



(Picture credit: Reuters.)

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How to manipulate standards

Daniel J. Bernstein

University of Illinois at Chicago &

Technische Universiteit Eindhoven



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is under attack from  
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ECDL/ECDH difficulty depends on curve  $E$ .

How did terrorists decide which curve  $E$  to use?

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Did they screw up? (See TLS.)

**Can we influence this?**

Move towards more accurate model of cryptography.

e.g. protocol  $\text{ECDH}_V$ :

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1. Alice generates secret  $a$ , sends  $aP$  on  $E$ .
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New: Optimized this computation using Keccak on cluster of 41 GTX780 GPUs. In 7 hours found “secure+twist-secure”  $b = 0x$

**BADA55EC**D8BBEAD3ADD6C534F92197DE  
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PS 186-4 (2013) requires

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New: Optimized this computation using Keccak on cluster of 41 GTX780 GPUs. In 7 hours found “secure+twist-secure”  $b = 0x$

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