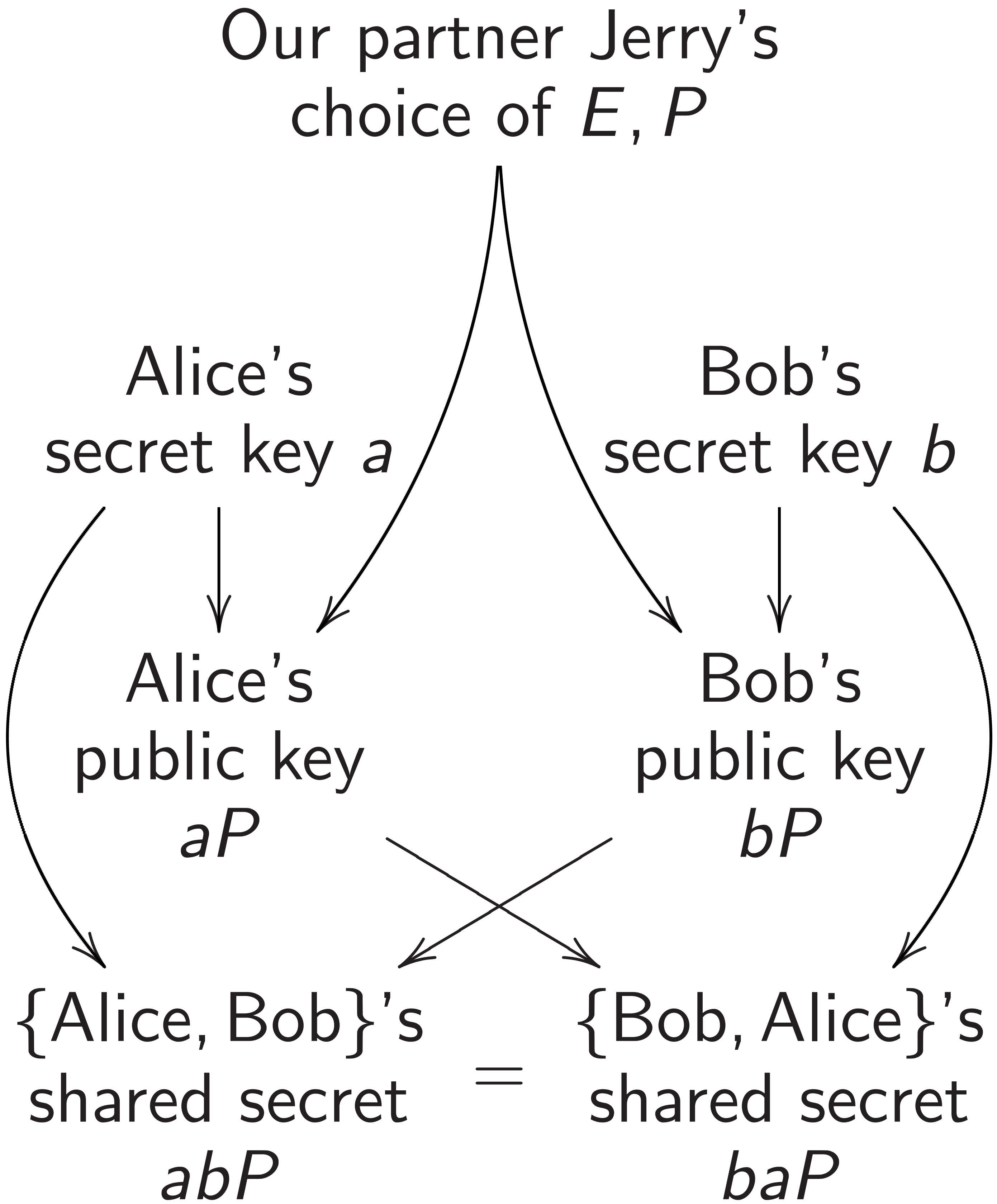


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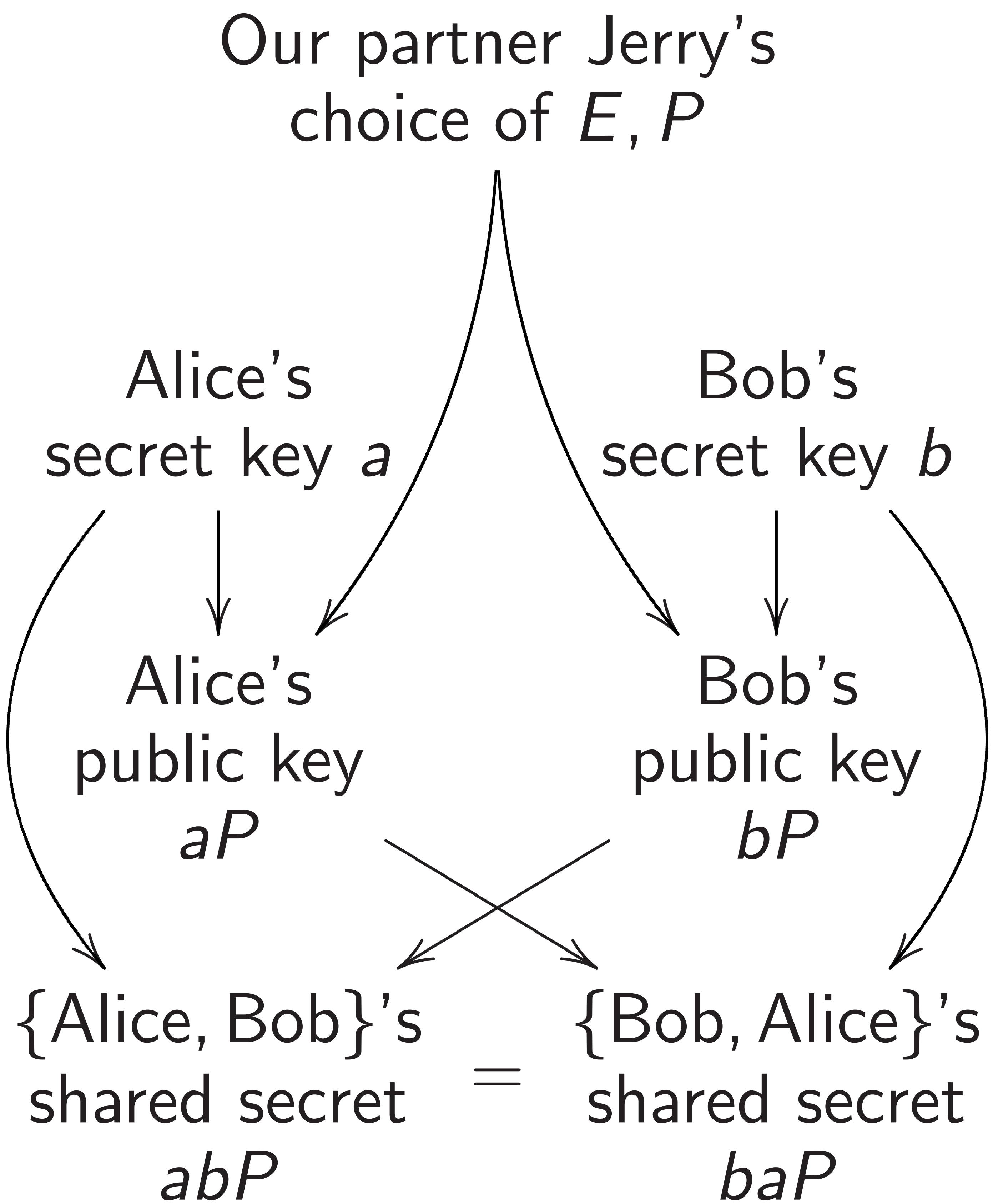


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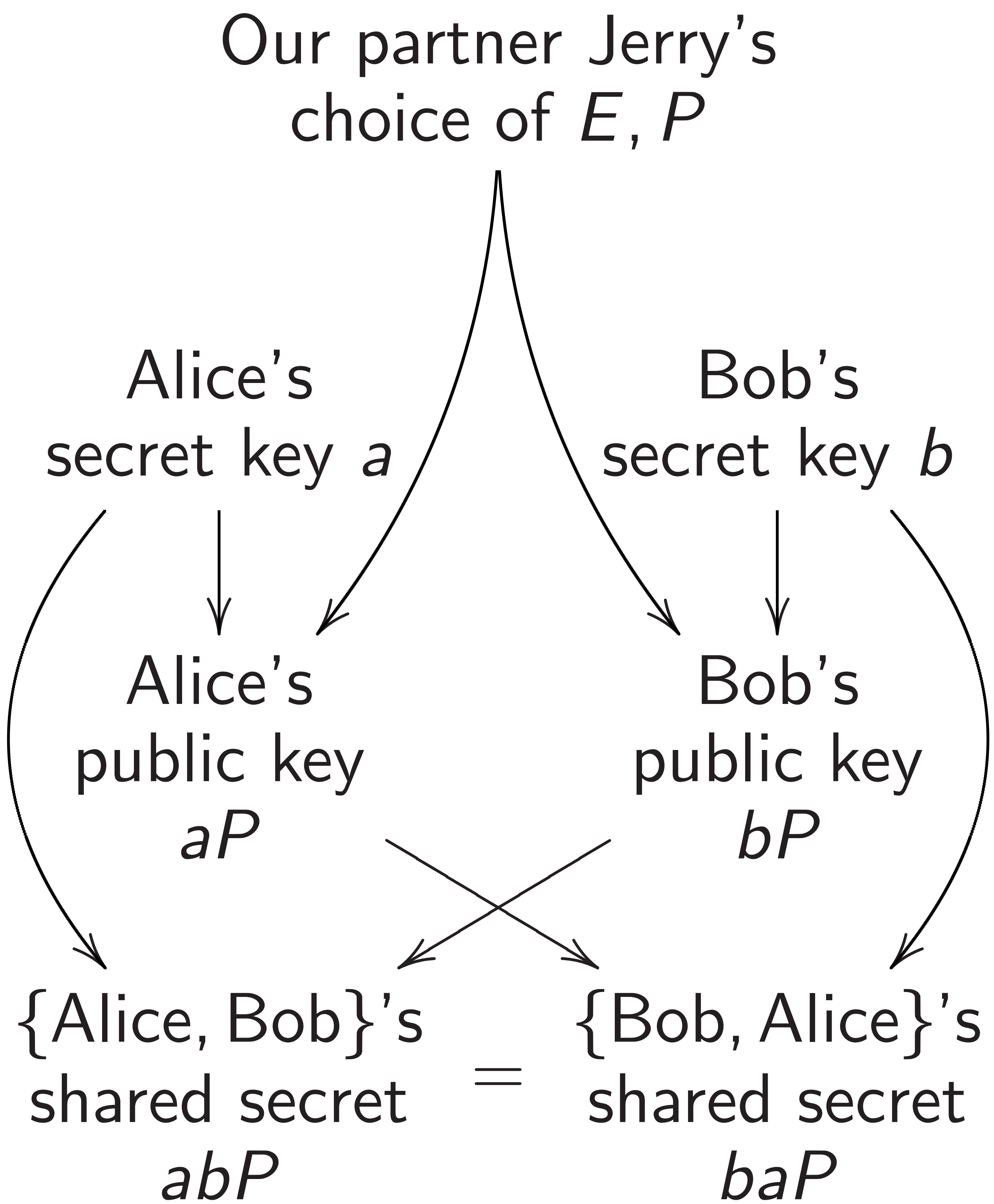


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"The choice of the seeds from which the [NIST] curve parameters have been derived is not motivated leaving an essential part of the security analysis open.
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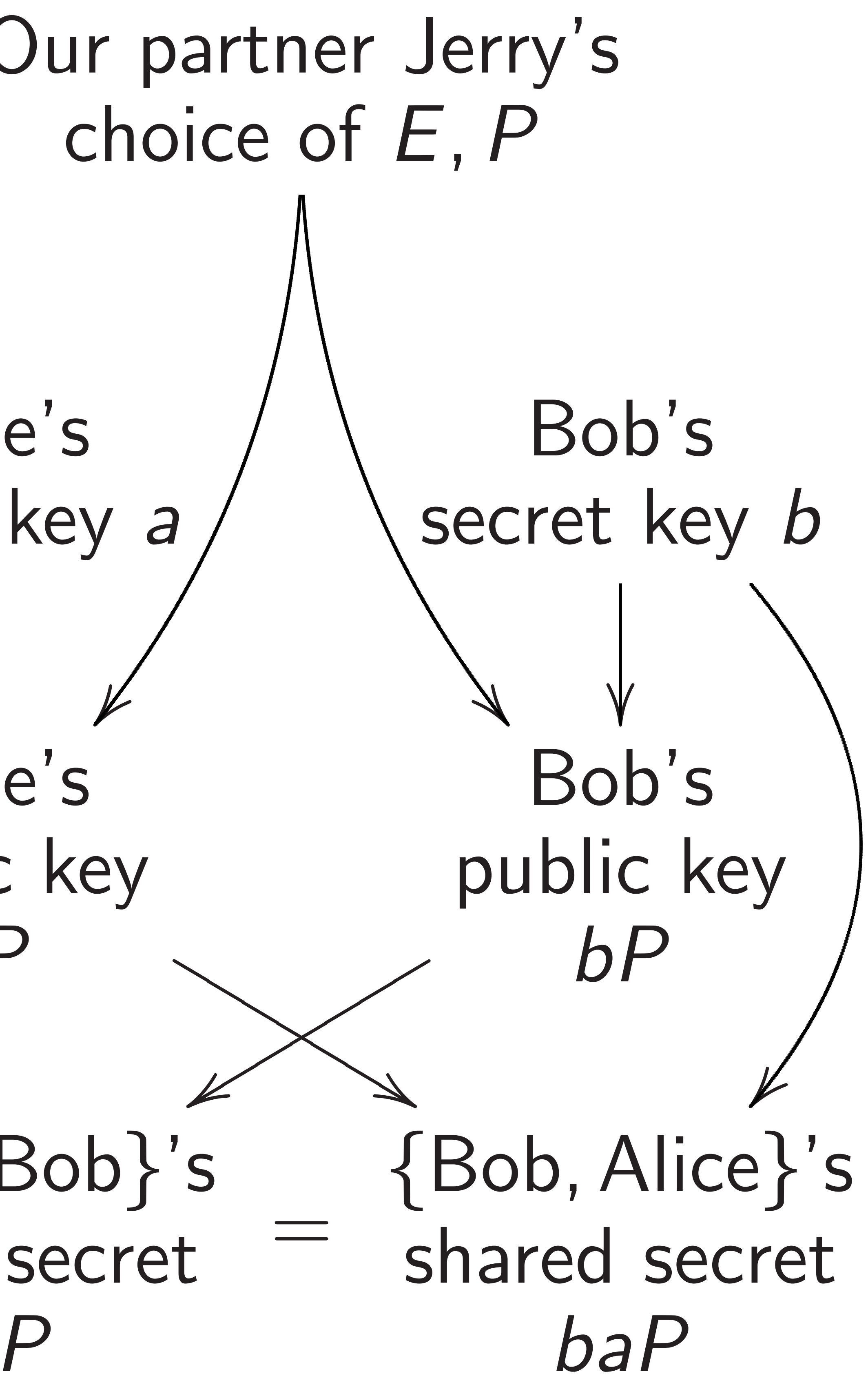


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```
import hashlib
def hash(seed): h
seedbytes = 20

p = 0xD7C134AA264
k = GF(p); R.<x> =

def secure(A,B):
  if k(B).is_squa
  n = EllipticCur
  return (n < p a
    and Integers(

def int2str(seed,
  return ''.join(

def str2int(seed)
  return Integer(

def update(seed):
  return int2str(

def fullhash(seed
  return str2int(

def real2str(seed
  return int2str(

nums = real2str(e
S = nums[2*seedby
while True:
  A = fullhash(S)
  if not (k(A)*x^
  S = update(S)
  B = fullhash(S)
  if not secure(A
  print 'p',hex(p
  print 'A',hex(A
  print 'B',hex(B
  break
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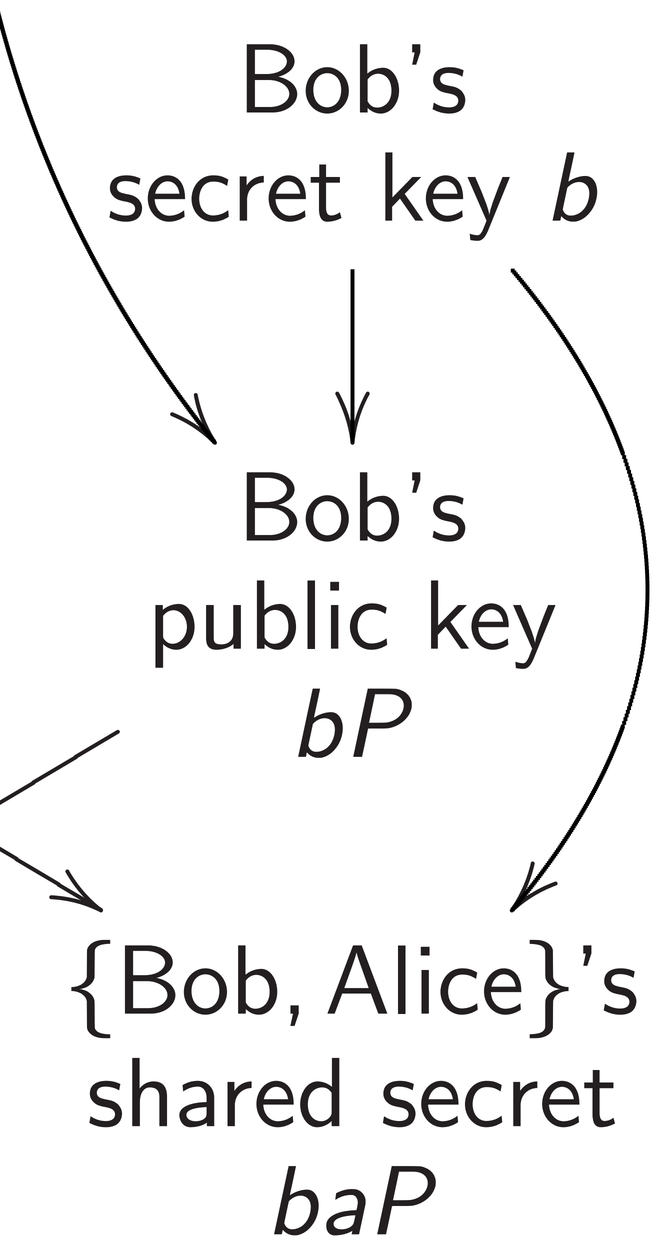


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import hashlib
def hash(seed): h = hashlib.sha1(); h.
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardi
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%25

def str2int(seed):
  return Integer(seed.encode('hex'),16

def update(seed):
  return int2str(str2int(seed) + 1,len

def fullhash(seed):
  return str2int(hash(seed) + hash(upd

def real2str(seed,bytes):
  return int2str(Integer(floor(RealFie

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = upd
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); c
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

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2005 Brainpool standard:
“The choice of the seeds from which the [NIST] curve parameters have been derived is not motivated leaving an essential part of the security analysis open.
. . . **Verifiably pseudo-random.** The [Brainpool] curves shall be generated in a pseudo-random manner using seeds that are generated in a systematic and comprehensive way.”



```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*2

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

# One final example

2005 Brainpool standard:
“The choice of the seeds from which the [NIST] curve parameters have been derived is not motivated leaving an essential part of the security analysis open.
. . . **Verifiably pseudo-random.** The [Brainpool] curves shall be generated in a pseudo-random manner using seeds that are generated in a systematic and comprehensive way.”



```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

## l example

ainpool standard:
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ich the [NIST] curve
ers have been derived is
ivated leaving an essential
he security analysis open.
**ifiably pseudo-random.**
ainpool] curves shall be
ed in a pseudo-random
using seeds that are
ed in a systematic and
ensive way."



```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

2015: W
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from the
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Output

```
p D7C134AA2643
A 2B98B906DC24
B 68AEC4BFE84C
```

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e seeds
IST] curve
een derived is
ing an essential
y analysis open.
**eudo-random.**
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```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

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k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

2015: We carefully
the curve-generati
from the Brainpoo
Previous slide: 22

Output of this pro

```
p D7C134AA264366862A18302575D1D7
A 2B98B906DC245F2916C03A2F953EA9
B 68AEC4BFE84C659EBB8B81DC39355A
```

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```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

2015: We carefully impleme

the curve-generation proced

from the Brainpool standard

Previous slide: 224-bit proce

Output of this procedure:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F
A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4B
B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA
```

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```



2015: We carefully implemented the curve-generation procedure from the Brainpool standard.

Previous slide: 224-bit procedure.

Output of this procedure:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```



2015: We carefully implemented the curve-generation procedure from the Brainpool standard. Previous slide: 224-bit procedure.

Output of this procedure:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

The standard 224-bit Brainpool curve **is not the same curve**:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
B 2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B
```

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  if k(B).is_square(): return False
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  S = update(S)
  B = fullhash(S)
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```



2015: We carefully implemented the curve-generation procedure from the Brainpool standard. Previous slide: 224-bit procedure.

Output of this procedure:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

The standard 224-bit Brainpool curve **is not the same curve**:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
B 2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B
```

Next slide: a procedure that **does** generate the standard Brainpool curve.

```
 = hashlib.sha1(); h.update(seed); return h.digest()

366862A18302575D1D787B09F075797DA89F57EC8C0FF
= k[]

re(): return False
ve([k(A),k(B)]).cardinality()
nd n.is_prime()
n)(p).multiplicative_order() * 100 >= n-1)

bytes):
[chr((seed//256^i)%256) for i in reversed(range(bytes))])

:
seed.encode('hex'),16)

str2int(seed) + 1,len(seed))

):
hash(seed) + hash(update(seed))) % 2^223

,bytes):
Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

xp(1)/16,7*seedbytes)
tes:3*seedbytes]

4+3).roots(): S = update(S); continue

,B): S = update(S); continue
).upper()
).upper()
).upper()
```



2015: We carefully implemented the curve-generation procedure from the Brainpool standard.

Previous slide: 224-bit procedure.

Output of this procedure:

```
p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A  2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B  68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

The standard 224-bit Brainpool curve **is not the same curve**:

```
p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A  68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
B  2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B
```

Next slide: a procedure that **does** generate the standard Brainpool curve.

```
import hashlib
def hash(seed): h
seedbytes = 20

p = 0xD7C134AA264
k = GF(p); R.<x> =

def secure(A,B):
  n = EllipticCur
  return (n < p a
    and Integers(

def int2str(seed,
  return ''.join(

def str2int(seed)
  return Integer(

def update(seed):
  return int2str(

def fullhash(seed
  return str2int(

def real2str(seed
  return int2str(

nums = real2str(e
S = nums[2*seedby
while True:
  A = fullhash(S)
  if not (k(A)*x^
  while True:
    S = update(S)
    B = fullhash(
    if not k(B).i
  if not secure(A
  print 'p',hex(p
  print 'A',hex(A
  print 'B',hex(B
  break
```

```
update(seed); return h.digest()

B09F075797DA89F57EC8C0FF

nality()

order() * 100 >= n-1)

6) for i in reversed(range(bytes))])

)

(seed))

ate(seed))) % 2^223

ld(8*bytes+8)(seed)*256^bytes)),bytes)

ate(S); continue

ontinue
```



2015: We carefully implemented the curve-generation procedure from the Brainpool standard.

Previous slide: 224-bit procedure.

Output of this procedure:

```
p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A  2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B  68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

The standard 224-bit Brainpool curve **is not the same curve**:

```
p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A  68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
B  2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B
```

Next slide: a procedure that **does** generate the standard Brainpool curve.

```
import hashlib
def hash(seed): h = hashlib.sha1(); h.
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787
k = GF(p); R.<x> = k[]

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardi
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%25

def str2int(seed):
  return Integer(seed.encode('hex'),16

def update(seed):
  return int2str(str2int(seed) + 1,len

def fullhash(seed):
  return str2int(hash(seed) + hash(upd

def real2str(seed,bytes):
  return int2str(Integer(floor(RealFie

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = upd
  while True:
    S = update(S)
    B = fullhash(S)
    if not k(B).is_square(): break
  if not secure(A,B): S = update(S); c
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

```
h.digest()

C0FF

range(bytes))])

56^bytes)),bytes)
```



2015: We carefully implemented the curve-generation procedure from the Brainpool standard. Previous slide: 224-bit procedure.

Output of this procedure:

```
p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A  2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B  68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

The standard 224-bit Brainpool curve **is not the same curve**:

```
p  D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A  68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
B  2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B
```

Next slide: a procedure that **does** generate the standard Brainpool curve.



```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C
k = GF(p); R.<x> = k[]

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*2

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  while True:
    S = update(S)
    B = fullhash(S)
    if not k(B).is_square(): break
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

2015: We carefully implemented the curve-generation procedure from the Brainpool standard.

Previous slide: 224-bit procedure.

Output of this procedure:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
B 68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D
```

The standard 224-bit Brainpool curve **is not the same curve**:

```
p D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
A 68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43
B 2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B
```

Next slide: a procedure that **does** generate the standard Brainpool curve.



```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n < p and n.is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def update(seed):
  return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
  return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
  return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
  A = fullhash(S)
  if not (k(A)*x^4+3).roots(): S = update(S); continue
  while True:
    S = update(S)
    B = fullhash(S)
    if not k(B).is_square(): break
  if not secure(A,B): S = update(S); continue
  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

/e carefully implemented
e-generation procedure
e Brainpool standard.
s slide: 224-bit procedure.

of this procedure:

66862A18302575D1D787B09F075797DA89F57EC8C0FF
5F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E
659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D

ndard 224-bit Brainpool
**not the same curve**:

66862A18302575D1D787B09F075797DA89F57EC8C0FF
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4138870713B1A92369E33E2135D266DBB372386C400B

de: a procedure
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dard Brainpool curve.



```
import hashlib
def hash(seed): h = hashlib.sha1(); h.update(seed); return h.digest()
seedbytes = 20

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def secure(A,B):
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def int2str(seed,bytes):
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def real2str(seed,bytes):
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nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
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  if not (k(A)*x^4+3).roots(): S = update(S); continue
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  print 'p',hex(p).upper()
  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```

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787B09F075797DA89F57EC8C0FF
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-bit Brainpool
**ame curve**:

787B09F075797DA89F57EC8C0FF
3514E182AD8B0042A59CAD29F43
9E33E2135D266DBB372386C400B

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npool curve.



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  print 'p',hex(p).upper()
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```

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  print 'A',hex(A).upper()
  print 'B',hex(B).upper()
  break
```



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Interesting Brainpool quote: “It is envisioned to provide additional curves on a regular basis.”

```
 = hashlib.sha1(); h.update(seed); return h.digest()

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):
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update(seed); return h.digest()

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To avoid Brainpool's complications of concatenat hash outputs: We upgraded from SHA-1 to state-of-the- maximum-security SHA3-51 Also upgraded to requiring maximum twist security.

Brainpool uses $\exp(1) = e$ and $\arctan(1) = \pi/4$, and N uses $\sin(1)$, so we used cos( We also used much simpler pattern of searching for seed

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We made a new 224-bit curve using standard NIST P-224 prime.

To avoid Brainpool's complications of concatenating hash outputs: We upgraded from SHA-1 to state-of-the-art maximum-security SHA3-512. Also upgraded to requiring maximum twist security.

Brainpool uses $\exp(1) = e$ and $\arctan(1) = \pi/4$, and MD5 uses $\sin(1)$, so we used $\cos(1)$. We also used much simpler pattern of searching for seeds.

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```
import simplesha3
hash = simplesha3

p = 2^224 - 2^96 
k = GF(p)
seedbytes = 20

def secure(A,B):
  n = EllipticCur
  return (n.is_pr
    and Integers(
    and Integers(

def int2str(seed,
  return ''.join(

def str2int(seed)
  return Integer(

def complement(se
  return ''.join(

def real2str(seed
  return int2str(

sizeofint = 4
nums = real2str(c
for counter in xr
  S = int2str(cou
  T = complement(
  A = str2int(has
  B = str2int(has
  if secure(A,B):
    print 'p',hex
    print 'A',hex
    print 'B',hex
    break
```

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```
import simplesha3
hash = simplesha3.sha3512

p = 2^224 - 2^96 + 1
k = GF(p)
seedbytes = 20

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardi
  return (n.is_prime() and (2*p+2-n).i
    and Integers(n)(p).multiplicative_
    and Integers(2*p+2-n)(p).multiplic

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%25

def str2int(seed):
  return Integer(seed.encode('hex'),16

def complement(seed):
  return ''.join([chr(255-ord(s)) for

def real2str(seed,bytes):
  return int2str(Integer(RealField(8*b

sizeofint = 4
nums = real2str(cos(1),seedbytes - siz
for counter in xrange(0,256^sizeofint)
  S = int2str(counter,sizeofint) + num
  T = complement(S)
  A = str2int(hash(S))
  B = str2int(hash(T))
  if secure(A,B):
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
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```

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  return ''.join([chr((seed//256^i)%256) for i in reversed(

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def complement(seed):
  return ''.join([chr(255-ord(s)) for s in seed])

def real2str(seed,bytes):
  return int2str(Integer(RealField(8*bytes)(seed)*256^bytes

sizeofint = 4
nums = real2str(cos(1),seedbytes - sizeofint)
for counter in xrange(0,256^sizeofint):
  S = int2str(counter,sizeofint) + nums
  T = complement(S)
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def real2str(seed,bytes):
  return int2str(Integer(RealField(8*bytes)(seed)*256^bytes),bytes)

sizeofint = 4
nums = real2str(cos(1),seedbytes - sizeofint)
for counter in xrange(0,256^sizeofint):
  S = int2str(counter,sizeofint) + nums
  T = complement(S)
  A = str2int(hash(S))
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  if secure(A,B):
    print 'p',hex(p).upper()
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  return Integer(seed.encode('hex'),16)

def complement(seed):
  return ''.join([chr(255-ord(s)) for s in seed])

def real2str(seed,bytes):
  return int2str(Integer(RealField(8*bytes)(seed)*256^bytes),bytes)

sizeofint = 4
nums = real2str(cos(1),seedbytes - sizeofint)
for counter in xrange(0,256^sizeofint):
  S = int2str(counter,sizeofint) + nums
  T = complement(S)
  A = str2int(hash(S))
  B = str2int(hash(T))
  if secure(A,B):
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
```

Output:

24-bit curve
ST P-224 prime.

ol's
oncatenating
upgraded
ate-of-the-art
SHA3-512.
requiring
curity.

o(1) = e
π/4, and MD5
used cos(1).
h simpler
ng for seeds.



```
import simplesha3
hash = simplesha3.sha3512

p = 2^224 - 2^96 + 1
k = GF(p)
seedbytes = 20

def secure(A,B):
  n = EllipticCurve([k(A),k(B)]).cardinality()
  return (n.is_prime() and (2*p+2-n).is_prime()
    and Integers(n)(p).multiplicative_order() * 100 >= n-1
    and Integers(2*p+2-n)(p).multiplicative_order() * 100 >= 2*p+2-n-1)

def int2str(seed,bytes):
  return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
  return Integer(seed.encode('hex'),16)

def complement(seed):
  return ''.join([chr(255-ord(s)) for s in seed])

def real2str(seed,bytes):
  return int2str(Integer(RealField(8*bytes)(seed)*256^bytes),bytes)

sizeofint = 4
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  A = str2int(hash(S))
  B = str2int(hash(T))
  if secure(A,B):
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
```

Output: 7144BA12CE8A

ve

prime.

ing

art

2.

MD5

1).

ls.



```
import simplesha3
hash = simplesha3.sha3512

p = 2^224 - 2^96 + 1
k = GF(p)
seedbytes = 20

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  if secure(A,B):
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
```

Output: `7144BA12CE8A0C3BEFA053EDB`

```
import simplesha3
hash = simplesha3.sha3512

p = 2^224 - 2^96 + 1
k = GF(p)
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  T = complement(S)
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  B = str2int(hash(T))
  if secure(A,B):
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
```



Output: 7144BA12CE8A0C3BEFA053EDBADA55...

```
import simplesha3
hash = simplesha3.sha3512

p = 2^224 - 2^96 + 1
k = GF(p)
seedbytes = 20

def secure(A,B):
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    and Integers(2*p+2-n)(p).multiplicative_order() * 100 >= 2*p+2-n-1)

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  B = str2int(hash(T))
  if secure(A,B):
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
```



Output: `7144BA12CE8A0C3BEFA053EDBADA55...`

We actually generated >1000000 curves for this prime, each having a Brainpool-like explanation, even without complicating hashing, seed search, etc.; e.g., `BADA55-VPR2-224` uses exp(1).

```
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    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
```



Output: `7144BA12CE8A0C3BEFA053EDBADA55...`

We actually generated >1000000 curves for this prime, each having a Brainpool-like explanation, even without complicating hashing, seed search, etc.; e.g., BADA55-VPR2-224 uses exp(1).

See bada55.cr.yp.to for much more: full paper; scripts; detailed Brainpool analysis; manipulating "minimal" primes and curves (Microsoft "NUMS"); manipulating security criteria.