McBits:
fast constant-time
code-based cryptography

D. J. Bernstein
University of Illinois at Chicago &
Technische Universiteit Eindhoven

Joint work with:

Tung Chou
Technische Universiteit Eindhoven

Peter Schwabe
Radboud University Nijmegen

Objectives

Set new speed records
for public-key cryptography.

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Some cycle counts on `h9ivy` (Intel Core i5-3210M, Ivy Bridge) from `bench.cr.yp.to`:

`mceliece` encrypt	73092
(2008 Biswas–Sendrier, $\approx 2^{80}$)	
`gls254` DH	76212
(binary elliptic curve; CHES 2013)	
`kummer` DH	88448
(hyperelliptic; Asiacrypt 2014)	
`curve25519` DH	182708
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Talk will focus on this case.

(Decryption is slightly slower: includes hash, cipher, MAC.)

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Similar improvements for CFS.

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Low-end smartphone CPU: 128-bit XOR every cycle.

Ivy Bridge: 256-bit XOR every cycle, or three 128-bit XORs.

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Yes, we are.

Not as slow as it sounds!
On a typical 32-bit CPU,
the XOR instruction
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operating in parallel
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Low-end smartphone CPU:
128-bit XOR every cycle.

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The additive FFT

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41 adds, 41 mults.

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Or use Chien search: compute c_ig^i, c_ig^{2i}, c_ig^{3i}, etc. Cost per point: again 41 adds, 41 mults.

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Isn't this better than $n^2/\lg n$?

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Write f as $f_0(x^2)$
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at all the nth roots of 1.

Write f as $f_0(x^2) + x f_1(x^2)$.

Observe big overlap betwee

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and $f(\alpha + 1) =$
$f_0(\alpha^2 + \alpha) + (\alpha + 1) f_1(\alpha^2 + \alpha)$.

"Twist" to ensure $1 \in$ space.
Then $\{\alpha^2 + \alpha\}$ is a
size-$(n/2)$ $\mathbf{F}_2$-linear space.
Apply same idea recursively.

Results

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Gao and Mateer evaluate
$f = c_0 + c_1 x + \cdots + c_{n-1} x^{n-1}$
on a size-n $\mathbf{F}_2$-linear space.

Their main idea: Write f as
$f_0(x^2 + x) + x f_1(x^2 + x)$.

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Mateer evaluate

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What you find in paper:

Cryptosystem specification.

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sorting networks,
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