Timing attacks

1970s: TENEX operating system compares user-supplied string against secret password one character at a time, stopping at first difference:

- AAAAAAA vs. SECRET: stop at 1.
- SAAAAAA vs. SECRET: stop at 2.
- SEAAAAA vs. SECRET: stop at 3.

Attacker sees comparison time, deduces position of difference. A few hundred tries reveal secret password.
How typical software checks 16-byte authenticator:

```c
for (i = 0; i < 16; ++i)
    if (x[i] != y[i]) return 0;
return 1;
```

Fix, eliminating information flow from secrets to timings:

```c
uint32 diff = 0;
for (i = 0; i < 16; ++i)
    diff |= x[i] ^ y[i];
return 1 & ((diff-1) >> 8);
```

Notice that the language makes the wrong thing simple and the right thing complex.
Language designer’s notion of “right” is too weak for security.

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One of many current examples, part of the reference software for CAESAR candidate CLOC:

```c
/* compare the tag */
int i;
for(i = 0; i < CRYPTO_ABYTES; i++)
  if(tag[i] != c[(*mlen) + i]){
    return RETURN_TAG_NO_MATCH;
  }
return RETURN_SUCCESS;
```
Do timing attacks really work?

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To guarantee security, defender must block all information flow.

Answer #2: Attacker uses statistics to eliminate noise.

Answer #3, what the 1970s attackers actually did:
Cross page boundary, inducing page faults, to amplify timing signal.
Examples of successful attacks:

2005 Tromer–Osvik–Shamir: 65ms to steal Linux AES key used for hard-disk encryption.

2013 AlFardan–Paterson “Lucky Thirteen: breaking the TLS and DTLS record protocols” steals plaintext using decryption timings.

2014 van de Pol–Smart–Yarom steals Bitcoin key from timings of 25 OpenSSL signatures.

2016 Yarom–Genkin–Heninger “CacheBleed” steals RSA secret key via timings of OpenSSL.
Constant-time ECC

ECDH computation: \( a, P \mapsto aP \)
where \( a \) is your secret key.

Key generation: \( a \mapsto aB \).

Signing: \( r \mapsto rB \).

All of these use secret data. Does timing leak this data?

Are there any branches in ECC ops? Point ops? Field ops? Do the underlying machine insns take variable time?
Recall left-to-right binary method to compute $n, P \mapsto nP$ using point addition:

```python
def scalarmult(n,P):
    if n == 0: return 0
    if n == 1: return P
    R = scalarmult(n//2,P)
    R = R + R
    if n % 2: R = R + P
    return R
```

Many branches here.
NAF etc. also use many branches.
Even if each point addition takes the same amount of time (certainly not true in Python), total time depends on $n$.

If $2^{e-1} \leq n < 2^e$ and $n$ has exactly $w$ bits set: number of additions is $e + w - 2$.

Particularly fast total time usually indicates very small $n$.

“Lattice attacks” on signatures compute the secret key given positions of very small nonces $r$. 
Even worse:
CPUs do not try to protect metadata regarding branches.

Actual time for a branch affects, and is affected by, detailed state of code cache, branch predictor, etc.

Attacker interacts with this state, often sees pattern of branches. Exploited in, e.g., Bitcoin attack.
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Confidence-inspiring solution: Avoid all data flow from secrets to branch conditions.
Double-and-add-always

Eliminate branches by always computing both results:

```python
def scalarmult(n,b,P):
    if b == 0: return 0
    R = scalarmult(n//2,b-1,P)
    R2 = R + R
    S = [R2,R2 + P]
    return S[n % 2]
```

Works for $0 \leq n < 2^b$.
Always takes $2b$ additions (including $b$ doublings).
Use public $b$: bits allowed in $n$. 
Another big problem: CPUs do not try to protect metadata regarding *array indices*. Actual time for $x[i]$ affects, and is affected by, detailed state of data cache, store-to-load forwarder, etc. Exploited in, e.g., CacheBleed, despite Intel and OpenSSL claiming their code was safe.
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Confidence-inspiring solution: **Avoid all data flow from secrets to memory addresses.**
Table lookups via arithmetic

Always read all table entries. Use bit operations to select the desired table entry:

```python
def scalarmult(n,b,P):
    if b == 0: return 0

    R = scalarmult(n//2,b-1,P)
    R2 = R + R
    S = [R2,R2 + P]

    mask = -(n % 2)
    return S[0]^((mask&(S[1]^S[0])))
```
Width-2 unsigned fixed windows

def fixwin2(n,b,table):
    if b <= 0: return 0
    T = table[0]
    mask =(-(1 ^ (n % 4))) >> 2
    T ^= ~mask & (T^table[1])
    mask =(-(2 ^ (n % 4))) >> 2
    T ^= ~mask & (T^table[2])
    mask =(-(3 ^ (n % 4))) >> 2
    T ^= ~mask & (T^table[3])
    R = fixwin2(n//4,b-2,table)
    R = R + R
    R = R + R
    return R + T
def scalarmult(n,b,P):
    P2 = P+P
    table = [0,P,P2,P2+P]
    return fixwin2(n,b,table)

Public branches, public indices.

For $b \in \mathbb{Z}$:
Always $b$ doublings.
Always $b/2$ additions of $T$.
Always 2 additions for table.

Can similarly protect larger-width fixed windows.
Unsigned is slightly easier.
Signed is slightly faster.
Fixed-base scalar multiplication

Obvious way to handle keygen $a \mapsto aB$ and signing $r \mapsto rB$: reuse $n, P \mapsto nP$ from ECDH.
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Can do much better since $B$ is a constant: standard base point.

e.g. For $b = 256$: Compute $(2^{128} n_1 + n_0)B$ as $n_1 B_1 + n_0 B$ using double-scalar fixed windows, after precomputing $B_1 = 2^{128} B$.

Fun exercise: For each $k$, try to minimize number of additions using $k$ precomputed points.
Recall Chou timings:
57164 cycles for keygen,
63526 cycles for signature,
205741 cycles for verification,
159128 cycles for ECDH.

ECDH is single-scalar mult.

Verification is double-scalar mult, somewhat slower than ECDH. (But batch verification is faster.)

Keygen is fixed-base scalar mult, much faster than ECDH.

Signing is keygen plus overhead depending on message length.
Let’s move down a level:

**ECC ops:** e.g., verify $SB = R + hA$

windowing etc.

**Point ops:** e.g., $P, Q \mapsto P + Q$

faster doubling etc.

**Field ops:** e.g., $x_1, x_2 \mapsto x_1x_2$ in $\mathbb{F}_p$

delayed carries etc.

**Machine insns:** e.g., 32-bit multiplication

pipelining etc.

**Gates:** e.g., AND, OR, XOR
Eliminating divisions

Have to do many additions of curve points: $P, Q \mapsto P + Q$. How to efficiently decompose additions into field ops?

Addition $(x_1, y_1) + (x_2, y_2) = ((x_1 y_2 + y_1 x_2)/(1 + dx_1 x_2 y_1 y_2), (y_1 y_2 - x_1 x_2)/(1 - dx_1 x_2 y_1 y_2))$ uses expensive divisions.
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Addition \((x_1, y_1) + (x_2, y_2) = ((x_1y_2 + y_1x_2)/(1 + dx_1x_2y_1y_2),
(y_1y_2 - x_1x_2)/(1 - dx_1x_2y_1y_2))\) uses expensive divisions.

Better: postpone divisions and work with fractions.

Represent \((x, y)\) as \((X : Y : Z)\) with \( x = X/Z, y = Y/Z, Z \neq 0 \).
Addition now has to handle fractions as input:

\[
\left( \frac{X_1}{Z_1}, \frac{Y_1}{Z_1} \right) + \left( \frac{X_2}{Z_2}, \frac{Y_2}{Z_2} \right) = \\
\left( \frac{X_1}{Z_1} \frac{Y_2}{Z_2} + \frac{Y_1}{Z_1} \frac{X_2}{Z_2} \right. \\
\left. \frac{1 + d \frac{X_1}{Z_1} \frac{X_2}{Z_2} \frac{Y_1}{Z_1} \frac{Y_2}{Z_2}}{1 - d \frac{X_1}{Z_1} \frac{X_2}{Z_2} \frac{Y_1}{Z_1} \frac{Y_2}{Z_2}} \right)
\]
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\left( \frac{X_1}{Z_1} \frac{Y_2}{Z_2} + \frac{Y_1}{Z_1} \frac{X_2}{Z_2} \right) \left( 1 + d \frac{X_1}{Z_1} \frac{X_2}{Z_2} \frac{Y_1}{Z_1} \frac{Y_2}{Z_2} \right), \\
\left( \frac{Y_1}{Z_1} \frac{Y_2}{Z_2} - \frac{X_1}{Z_1} \frac{X_2}{Z_2} \right) \left( 1 - d \frac{X_1}{Z_1} \frac{X_2}{Z_2} \frac{Y_1}{Z_1} \frac{Y_2}{Z_2} \right) = \\
\left( \frac{Z_1 Z_2 (X_1 Y_2 + Y_1 X_2)}{Z_1^2 Z_2^2 + d X_1 X_2 Y_1 Y_2}, \frac{Z_1 Z_2 (Y_1 Y_2 - X_1 X_2)}{Z_1^2 Z_2^2 - d X_1 X_2 Y_1 Y_2} \right)
\]
i.e. \( \left( \frac{X_1}{Z_1}, \frac{Y_1}{Z_1} \right) + \left( \frac{X_2}{Z_2}, \frac{Y_2}{Z_2} \right) \)

\[ = \left( \frac{X_3}{Z_3}, \frac{Y_3}{Z_3} \right) \]

where

\[
F = Z_1^2 Z_2^2 - dX_1 X_2 Y_1 Y_2, \\
G = Z_1^2 Z_2^2 + dX_1 X_2 Y_1 Y_2, \\
X_3 = Z_1 Z_2 (X_1 Y_2 + Y_1 X_2) F, \\
Y_3 = Z_1 Z_2 (Y_1 Y_2 - X_1 X_2) G, \\
Z_3 = FG.
\]

Input to addition algorithm:
\(X_1, Y_1, Z_1, X_2, Y_2, Z_2\).

Output from addition algorithm:
\(X_3, Y_3, Z_3\). No divisions needed!
Eliminate common subexpressions to save multiplications:

\[ A = Z_1 \cdot Z_2; \quad B = A^2; \]
\[ C = X_1 \cdot X_2; \]
\[ D = Y_1 \cdot Y_2; \]
\[ E = d \cdot C \cdot D; \]
\[ F = B - E; \quad G = B + E; \]
\[ X_3 = A \cdot F \cdot (X_1 \cdot Y_2 + Y_1 \cdot X_2); \]
\[ Y_3 = A \cdot G \cdot (D - C); \]
\[ Z_3 = F \cdot G. \]

Cost: \(11M + 1S + 1M_d\) where \(M, S\) are costs of mult, square. Choose small \(d\) for cheap \(M_d\).
Can do better: $10M + 1S + 1M_d$.

Obvious 4M method to compute product $C + Mt + Dt^2$ of polys $X_1 + Y_1 t$, $X_2 + Y_2 t$:

\[
C = X_1 \cdot X_2; \\
D = Y_1 \cdot Y_2; \\
M = X_1 \cdot Y_2 + Y_1 \cdot X_2.
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Karatsuba’s $3M$ method:

$$C = X_1 \cdot X_2;$$
$$D = Y_1 \cdot Y_2;$$
$$M = (X_1 + Y_1) \cdot (X_2 + Y_2) - C - D.$$
Faster doubling

\((x_1, y_1) + (x_1, y_1) = ((x_1y_1 + y_1x_1)/(1 + dx_1x_1y_1y_1),
(y_1y_1 - x_1x_1)/(1 - dx_1x_1y_1y_1)) = ((2x_1y_1)/(1 + dx_1^2y_1^2),
(y_1^2 - x_1^2)/(1 - dx_1^2y_1^2)).\)
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(y_1^2 - x_1^2)/(2 - x_1^2 - y_1^2))).
\]

Again eliminate divisions

using \((X : Y : Z)\): only \(3M + 4S\).

Much faster than addition.
More addition strategies

Dual addition formula:

\[(x_1, y_1) + (x_2, y_2) = ((x_1y_1 + x_2y_2)/(x_1x_2 + y_1y_2),

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Low degree, no need for \(d\).
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Most EC formulas have failures.
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Can test for failure cases.
Can produce constant-time code by eliminating branches.
For some ECC ops, can prove that failure cases never happen.
More coordinate systems: e.g.,

• inverted: $x = Z/X, y = Z/Y$.
• extended: $x = X/Z, y = Y/T$.
• completed: $x = X/Z, y = Y/Z, \ xy = T/Z$.

“−1-twisted Edwards curves”

$-x^2 + y^2 = 1 + dx^2y^2$:

further speedups related to

$-x^2 + y^2 = (y - x)(y + x)$.

Inside modern ECC operations:

$8M$ for addition,

$3M + 4S$ for doubling.
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How did Curve25519 obtain good speeds for ECDH? "Montgomery curve with the Montgomery ladder."

Why did NIST not choose Montgomery curves? Unclear.