McBits:

fast constant-time code-based cryptography

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... using code-based crypto with a solid track record.

... all of the above at once.

#### The track record

1978 McEliece proposed public-key code-based crypto.

Has held up well after extensive optimization of attack algorithms:

1962 Prange. 1981 Omura.

1988 Lee-Brickell. 1988 Leon.

1989 Krouk. 1989 Stern.

1989 Dumer.

1990 Coffey-Goodman.

1990 van Tilburg. 1991 Dumer.

1991 Coffey-Goodman-Farrell.

1993 Chabanne-Courteau.

1993 Chabaud.

- 1994 van Tilburg.
- 1994 Canteaut-Chabanne.
- 1998 Canteaut-Chabaud.
- 1998 Canteaut-Sendrier.
- 2008 Bernstein-Lange-Peters.
- 2009 Bernstein-Lange-
- Peters-van Tilborg.
- 2009 Bernstein (post-quantum).
- 2009 Finiasz-Sendrier.
- 2010 Bernstein-Lange-Peters.
- 2011 May-Meurer-Thomae.
- 2011 Becker-Coron-Joux.
- 2012 Becker–Joux–May–Meurer.
- 2013 Bernstein-Jeffery-Lange-
- Meurer (post-quantum).

### Examples of the competition

Some cycle counts on h9ivy (Intel Core i5-3210M, Ivy Bridge) from bench.cr.yp.to:

```
73092
mceliece encrypt
(2008 Biswas–Sendrier, \approx 2^{80})
gls254 DH
                            76212
(binary elliptic curve; CHES 2013)
kummer DH
                            88448
(hyperelliptic; Asiacrypt 2014)
curve25519 DH
                           182708
(conservative elliptic curve)
                         1130908
mceliece decrypt
ronald1024 decrypt
                         1313324
```

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All load/store addresses and all branch conditions are public. Eliminates cache-timing attacks etc.

Similar improvements for CFS.

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"How can this be competitive in speed? Are you really simulating field multiplication with hundreds of bit operations instead of simple log tables?"

Yes, we are.

Not as slow as it sounds! On a typical 32-bit CPU, the XOR instruction is actually 32-bit XOR, operating in parallel on vectors of 32 bits. Yes, we are.

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Low-end smartphone CPU: 128-bit XOR every cycle.

Ivy Bridge: 256-bit XOR every cycle, or three 128-bit XORs.

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Typical decoding algorithms have add, mult roughly balanced.

Coming next: how to save many adds and *most* mults. Nice synergy with bitslicing.

#### The additive FFT

Fix 
$$n = 4096 = 2^{12}$$
,  $t = 41$ .

Big final decoding step is to find all roots in  $\mathbf{F}_{2^{12}}$  of  $f = c_{41}x^{41} + \cdots + c_0x^0$ .

For each  $\alpha \in \mathbf{F}_{2^{12}}$ , compute  $f(\alpha)$  by Horner's rule: 41 adds, 41 mults.

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Our cost: 6.01 adds, 2.09 mults.

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Wait a minute. Didn't we learn in school that FFT evaluates an n-coeff polynomial at n points using  $n^{1+o(1)}$  operations? Isn't this better than  $n^2/\lg n$ ?

#### Standard radix-2 FFT:

Want to evaluate

$$f = c_0 + c_1 x + \cdots + c_{n-1} x^{n-1}$$
  
at all the *n*th roots of 1.

Write 
$$f$$
 as  $f_0(x^2) + x f_1(x^2)$ .  
Observe big overlap between  $f(\alpha) = f_0(\alpha^2) + \alpha f_1(\alpha^2)$ ,  $f(-\alpha) = f_0(\alpha^2) - \alpha f_1(\alpha^2)$ .

 $f_0$  has n/2 coeffs; evaluate at (n/2)nd roots of 1 by same idea recursively. Similarly  $f_1$ . Useless in char 2:  $\alpha = -\alpha$ . Standard workarounds are painful. FFT considered impractical.

1988 Wang–Zhu, independently 1989 Cantor: "additive FFT" in char 2. Still quite expensive.

1996 von zur Gathen-Gerhard: some improvements.

2010 Gao-Mateer: much better additive FFT.

We use Gao–Mateer, plus some new improvements.

Gao and Mateer evaluate  $f = c_0 + c_1x + \cdots + c_{n-1}x^{n-1}$  on a size-n  $\mathbf{F}_2$ -linear space.

Their main idea: Write 
$$f$$
 as  $f_0(x^2 + x) + x f_1(x^2 + x)$ .

Big overlap between 
$$f(\alpha) = f_0(\alpha^2 + \alpha) + \alpha f_1(\alpha^2 + \alpha)$$
  
and  $f(\alpha + 1) = f_0(\alpha^2 + \alpha) + (\alpha + 1)f_1(\alpha^2 + \alpha)$ .

"Twist" to ensure  $1 \in \text{space}$ . Then  $\{\alpha^2 + \alpha\}$  is a

size-(n/2) **F**<sub>2</sub>-linear space.

Apply same idea recursively.

### Results

60493 Ivy Bridge cycles:

8622 for permutation.

20846 for syndrome.

7714 for BM.

14794 for roots.

8520 for permutation.

Code will be public domain.

We're still speeding it up.

Also  $10\times$  speedup for CFS.

More information:

cr.yp.to/papers.html#mcbits

What you find in paper:

Cryptosystem specification.

Our speedups to additive FFT. (We now have more speedups: cr.yp.to/papers.html#auth256.)

Fast syndrome computation without big precomputed matrix. Important for lightweight!

Fast secret permutation using bit operations: sorting networks, permutation networks.