Stronger security bounds for Wegman-Carter-Shoup authenticators

D. J. Bernstein

Thanks to: University of Illinois at Chicago NSF CCR-9983950 Alfred P. Sloan Foundation

Standard polynomial-evaluation MAC: sender sends $(1, m_1, m_1(r) + s_1);$ $(2, m_2, m_2(r) + s_2);$ $(3, m_3, m_3(r) + s_3).$

univariate; degree $< 2^{16}$; constant coefficient 0.

r, *s*₁, *s*₂, *s*₃: elements of *F*; secret; known to sender, receiver.

F: field of size 2^{128} .

- m_1, m_2, m_3 : polynomials over F;

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Wegman-Carter version: (r, s_1, s_2, s_3) is a uniform random element of F^4 . 2^{512} possibilities. each equally likely. Wegman-Carter-Shoup version: $s_1 \neq s_2; s_1 \neq s_3; s_2 \neq s_3;$ otherwise uniform. $2^{256}(2^{128}-1)(2^{128}-2)$ possibilities, each equally likely.

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How secure are these MACs?

Standard security bounds for Wegman-Carter: "Authenticators reveal no information about r." Conditional distribution of r, given $(1, m_1, a_1)$, $(2, m_2, a_2)$, $(3, m_3, a_3)$, is uniform. There are 2^{128} possible r's, each consistent with a $s_2 = a_2 - m_2(r), \ s_3 = a_3 - m_3(r).$

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Say attacker atten (1, m, a) with $m = \frac{1}{7}$ m(0) = 0; degree Forgery is successf $a = m(r) + s_1 \iff$ $a = m(r) + a_1 - \frac{1}{7}$ r is a root of m =

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Say attacker attempts forgery (1, m, a) with $m \neq m_1$; m(0) = 0; degree $< 2^{16}$. Forgery is successful \iff $a=m(r)+s_1\iff$ $a=m(r)+a_1-m_1(r)\iff$ r is a root of $m - m_1 + a_1 - a_1$. $m-m_1+a_1-a$ is a nonzero polynomial of degree $< 2^{16}$ so it has $< 2^{16}$ roots. Attempted forgery has $< 2^{16}/2^{128}$ chance of success.



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 $a_1 = a_1 - m_1(r),$ $s_3 = a_3 - m_3(r).$ Say attacker attempts forgery (1, m, a) with $m \neq m_1$; m(0) = 0; degree $\leq 2^{16}$. Forgery is successful \iff $a = m(r) + s_1 \iff$ $a = m(r) + a_1 - m_1(r) \iff$ r is a root of $m - m_1 + a_1 - a$.

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Fix a deterministic attack A that generates m_1 ; sees $m_1(r) + s_1$; generates m_2 ; sees $m_2(r) + s_2$; generates m_3 ; sees $m_3(r) + s_3$; generates forgery attempt (n, m, a) with $n \in \{1, 2, 3\}$, $m \neq m_n$, m(0) = 0, deg $< 2^{16}$. (Generalizations: randomized A;

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Apply A to Wegman-Carter. $\Pr[a = m(r) + s_n] \le 1/2^{112}.$ Proved this earlier. For each $S \in F^3$: Define p(S) as conditional probability that $a = m(r) + s_n$ given that $(s_1, s_2, s_3) = S$. $\Pr[a = m(r) + s_n]$ $= \sum_{S} \Pr[(s_1, s_2, s_3) = S] p(S)$ $=\sum_{S} 2^{-384} p(S).$

- Thus $\sum_{S} 2^{-384} p(S) \le 1/2^{112}$.

c attack A that s $m_1(r) + s_1;$ s $m_2(r) + s_2;$ s $m_3(r) + s_3;$ attempt $\in \{1, 2, 3\},$ 0, deg $\leq 2^{16}$.

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Apply A to Wegm $\Pr[(s_1, s_2, s_3) = S$ $\delta = 2^{384}/2^{128}(2^{128})$ For $S \in F^3$: Cond that a = m(r) + s $(s_1, s_2, s_3) = S$, is so $\Pr[a = m(r) +$ $\leq \sum_{S} 2^{-384} \delta p(S)$ This is the stronge Could take careles use Pr < 1 to get $\Pr < 1/2^{112} + 3/2$

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that $a = m(r) + s_n$, given that $(s_1, s_2, s_3) = S$, is the same p(S), so $\Pr[a = m(r) + s_n]$ $\leq \sum_{S} 2^{-384} \delta p(S) \leq \delta/2^{112}$. Could take careless extra step: use Pr < 1 to get weaker bound $\Pr \le 1/2^{112} + 3/2^{128}$.

- Apply A to Wegman-Carter-Shoup.
- $\Pr[(s_1, s_2, s_3) = S] < 2^{-384} \delta$ where $\delta = \frac{2^{384}}{2^{128}} (2^{128} - 1)(2^{128} - 2).$
- For $S \in F^3$: Conditional probability
- This is the stronger security bound.

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Wegman-Carter-S after 240 chosen n and D forgery atte Stronger: $\leq \approx D/($ Careless: $\leq \approx (D/$ Original: $\leq \approx D/($ 2^{60} instead of 2^{40} : Stronger: $\leq \approx D/($ Careless: $\leq \approx (D/$ Original: $< \approx \infty$.

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For $S \in F^3$: Conditional probability that $a = m(r) + s_n$, given that $(s_1, s_2, s_3) = S$, is the same p(S), so $\Pr[a = m(r) + s_n]$ $\leq \sum_{S} 2^{-384} \delta p(S) \leq \delta/2^{112}$. This is the stronger security bound.

Could take careless extra step: use Pr < 1 to get weaker bound $\Pr < 1/2^{112} + 3/2^{128}$.

Wegman-Carter-Shoup bounds after 2⁴⁰ chosen messages and *D* forgery attempts: Stronger: $< \approx D/(2^{112} - 2^{63})$. Careless: $\leq \approx (D/2^{112}) + (1/2^{49})$. Original: $\leq \approx D/(2^{112} - 2^{79})$. 2^{60} instead of 2^{40} : Stronger: $\leq \approx D/(2^{112} - 2^{103})$. Careless: $<\approx (D/2^{112}) + (1/2^9)$. Original: $\leq \approx \infty$.

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 $[S] \leq 2^{-384} \delta$ where $[S^8 - 1)(2^{128} - 2).$

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Generalize $m_i(r)$ $h(m_i) + s_i$ where small differential p $\Pr[h(m) - h(m')]$

Original bound \approx for *C* as large as $\sqrt{}$ where *C* is # chose Proof strategy is c for larger *C*.

Stronger bound \approx for *C* as large as $\sqrt{}$ Careless bound \approx Wegman-Carter-Shoup bounds after 2⁴⁰ chosen messages and D forgery attempts: Stronger: $< \approx D/(2^{112} - 2^{63})$. Careless: $\leq \approx (D/2^{112}) + (1/2^{49})$. Original: $\leq \approx D/(2^{112} - 2^{79})$. 2^{60} instead of 2^{40} : Stronger: $< \approx D/(2^{112} - 2^{103})$. Careless: $<\approx (D/2^{112}) + (1/2^9)$. Original: $< \approx \infty$.

Generalize $m_i(r) + s_i$ to any $h(m_i) + s_i$ where h has small differential probabilities: $\Pr[h(m) - h(m') = g] \leq \epsilon.$ Original bound $\approx D\epsilon$ for C as large as $\sqrt{1/\epsilon}$, where C is # chosen messages. Proof strategy is doomed for larger C.

Stronger bound $\approx D\epsilon$ for C as large as $\sqrt{2^{128}}$.

Careless bound $\approx D\epsilon + C^2/2^{129}$.

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Wegman-Carter-Shoup security implies $h(m_i) + AES_k(i)$ security if AES is secure.

Explicit AES security goal: $AES_k(1), AES_k(2), ...$ indistinguishable from s_1, s_2, \ldots

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http://cr.yp.to/mac.html. Poly1305-AES bound on ϵ is $[L/16]/2^{103}$ for *L*-byte messages. e.g., $\epsilon < 2^{-92}$ for L = 2048. Security gap compared to AES $< 1.7D/2^{92}$ if $C < 2^{64}$. With old security bound, C was limited to about 2^{46} .

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Improved security bounds

/papers.html#permutations

Stronger than "game-playing."

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/papers.html#countermode, coming soon.

apply far beyond the MAC context.

"Stronger security bounds for

- permutations": http://cr.yp.to
- Another application: Counter mode

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and on ϵ

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Improved security bounds apply far beyond the MAC context. "Stronger security bounds for permutations": http://cr.yp.to /papers.html#permutations Stronger than "game-playing." Another application: Counter mode is provably stronger than CBC. /papers.html#countermode, coming soon.

AES security problem 16-byte block inverse Partly fixed in this but still annoying.

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Another application: Counter mode is provably stronger than CBC. /papers.html#countermode, coming soon.

AES security problems from 16-byte block invertibility: Partly fixed in this talk, but still annoying.

AES security problems from secret-index table lookups: was wrong. Very hard to fix without extreme slowdowns. /papers.html#cachetiming

Many fast stream ciphers don't have these problems. Do we want to keep AES?

- "Not vulnerable to timing attacks"