Standardization for the black hat

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1 bada55.cr.yp.to “BADA55 Crypto” including “How to manipulate curve standards: a white paper for the black hat.”

2 projectbullrun.org including “Dual EC: a standardized back door.”
Includes joint work with (in alphabetical order):

Tung Chou ①
Chitchanok Chuengsatiansup ①
Andreas Hülsing ①
Eran Lambooij ①
Tanja Lange ① ②
Ruben Niederhagen ① ②
Christine van Vredendaal ①

Inspirational previous work:
ANSI, ANSSI, Brainpool, IETF, ISO, NIST, OSCCA, SECG, and especially our buddies at NSA.
The DES key size

IBM: 128! NSA: 32!
IBM: 64! NSA: 48!
Final compromise: 56.
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Crypto community to NSA+NBS:
Your key size is too small.
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NBS (now NIST) continues to promote DES for two decades, drastically increasing cost of the inevitable upgrade.
Random nonces in DSA/ECDSA

1992 Rivest: “The poor user is given enough rope with which to hang himself—something a standard should not do.”

Standardize anyway.
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Add complicated options for deterministic nonces, while preserving old options.
Denial of service via flooding

Suspected terrorists Alice and Bob are aided and abetted by “auditors” (= “cryptanalysts” = “reviewers”) checking for exploitable security problems in cryptographic systems.

Example: SHA-3 competition involved 200 cryptographers around the world and took years of sustained public effort. How can we slip a security problem past all of them?
During the same period, NIST also published FIPS 186-3 (signatures), FIPS 198-1 (authentication), SP 800-38E (disk encryption), SP 800-38F (key wrapping), SP 800-56C (key derivation), SP 800-57 (key management), SP 800-67 (block encryption), SP 800-108 (key derivation), SP 800-131A (key lengths), SP 800-133 (key generation), SP 800-152 (key management), and related protocol documents such as SP 800-81r1.
Attention of auditors was not entirely on SHA-3.

Auditors caught a severe security flaw in EAX Prime just before NIST standardization.
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Why did this take years? Scientific advances? No!

We successfully denied service.
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Why did this take years? Scientific advances? No! **We successfully denied service.**

And NIST is just the tip of the crypto standardization iceberg.
Flooding via dishonesty

If we were honest then we would tell Alice+Bob to reuse ciphers/hasheas as PRNGs.
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But why should we be honest? Let’s build PRNGs from scratch!
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But why should we be honest? Let’s build PRNGs from scratch!

2004: Number-theoretic RNGs provide “increased assurance.”

2006: Dual EC “is the only DRBG mechanism in this Recommendation whose security is related to a hard problem in number theory.”
Denial of service via hoops

2006 Gjøsteen, independently
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Are *all* applications broken? Obviously not! Standardize!
Denial of service via hoops

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Are all applications broken? Obviously not! Standardize!

2007 Shumow–Ferguson: Dual EC has a back door. Would have been easy to build $Q$ with the key.

2007 Schneier: Never use Dual EC. “Both NIST and the NSA have some explaining to do.”
Did Shumow and Ferguson show us the key? No!

Maintain and promote Dual EC standard. Pay people to use it.

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Maintain and promote Dual EC standard. Pay people to use it.


Even after being caught, continue to burn auditors’ time by demanding that they jump higher.

NSA’s Dickie George, 2014: Gee, Dual EC is really hard to exploit!
System vs. ecosystem

Traditional RNG auditing:
Auditor looks at one system, an RNG. Tries to find weakness.

Auditor’s starting assumption: random numbers for Alice and Bob are created by an RNG.
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Auditor’s starting assumption: random numbers for Alice and Bob are created by an RNG.

Reality: random numbers are created by a much more complicated ecosystem that designs, evaluates, standardizes, selects, implements, and deploys RNGs. (Same for other crypto.)
This is a critical change in perspective. Auditor is stuck defending the wrong targets!

The ecosystem has many weaknesses that are not visible inside any particular system.

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e.g. Propose 20 weak standards. Some will survive auditing. Then manipulate selection.
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E.g. Propose 20 weak standards. Some will survive auditing. Then manipulate selection.

Deter publication of weaknesses: “This attack is trivial. Reject.”
Textbook key exchange using standard point $P$ on a standard elliptic curve $E$:

Alice’s secret key $a$  
↓  
Alice’s public key $aP$  
↓  
{Alice, Bob}’s shared secret $abP$  

Bob’s secret key $b$  
↓  
Bob’s public key $bP$  

{Bob, Alice}’s shared secret $baP$
Textbook key exchange using standard point $P$ on a standard elliptic curve $E$:

Alice’s secret key $a$

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Bob’s secret key $b$

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Bob’s public key $bP$

{Alice, Bob}’s shared secret $abP$ = {Bob, Alice}’s shared secret $baP$

Security depends on choice of $E$. 
Our partner Jerry’s choice of $E, P$

- Alice’s secret key $a$
- Bob’s secret key $b$

- Alice’s public key $aP$
- Bob’s public key $bP$

$\{\text{Alice, Bob}\}$’s shared secret $abP = \{\text{Bob, Alice}\}$’s shared secret $baP$

This is not the same picture!
One final example

2005 Brainpool standard: “The choice of the seeds from which the [NIST] curve parameters have been derived is not motivated leaving an essential part of the security analysis open. . . . **Verifiably pseudo-random.** The [Brainpool] curves shall be generated in a pseudo-random manner using seeds that are generated in a systematic and comprehensive way.”
import hashlib

def hash(seed):
    h = hashlib.sha1();
    h.update(seed);
    return h.digest()

seedbytes = 20

p = 0xD7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF
k = GF(p); R.<x> = k[]

def secure(A,B):
    if k(B).is_square(): return False
    n = EllipticCurve([k(A),k(B)]).cardinality()
    return (n < p and n.is_prime()
            and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
    return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
    return Integer(seed.encode('hex'),16)

def update(seed):
    return int2str(str2int(seed) + 1,len(seed))

def fullhash(seed):
    return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
    return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]
while True:
    A = fullhash(S)
    if not (k(A)*x^4+3).roots():
        S = update(S);
        continue
    S = update(S)
    B = fullhash(S)
    if not secure(A,B): 
        S = update(S);
        continue
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
2015: We carefully implemented the curve-generation procedure from the Brainpool standard. Previous slide: 224-bit procedure.

Output of this procedure:

\[\begin{align*}
  p & \quad \text{D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF} \\
  A & \quad \text{2B98B906DC245F2916C03A2F953EA9AE565C3253E8AEC4BFE84C659E} \\
  B & \quad \text{68AEC4BFE84C659EBB8B81DC39355A2EBFA3870D98976FA2F17D2D8D}
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The standard 224-bit Brainpool curve is not the same curve:

\[
\begin{align*}
p & = \text{D7C134AA264366862A18302575D1D787B09F075797DA89F57EC8C0FF} \\
A & = \text{68A5E62CA9CE6C1C299803A6C1530B514E182AD8B0042A59CAD29F43} \\
B & = \text{2580F63CCFE44138870713B1A92369E33E2135D266DBB372386C400B}
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Next slide: a procedure that **does** generate the standard Brainpool curve.
import hashlib

def hash(seed):
    h = hashlib.sha1();
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    return h.digest()

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k = GF(p); R.<x> = k[]

def secure(A,B):
    n = EllipticCurve([k(A),k(B)]).cardinality()
    return (n < p and n.is_prime() and Integers(n)(p).multiplicative_order() * 100 >= n-1)

def int2str(seed,bytes):
    return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
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def update(seed):
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def fullhash(seed):
    return str2int(hash(seed) + hash(update(seed))) % 2^223

def real2str(seed,bytes):
    return int2str(Integer(floor(RealField(8*bytes+8)(seed)*256^bytes)),bytes)

nums = real2str(exp(1)/16,7*seedbytes)
S = nums[2*seedbytes:3*seedbytes]

while True:
    A = fullhash(S)
    if not (k(A)*x^4+3).roots(): S = update(S); continue
    while True:
        S = update(S)
        B = fullhash(S)
        if not k(B).is_square(): break
    if not secure(A,B): S = update(S); continue
    print 'p',hex(p).upper()
    print 'A',hex(A).upper()
    print 'B',hex(B).upper()
    break
Did Brainpool check before publication? After publication? Did they know before 2015?

Brainpool procedure is advertised as “systematic”, “comprehensive”, “completely transparent”, etc. Surely we can say the same for both procedures.
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Can quietly manipulate choice to take the weaker procedure.

Interesting Brainpool quote: “It is envisioned to provide additional curves on a regular basis.”
We made a new 224-bit curve using standard NIST P-224 prime.

To avoid Brainpool’s complications of concatenating hash outputs: We upgraded from SHA-1 to state-of-the-art maximum-security SHA3-512. Also upgraded to requiring maximum twist security.

Brainpool uses \( \exp(1) = e \) and \( \arctan(1) = \pi/4 \), and MD5 uses \( \sin(1) \), so we used \( \cos(1) \). We also used much simpler pattern of searching for seeds.
import simplesha3
hash = simplesha3.sha3512

p = 2^224 - 2^96 + 1
k = GF(p)
seedbytes = 20

def secure(A,B):
    n = EllipticCurve([k(A),k(B)]).cardinality()
    return (n.is_prime() and (2*p+2-n).is_prime()
        and Integers(n)(p).multiplicative_order() * 100 >= n-1
        and Integers(2*p+2-n)(p).multiplicative_order() * 100 >= 2*p+2-n-1)

def int2str(seed,bytes):
    return ''.join([chr((seed//256^i)%256) for i in reversed(range(bytes))])

def str2int(seed):
    return Integer(seed.encode('hex'),16)

def complement(seed):
    return ''.join([chr(255-ord(s)) for s in seed])

def real2str(seed,bytes):
    return int2str(Integer(RealField(8*bytes)(seed)*256^bytes),bytes)

sizeofint = 4
nums = real2str(cos(1),seedbytes - sizeofint)
for counter in xrange(0,256^sizeofint):
    S = int2str(counter,sizeofint) + nums
    T = complement(S)
    A = str2int(hash(S))
    B = str2int(hash(T))
    if secure(A,B):
        print 'p',hex(p).upper()
        print 'A',hex(A).upper()
        print 'B',hex(B).upper()
        break
Output: 7144BA12CE8A0C3BEFA053EDBADA55...
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We actually generated $>1000000$ curves for this prime, each having a Brainpool-like explanation, even without complicating hashing, seed search, etc.; e.g., BADA55-VPR2-224 uses $\exp(1)$. 
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See bada55.cr.yp.to for much more: full paper; scripts; detailed Brainpool analysis; manipulating “minimal” primes and curves (Microsoft “NUMS”); manipulating security criteria.