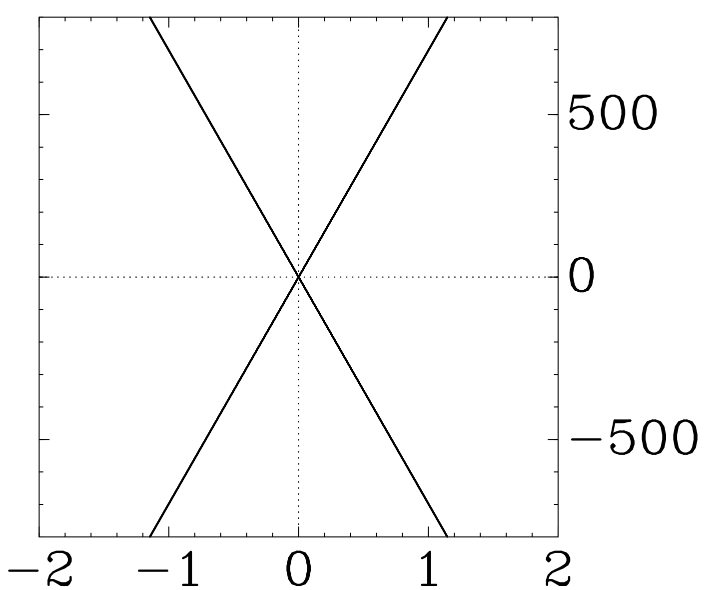
New speed records for point multiplication

D. J. Bernstein



Thanks to:
University of Illinois at Chicago
NSF CCR–9983950
Alfred P. Sloan Foundation

640838 Pentium M cycles to compute a 32-byte secret shared by Dan and Tanja, given Dan's 32-byte secret key n and Tanja's 32-byte public key K.

All known attacks: $> 2^{128}$ cycles.

This is the new speed record for high-security Diffie-Hellman.

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Curve25519 is the elliptic curve
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1. For $b \in \{0, 1\}$, compute $x[b]$ as $bx[1] + (1 - b)x[0]$ or similar. Avoids data-dependent indexing. Costs 36210 fp ops (6%).

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Reciprocal: 43821

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$2673 = 11 \cdot 243$ for 11 more mults.

Additional work: 304 fp ops.

Inside one main-loop iteration:
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Mult: poly mult using
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reduce using 9 fp mults, 9 fp adds;
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overall $2 \cdot 10^2 + 4 \cdot 10 + 3$ fp ops.

Squaring: start with 9 fp doublings;
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Use prime close to power of 2 to save time in field operations.

Also reduces NFS exponent, so would need larger prime for traditional discrete-log systems; but doesn't seem to affect ECDL.

Use prime not far below 2^{32k} to avoid wasting bandwidth.

Comfortable security, $k = 8$: $2^{253} + 39$, $2^{253} + 51$, $2^{254} + 79$, $2^{255} - 31$, $2^{255} - 19$, $2^{255} + 95$.

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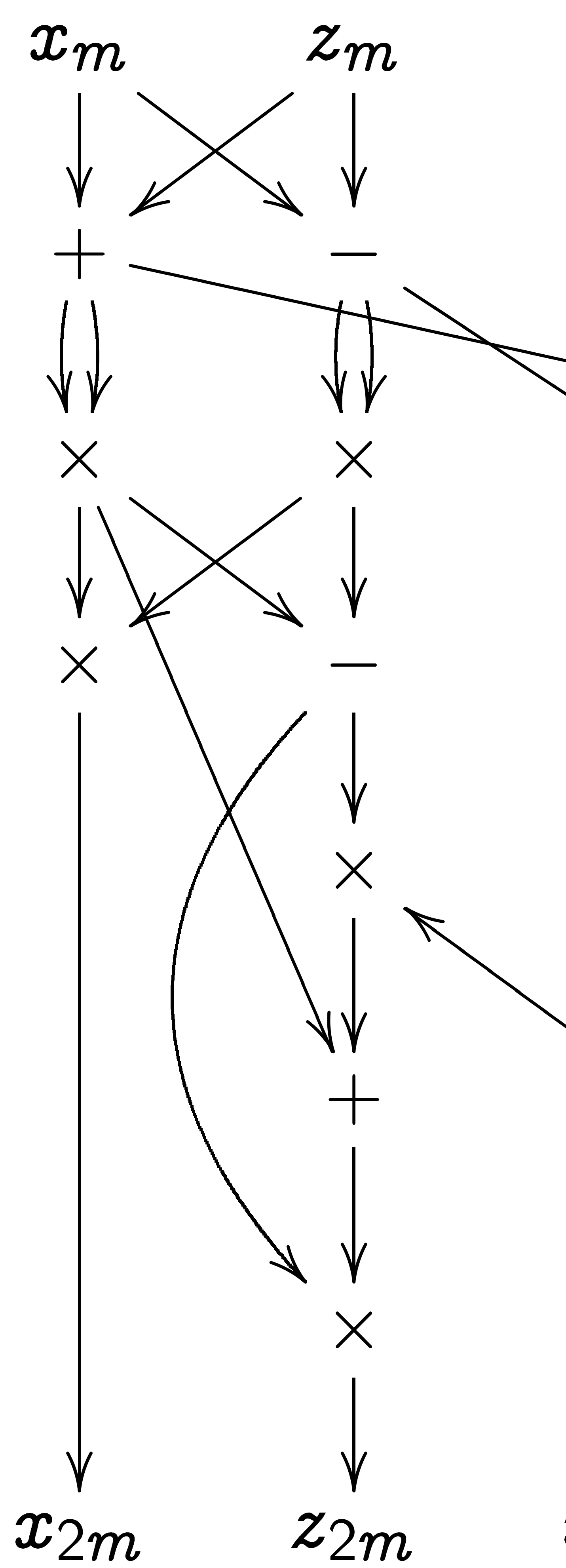

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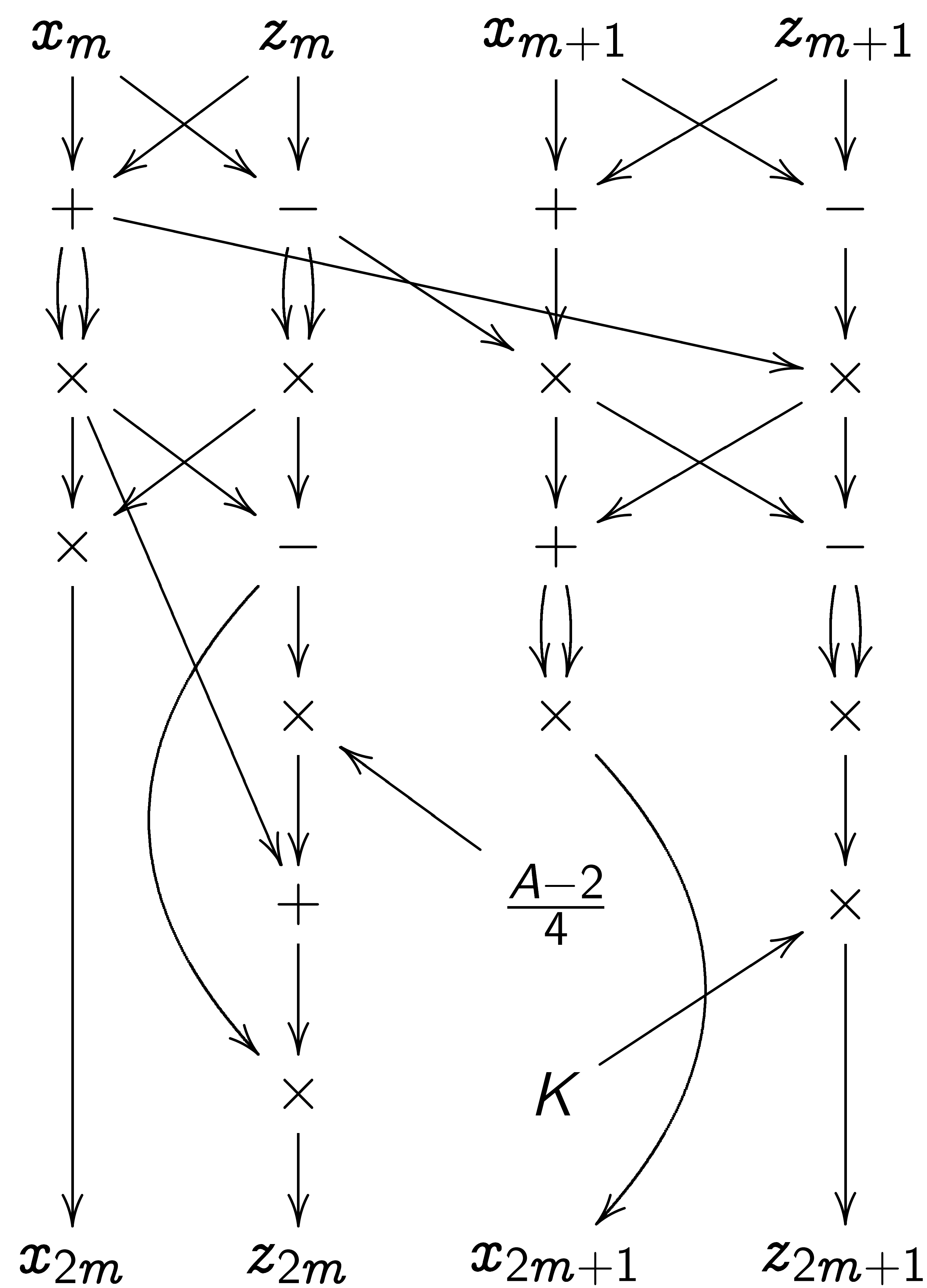

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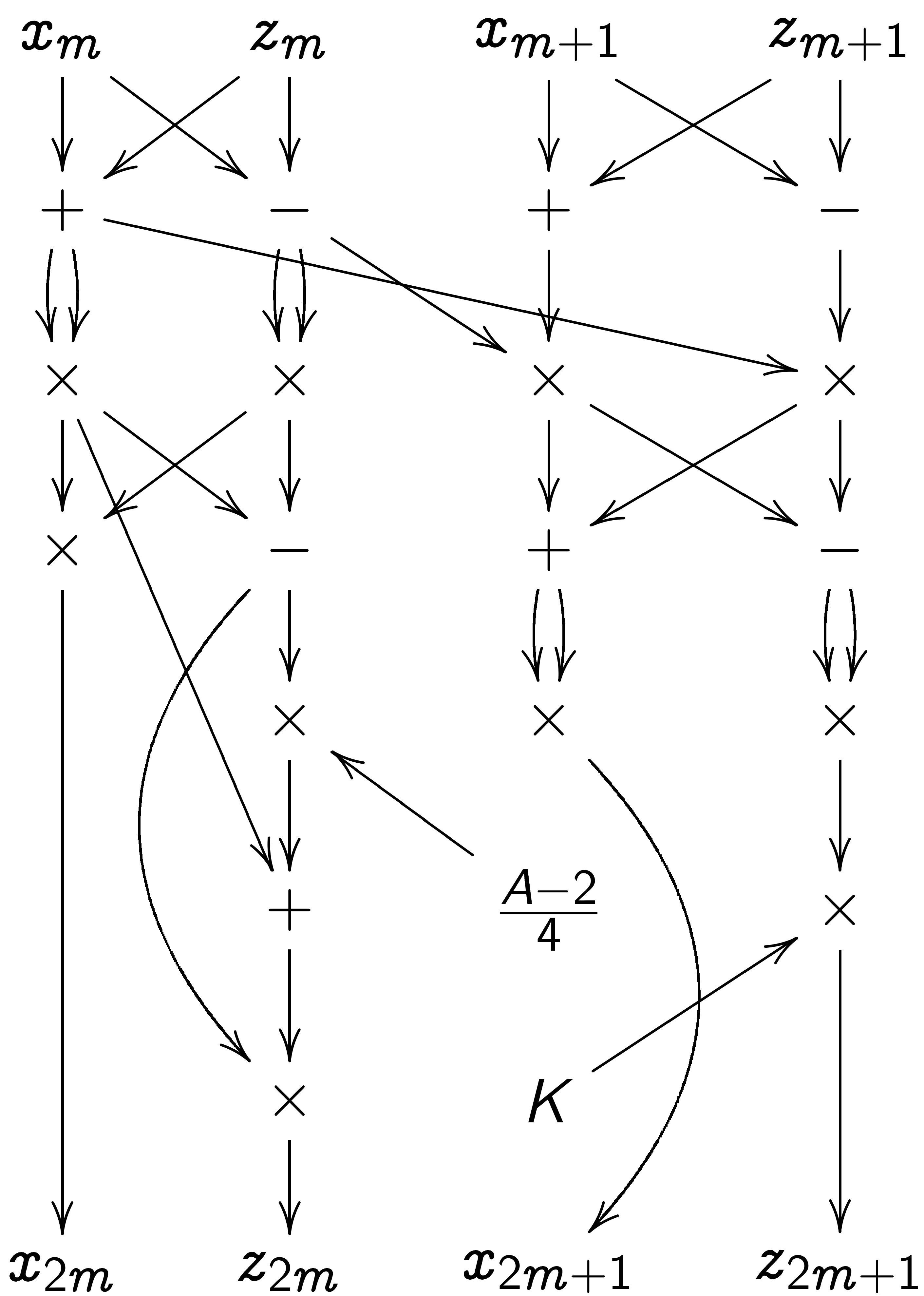

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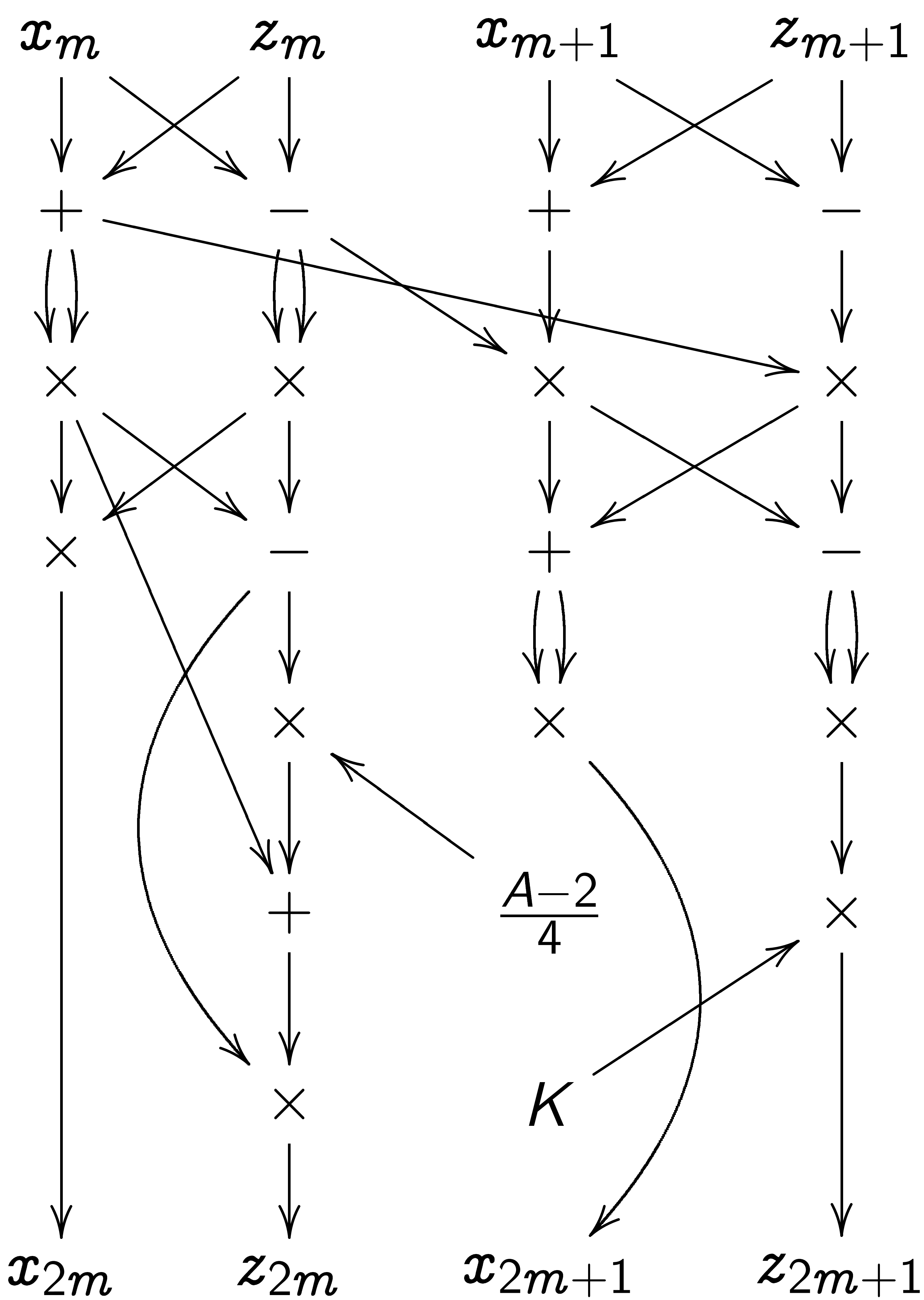

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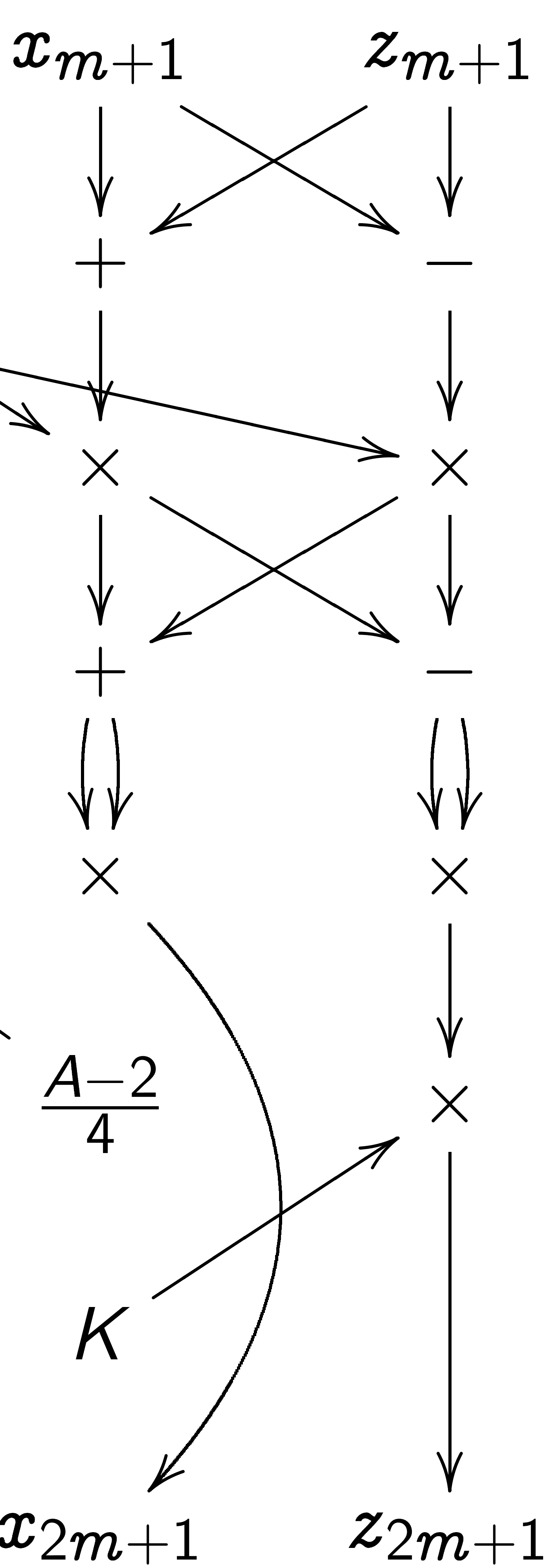

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How was the key r

Public key for secr
is x-coordinate of
of standard base p

Base-point order is
so uniform random
$2^{251} + \{0, 1, 2, \ldots$
produces almost ex
random public key
among $\approx 2^{251}$ pos

The addition of 2^{2}
and avoids timing

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Reality: mult by small constant is as expensive as several adds.

Reality: square-to-multiply ratio is $2/3$ for this field, not $4/5$.

Reality: $a^2 + b^2 + c^2$ is faster than (a^2, b^2, c^2).

<u>How was the key range chosen?</u>

Public key for secret key n is x-coordinate of nth multiple of standard base point $(9, \ldots)$.

Base-point order is $p_1 \approx 2^{252}$, so uniform random n in $2^{251} + \{0, 1, 2, \ldots, 2^{251} - 1\}$ produces almost exactly uniform random public key from among $\approx 2^{251}$ possibilities.

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Curve25519 implementation is actually in `qhasm`, new programming language for high-speed computations.

Language allows declaration and propagation of fp ranges; guided register allocation; et al.

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Culmination of extensive work on eliminating field mults for genus-2 hyperelliptic curves: 25 mults per bit. Gaudry, `eprint.iacr.org/2005/314`

Half-size prime: e.g., $2^{127} - 1$. Select curve to make some mults easier, like choosing A.

Should count fp ops instead. Prediction: this will beat genus 1.