

# “HILA5 Pindakaas”:<sup>†</sup> On the CCA security of lattice-based encryption with error correction

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**Abstract.** We show that HILA5 is not secure against chosen-ciphertext attacks. Specifically, we demonstrate a key-recovery attack on HILA5 using an active attack on reused keys. The attack works around the error correction in HILA5. The attack applies to the HILA5 key-encapsulation mechanism (KEM), and also to the public-key encryption mechanism (PKE) obtained by NIST’s procedure for combining the KEM with authenticated encryption. This contradicts the most natural interpretation of the IND-CCA security claim for HILA5.

**Keywords:** Post-quantum cryptography, KEM, RLWE, reaction attack.

## 1 Introduction

HILA5 [13] is a public-key scheme designed by Saarinen. HILA5 was submitted as a “Key Encapsulation Mechanism and Public Key Encryption Algorithm” [12] to NIST’s call [10] for post-quantum proposals. HILA5’s design is based on Ring Learning With Errors (RLWE) over NTRU NTT rings. HILA5 takes the same ring parameters as New Hope [2] and changes the reconciliation method by which Alice and Bob achieve the same key to get a much lower chance of decryption failures.

The HILA5 submission [12] states

*This design also provides IND-CCA secure KEM-DEM [CS03] public key encryption if used in conjunction with an appropriate AEAD [Rog02] such as NIST approved AES256-GCM [FIP01, Dwo07].*

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<sup>†</sup> “Helaas pindakaas” is a Dutch expression meaning “Oh well, too bad”.

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In this paper we show that HILA5 is not CCA secure: We compute Alice’s secret key by sending her multiple encapsulation messages and using her answers. Our attack works independently of whether an AEAD is used or not and despite the error correcting padding introduced in HILA5.

We have fully implemented our attack and experimentally verified that it works with high probability. We use the HILA5 reference implementation for Alice’s part and also to verify that the retrieved secret key works for decryption. We use a modified version of the same software for some of the computations in the attack.

## 1.1 Related work

Ajtai–Dwork [1] and NTRU [7] are the oldest lattice-based encryption systems. In 1999 Hall, Goldberg, and Schneier [6] developed a reaction attack which recovers the Ajtai–Dwork private key by observing decryption failures for suitably crafted encryptions to the public key. They wrote “We feel that the existence of these attacks effectively limits these ciphers to theoretical considerations only. That is, any implementation of the ciphers will be subject to the attacks we present and hence not safe.”

Hoffstein and Silverman [8] adapted the attack to NTRU. They suggested modifying NTRU to use the Fujisaki–Okamoto transform [5]. This complicates and slows down the cryptosystem, and also turns out to still allow attacks that exploit occasional decryption failures for *valid* ciphertexts. See [9].

New Hope is a key-encapsulation mechanism (KEM), presented as a key-exchange protocol. It allows occasional decryption failures for valid ciphertexts, and explicitly avoids the “changes” that would be required for the Fujisaki–Okamoto transform. To prevent reaction attacks and other chosen-ciphertext attacks by a malicious Bob, New Hope requires using ephemeral keys, meaning keys that change with every execution of the protocol. The New Hope paper warns that reusing a public key in multiple protocol runs (“key caching”) would be “disastrous for security”, although it does not describe an attack.

Fluhrer [4] showed the details of how to attack key reuse in a similar key-exchange protocol. Followup work [3] extended the attack to more key-exchange protocols.

HILA5 is similar to New Hope, and still does not use the Fujisaki–Okamoto transform. HILA5 includes an error-correction method that practically eliminates decryption failures for valid ciphertexts. HILA5 does not warn against key caching: on the contrary, the most natural interpretation of the HILA5 security claims is that HILA5 is secure against chosen-ciphertext attacks. See Section 5.

## 2 Data flow in the attack

A KEM is defined by three algorithms. Key generation produces a secret key and a public key. Encapsulation produces a ciphertext and a session key, given a public key. Decapsulation produces a session key or failure, given a ciphertext

and a secret key. The HILA5 submission document [12] gives details and reference code for a particular KEM, the “HILA5 KEM”.

Our attack is a key-recovery attack against the HILA5 KEM: the attacker, evil Bob, ends up computing the secret key of a target Alice. This secret key gives the attacker the ability to run the decapsulation algorithm using Alice’s secret key, and thus the ability to immediately decrypt legitimate ciphertexts sent by other users to Alice.

Our attack is a chosen-ciphertext attack: evil Bob chooses ciphertexts to provide to Alice (different from the legitimate ciphertexts), and learns something from observing the outputs of Alice decapsulating those ciphertexts. Formally, the attack shows that the HILA5 KEM does not provide IND-CCA2 security.

There are two important ways that the attack does not need the full power of a CCA2 decapsulation oracle. First, the attack is what is called a “reaction attack” in [6] or a “sloppy Alice attack” in [14]: evil Bob has a guess for the output of each decapsulation, and learns whether Alice’s actual decapsulation output matches this guess. Evil Bob does not need any further information.

Second, evil Bob chooses all of his ciphertexts, and learns the secret key from Alice’s reactions, before seeing the legitimate ciphertexts to decrypt. Formally, the attack shows not only that the HILA5 KEM does not provide IND-CCA2 security, but also that it does not provide IND-CCA1 security.

## 2.1 Hashing the secret key does not stop the attack

One can easily stop key-recovery attacks by defining HILA5Hash as follows. HILA5Hash key generation computes a uniform random 32-byte string  $s$ , and then runs HILA5 key generation to obtain a public key, hashing  $s$  to generate all randomness used in HILA5 key generation. The HILA5Hash secret key is  $s$ . HILA5Hash encapsulation is the same as HILA5 encapsulation. HILA5Hash decapsulation reconstructs the HILA5 secret key from  $s$  (again running the HILA5 key-generation algorithm; alternatively, the HILA5 secret key can be cached), and then runs the HILA5 decapsulation algorithm.

Unless the hash function is easy to invert, a key-recovery attack against HILA5 does not produce a key-recovery attack against HILA5Hash. However, this hashing does not prevent the attacker from decrypting legitimate ciphertexts sent by other users to Alice.

## 2.2 AEAD does not stop the attack

A PKE is defined by three algorithms. Key generation produces a secret key and a public key, as in a KEM. Encapsulation produces a ciphertext, given a plaintext and a public key. Decapsulation produces a plaintext or failure, given a ciphertext and a secret key.

The subtitle of the HILA5 submission is “Key Encapsulation Mechanism (KEM) and Public Key Encryption Algorithm”. The submission document does not include a definition of a PKE, but NIST had already stated before submission

that it would automatically convert each submitted KEM to a PKE using the following “standard conversion technique”: “appending to the KEM ciphertext, an AES-GCM ciphertext of the plaintext message” where the AES-GCM key is “the symmetric key output by the encapsulate function”. This is the standard Cramer–Shoup “KEM-DEM” construction, using AES-GCM as the DEM. We write “HILA5 PKE” for the PKE that NIST will automatically produce in this way from the HILA5 KEM.<sup>3</sup>

Breaking the IND-CCA2 security of a KEM does not necessarily imply breaking the IND-CCA2 security of a PKE obtained in this way. IND-CCA2 attacks against the KEM can see session keys produced by decapsulation, whereas IND-CCA2 attacks against the PKE are merely able to see the result of AES-GCM decryption using those keys.

However, our attack against the HILA5 KEM is also a key-recovery attack against the HILA5 PKE. It is important here that the attack is a reaction attack: what evil Bob needs to know is merely whether a guessed session key is correct. Starting from this guessed session key, evil Bob produces a valid AES-GCM ciphertext using this guess as an AES key. If decapsulation in fact produces this session key then AES-GCM decryption succeeds and produces the plaintext that evil Bob started with. If decapsulation produces a different session key then AES-GCM decryption is practically guaranteed to fail (anything else would be a surprising security flaw in AES-GCM), so evil Bob sees a decryption failure from the PKE.

To summarize, evil Bob sees decryption failures from the PKE, and learns from this which guesses were correct, which is the same information that evil Bob obtains from the KEM. Evil Bob then computes the secret key from this information. Consequently, the HILA5 PKE does not provide IND-CCA2 security, and does not even provide IND-CCA1 security.

### 2.3 Black holes would stop the attack

Like other chosen-ciphertext attacks, our attack is inapplicable to scenarios where the results of decapsulation and decryption are hidden from the attacker. For example, if ciphertexts are sent to NSA’s public key, and if NSA hides the results of applying its secret key to those ciphertexts, then an attacker outside NSA cannot use our attack to compute NSA’s secret key. However, if NSA reacts to those results in a way that leaks to the attacker which ciphertexts were valid, then the attacker can compute NSA’s secret key.

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<sup>3</sup> NIST actually deviates slightly from the KEM-DEM construction: it specifies a “randomly generated IV” for AES-GCM, while Cramer and Shoup use a deterministic DEM. For consistency with the ciphertext sizes mentioned in [12], we actually define “HILA5 PKE” to be the Cramer–Shoup construction using AES-GCM with an all-zero IV. Switching to NIST’s construction would expand ciphertext sizes by 12 bytes using the default IV sizes for AES-GCM, and would not affect our attack.

## 2.4 The Fujisaki–Okamoto transform would stop the attack

We briefly outline a more radical change to HILA5, which we call “HILA5FO”. HILA5FO ciphertexts are slightly larger than HILA5 ciphertexts, decapsulation is more complicated, and decapsulation is extrapolated (from reported HILA5 benchmarks) to be several times slower, but HILA5FO would stop our attack.

The idea of the HILA5FO KEM is to reapply the encapsulation algorithm as part of decapsulation, and check whether the resulting ciphertext is identical to the received ciphertext. This is not a new idea: it is used in many other submissions to NIST (with various differences in details), typically with credit to Fujisaki and Okamoto [5].

HILA5 does not provide any easy way to reconstruct the randomness used in encapsulation (most importantly Bob’s  $b$ ), so the HILA5FO KEM computes this randomness as a hash of a plaintext recovered as part of decapsulation. The HILA5 KEM does not transmit a plaintext, so the HILA5FO KEM is instead built from the HILA5 PKE.

Encapsulation in the HILA5FO KEM thus chooses a random plaintext, and encrypts this plaintext using the HILA5 PKE (the HILA5 KEM producing a session key for AES-GCM) using a hash of the plaintext to compute all randomness used inside the PKE. Decapsulation applies HILA5 PKE decryption (HILA5 KEM decapsulation producing a session key for AES-GCM decryption), and checks that the resulting plaintext produces the same ciphertext.

Deriving a PKE from the HILA5FO KEM would involve two layers of AES-GCM, which can be compressed to one layer as follows: place 32 bytes of randomness at the beginning of the user-supplied plaintext, and then encrypt this plaintext using the HILA5 PKE, again using a hash of the plaintext to compute all randomness used inside the PKE. The overall ciphertext size is the original plaintext size, plus 32 bytes (the randomness), plus the HILA5 KEM ciphertext size, plus 16 bytes (the AES-GCM authenticator), i.e., 32 bytes more than the HILA5 PKE. The main cost in HILA5FO decryption (for short messages) is reapplying HILA5 KEM encapsulation, which according to [12, Table 1] is five times slower than HILA5 KEM decapsulation.

## 3 Preliminaries

This section describes the HILA5 scheme and Fluhrer’s attack on RLWE schemes.

### 3.1 The HILA5 scheme

We describe the scheme as given in [12, Section 4.9] but leave out formatting and NTT conversions. These are used in the attack implementation to interface with the reference implementation but do not contribute to the security and hamper readability.

The major computations take place in the ring  $R = \mathbb{Z}_q[x]/(x^n + 1)$ , where  $n = 1024$  and  $q = 12289$ . Alice’s secret key is a small, random polynomial  $a \in R$ ,

where small (here and in the following) means that the coefficients are chosen from a narrow distribution around zero, more precisely the discrete binomial distribution  $\Psi_{16}$  which has integer values in  $[-16, 16]$ . To compute the public key she picks another small random polynomial  $e \in R$  and a random  $g \in R$  and computes  $A = ga + e$ . She publishes  $(g, A)$  and keeps  $a$  as her secret.

An honest Bob picks two random small polynomials  $b, e'$  and computes  $B = gb + e'$  and  $y = Ab$ . If Bob sends value  $B$  to Alice, the second value  $y = Ab = (ga + e)b = gab + eb \approx gab$  will be very close to what Alice can compute using her secret:  $x = aB = a(gb + e') = gab + e'a \approx gab$ , because  $a, b, e, e'$  are all small. A simple rounding operation, such as taking the top bits of each coefficient, to achieve a shared secret will induce differences between Alice's and Bob's version with too high probability. For example, Bob could take  $k[i] = \lfloor 2y[i]/q \rfloor$  and Alice could take  $k'[i] = \lfloor 2x[i]/q \rfloor$ , where we use  $t[i]$  to denote the  $i$ th coefficient of polynomial or vector  $t$ , but for indices with  $(gab)[i] \approx 0$  (or  $q/2$ ) the error-terms can cause the values to flip to a different bit, i.e.  $k[i] \neq k'[i]$ . For this rounding operation, we call elements of  $\{0, q/2\}$  'the "edges", as these are the values for which it is probable that errors occur. This is why Bob sends a second vector, a binary reconciliation vector  $c$ , to help Alice recover the same  $k$  as Bob. Basically, this means that the scheme uses two pairs of edges. If  $y[i]$  was close to one edge of a certain pair, Bob will choose the other pair of edges, so that Alice can still successfully recover the shared secret. In previous work [11], the reconciliation vector achieves a successful shared secret with high probability, as long as  $|x[i] - y[i]| < q/8$ .

HILA5 differs in how these reconciliation bits are computed. For each coefficient  $y[i]$  of  $y$  Bob computes  $k[i] = \lfloor 2y[i]/q \rfloor$ ,  $c[i] \equiv \lfloor 4y[i]/q \rfloor \pmod 2$ , and

$$d[i] = \begin{cases} 1 & \text{if } |(y[i] \bmod \lfloor q/4 \rfloor) - \lfloor q/8 \rfloor| \leq \beta \\ 0 & \text{otherwise,} \end{cases}$$

where  $\beta = 799$ .

He then selects the first 496 positions  $i$  for which  $d[i] = 1$ . Basically, these are the positions for which it is likely that Alice and Bob recover the same value. In other words, for these indices the value  $|x[i] - y[i]|$  is likely to be far away from the edge  $M$ , thus further reducing the probability of errors in the shared secret. (Note that the description suggests to discard some positions if there are more than 496 such positions while the code deterministically discards the later ones by setting  $d[j] = 0$  for them.)

The encapsulation consists of  $B, d, c$ , and an extra part  $r$  described below; here  $d$  covers the full  $n$  positions while  $c$  can compress to those positions with  $d[i] = 1$ .

Alice recovers the  $k[i]$  at the selected 496 positions by computing

$$k'[i] = \lfloor (2/(x[i] - c[i]\lfloor q/4 \rfloor + \lfloor q/8 \rfloor) \bmod q)/q \rfloor.$$

The HILA5 submission shows that with probability  $1 - 2^{-36}$   $k'[i] = k[i]$ . Let  $k$  (resp.  $k'$ ) be the string of 496 bits given by the concatenation of the  $k[i]$  (resp.  $k'[i]$ ).

The role of  $r$  is not well described but the HILA5 design overview says that is an encrypted encoding of a part of  $k$ . It is computed by splitting  $k$  as  $k = m||z$ , where  $m$  gets the first 256 bits and  $z$  the remaining 240 bits. HILA5 uses a custom-designed error-correcting code XE5 that corrects at least 5 errors to compute a 240-bit checksum  $s$  of  $m$  and then computes  $r = s \oplus z$ , where  $\oplus$  denotes bitwise addition (XOR).

Alice computes  $k' = m' || z'$ , the checksum  $s'$  on  $m'$  and uses the error correction in XE5 on  $m', s', z'$  and  $r$  to correct  $m'$  to  $m$ .

### 3.2 Fluhrer's attack

The chosen-ciphertext attack on HILA5 that we are going to present is a variant of the following attack against key reuse in RLWE-based key exchange protocols presented by Fluhrer in 2016 [4]. This section assumes that Bob computes the  $c[i]$  and  $k[i]$  in a way similar to the previous section. The  $d[i]$  were added in HILA5 and will be considered in the next section.

Recall that Alice's version of the shared secret key is

$$gab + e'a,$$

where  $g$  is some large public generator element,  $a$  and  $b$  are Alice and Bob's small private keys, and  $e'$  is a small noise vector chosen by Bob. This version of the shared secret differs from Bob's by some small error, hence they need to employ a reconciliation mechanism to arrive at the same secret bit string.

The general strategy of an evil Bob is to artificially force one (say, the first) coefficient of  $gab$  to be close to the edge  $M$  between the intervals that are mapped to bits 0 and 1 during reconciliation. An honest user would set the reconciliation bit  $c[0]$  in that case, such that Alice uses another mapping that is less likely to produce an error, but evil Bob does not. Since evil Bob proceeds honestly except for the first bit, he knows two possibilities for Alice's key, hence he can query Alice with one of these guesses and distinguish between 0 and 1 based on her reaction. As evil Bob is assumed to know  $(gab)[0]$ , this tells him that  $(e'a)[0]$  has to lie in a certain interval.

After a few queries using binary search with varying values for  $(gab)[0]$ , evil Bob knows the exact distance of  $(e'a)[0]$  from the edge, and if he sets  $e' = 1$ , this distance is nothing but the first coefficient of Alice's secret key  $a$ . Note that in Fluhrer's setting the edge  $M$  is at zero and he uses  $b$  with  $(gab)[0] = 1$ , hence evil Bob can just multiply  $b$  by small distances when searching for  $(e'a)[0]$ . In our adaptation of the attack to HILA5, this step is slightly more involved; see Section 4.2.

Of course, one could apply this method individually to each coefficient to extract Alice's full secret key, but in fact, being able to recover the coefficient at one position is enough: due to the structure of the underlying ring, evil Bob can shift any other coefficient of  $a$  into the constant term of  $aB$  by setting  $e'$  to  $x^{-i}$ , i. e., a vector with one entry  $\pm 1$  and 0 elsewhere.

Notice that evil Bob does not a priori know a value  $b$  such that  $(gab)[0] = 1$ , but he can still reasonably guess one: Alice's public key is  $ga + e$  for small vectors

$a$  and  $e$ , hence if  $b$  is a small low-weight vector such that  $(b \cdot (ga + e))[0]$  is close to 1, there is a good chance that in fact  $(gab)[0] = 1$ . Thus, while evil Bob does not have a deterministic method to find an “evil”  $b$ , he can still just make educated guesses until he finds one that works.

Finding  $b \in R$  with  $(b \cdot (ga + e))[0]$  is close to 1 is an offline computation using only Alice’s public key, testing for  $(gab)[0] = 1$  requires interaction with Alice.

There are several follow-ups to Fluhrer’s paper, e.g. the recently posted [3]), but a small and new generalization of Fluhrer’s attack is sufficient to attack HILA5.

## 4 Chosen-ciphertext attack on HILA5

In this section, we describe how we circumvent the error-correction code and how to adapt Fluhrer’s attack to the HILA5 case.

### 4.1 Working around error correction

Fluhrer’s original attack depends crucially on the attacker’s ability to detect single-bit errors in Alice’s version of the shared secret. However, the HILA5 construction includes XE5 as an error-correcting code that is applied to the shared secret after decapsulation, hiding any induced single-bit errors from the application and the attacker. Thus, to apply Fluhrer’s attack, we have to work around these error-correction abilities. On Alice’s side, a redundancy check  $r'$  (for error correcting) will be computed from her version of the shared secret. If evil Bob only induces a single bit-flip in  $m$  in applying Fluhrer’s attack, the redundancy part  $z$  will not contain errors. This means that if evil Bob does not flip any further bits, he still cannot apply Fluhrer’s attack as he will not see different outputs.

Based on evil Bob’s  $r$  (which is part of the ciphertext), Alice will compute  $r^\Delta = r \oplus r'$ , and from that derive weight value  $w_i^\Delta$  of  $r^\Delta$  for position  $i$  using a formula given in the paper ([12, Formula (2)]). XE5 corrects bit  $m[i]$  whenever  $w_i^\Delta \geq 6$  [12, Theorem 1]. Flipping a single non-zero bit  $m[i]$ , will cause value  $w_i = 10$  [12, Lemma 2], meaning at least 6 of the error correction bits in  $r$  should be correct (as this means  $w_i^\Delta \geq 6$ ). Therefore, flipping at least 5 bits in evil Bob’s version of  $r$ , means Alice will be unable to correct bit  $m[i]$  if she computes a different value  $m'[i] \neq m[i]$ .

In the attack we focus on the first bit  $m[0]$  of  $m$ , which means we need to flip  $w_0^\Delta$ . This depends on 10 positions in  $r$ , the positions labeled “ $r_{(0,0)}$ ” through “ $r_{(9,0)}$ ” in [12]; we can flip any 5 of these bits. Our attack flips the first 5 of these bits. This means in the following section we consider the issue of error-correction solved and can directly apply a modification of Fluhrer’s attack.

## 4.2 Details of the attack

This section elaborates evil Bob’s approach to recover Alice’s secret key. As mentioned before, the general procedure mimics Fluhrer’s attack (Section 3.2). The major steps are:

1. Guess a small low-weight secret  $b_0$  such that  $(gab_0)[0]$  is at the edge  $M$ .
2. For each  $\delta \in \{-16, \dots, 16\}$ , compute  $b_\delta$  such that  $(gab_\delta)[0] = M + \delta$ .
3. For each target coefficient of Alice’s secret:
  - (a) Choose  $e'$  such that  $(e'a)[0]$  is the target coefficient.
  - (b) Perform a binary search using the  $b_\delta$  to recover the target coefficient.  
(Alice’s coefficient  $(gab_\delta)[0] + (e'a)[0]$  maps to a 1 bit iff  $(-e'a)[0] > \delta$ .)
4. If the results look “bad” after recovering a few coefficients in this way, the guess for  $b_0$  was probably wrong and evil Bob should start over at step 1.

Note that for each oracle query, i.e., for every interaction with Alice, Bob proceeds honestly except for using specially crafted  $b_\delta$  and  $e'$ , setting  $d_0 = c_0 = 1$ , and flipping a few bits in the error correction as described in Section 4.1. We now explain and analyze the steps above in more detail.

*Forcing coefficients near the edge.* In HILA5’s reconciliation mechanism, there is no edge at zero for any choice of reconciliation bit, hence Fluhrer’s attack does not apply without modifications. We chose to set the reconciliation bit  $c_0$  to 1 and attack the edge at

$$M = \lfloor q/8 \rfloor = 1536.$$

To perform the binary search for Alice’s secret coefficients in the attack, we need to find small low-weight vectors  $b_\delta$  such that

$$(gab_\delta)[0] = M + \delta$$

for all  $\delta$  with  $|\delta| \leq 16$ . (As mentioned in Section 3.2, Fluhrer’s evil Bob attacked  $M = 0$ , thus he could guess  $b_1$  and set  $b_\delta = \delta \cdot b_1$ .) One could of course try to guess each  $b_\delta$  individually based on Alice’s public key, but we want to get all  $b_\delta$  right at the same time, which has exponentially low success probability. Instead, we make use of a special property of the  $M$  used in HILA5: The inverse

$$M^{-1} \bmod q = -8$$

is small.<sup>4</sup> Hence, as soon as evil Bob successfully guessed  $b_0$ , he may simply set

$$b_\delta = (\delta M^{-1} \bmod q) \cdot b_0 + b_0.$$

In our case, we choose  $b_0$  with only two non-zero coefficients from  $\{\pm 1\}$ , thus  $b_\delta$  will have only two non-zero coefficients bounded by  $1 + 8\delta$ . This property is necessary to make sure evil Bob can actually know what Alice’s version of the

<sup>4</sup> Note that this also holds for some other “natural” choices of  $M$  as rounded fractions of  $q$ , but it is not automatically true for any conceivable  $M$ .

shared secret will be (except for the target bit that leaks information): If the coefficients of  $b_\delta$  are too large, the error  $eb - e'a$  between Alice and Bob's shared secrets becomes too large to recover from and their secrets will mismatch no matter what the value of the attacked bit is. In theory, with these parameters we still expect a tiny possibility of unintended errors, but this happens so rarely that it is not an issue in practice. If it ever does occur, Bob can detect that his recovered secret key is wrong and simply start over with a new  $b_0$ .

When evil Bob chooses a random  $b_0$  with two non-zero coefficients in  $\{\pm 1\}$  and with  $(Ab_0)[0] = M$ , the probability that in fact  $(gab_0)[0] = M$  holds is just the probability that two  $\Psi_{16}$ -distributed values sum to zero:

$$\sum_{i=0}^{32} \binom{32}{i}^2 / 2^{64} \approx 9.9\%,$$

hence he can expect to find a good  $b_0$  after approximately 10 tries. Since  $A$  can be approximated by a uniformly distributed sequence over  $\mathbb{Z}_q$ , the expected number of  $\pm 1$ -combinations of two coefficients of  $A$  which equal  $M$  is

$$\binom{1024}{2} \cdot 4/q \approx 170.$$

Hence, the probability that evil Bob exhausts this pool of choices without finding a good  $b_0$  is about  $2^{-25}$ .

(If this ever happens, then evil Bob can still try a larger interval, i. e., search for  $b_0$  with  $|(Ab_0)[0] - M| \leq K$  for some small  $K$ . This would in theory work for a wider range of keys, but the expected number of wrong guesses grows slightly. We could also use three nonzero coefficients in  $b_0$ , although this increases the chance of other errors. We have not had any problems with  $K = 0$  in practice.)

*Detecting bad guesses.* After choosing  $b_0$  based on Alice's public key as described above, evil Bob may simply try to recover Alice's secret key using that  $b_0$ . If it is correct, he will of course find a sequence that looks like sampled from the  $\Psi_{16}$  distribution. If  $b_0$  is bad, say,  $(gab_0)[0] = M + \gamma$  for some small  $\gamma \neq 0$ , then

$$(gab_\delta)[0] = M + \delta + \gamma - 8\delta\gamma,$$

hence typically  $(gab_\delta)[0]$  is considerably smaller than  $M$  if  $\delta > 0$  and considerably larger if  $\delta < 0$ ; in both cases Alice's secret  $(e'a)[0]$  is dominated by  $\delta + \gamma - 8\delta\gamma$ , which means the oracle output does not depend on the secret. This implies the binary search will always converge to 0 or  $-1$  when  $b_0$  is bad. (For  $\delta = 0$ , the behavior *does* depend on  $(e'a)[0]$  since  $\gamma$  is small, so both cases really occur.) Evil Bob can detect this failure mode by determining a few coefficients and checking whether all of them are in  $\{0, -1\}$ . If this is the case, evil Bob simply starts over with a new  $b_0$ . The probability that an actual secret key starts with a sequence of  $k$  coefficients from  $\{0, -1\}$  is about  $0.27^k$ , hence setting  $k = 8$  reduces the probability of a false negative to roughly  $2^{-15}$ . There is a small probability of false positives if evil Bob uses only this heuristic (e. g., when  $|\gamma| = 1$ ), but this

can easily be detected using statistical methods (the recovered sequence will not be  $\Psi_{16}$ -distributed) or by simply testing the obtained secret key in the end and running the attack again if it failed. In practice the heuristic works fine.

*The number of queries.* Assuming we already have a good  $b_0$ , the binary search needs an expected  $5 + \varepsilon$  queries to the oracle to recover one coefficient.<sup>5</sup> Since evil Bob decides whether he has a good  $b_0$  based on the first few coefficients that he obtains using that  $b_0$ , he usually wastes a few hundred queries on guesses for  $b_0$  that turn out to be useless: If he looks at the first 8 coefficients obtained from each  $b_0$  as suggested above, this adds expected  $\approx 400$  queries to the 5,120 needed to recover all the coefficients. In summary, evil Bob will with overwhelming probability recover Alice’s secret key in less than 6,000 queries.

Evil Bob can trade computation for a smaller number of queries: retrieve some coefficients, and reduce the original lattice problem to low enough dimension to solve by computation.

### 4.3 Implementation

We implemented the attack in Python, reusing portions of a slightly patched version of the HILA5 reference implementation via the `ctypes` Python library. The complete attack script can be found at <https://helaas.org/hila5-20171218.tar.gz>. As expected, we have never observed the attack script failing to recover Alice’s key. The empirical number of queries matches the theoretical predictions in the previous section.

## 5 HILA5 security claims

In this section, we discuss our interpretation of security claims made by both the paper and NIST submission of HILA5, which motivated this paper.

NIST does not require IND-CCA2 security for KEM and PKE submissions. Instead it requires submissions to say whether they are aiming for IND-CCA2 security or merely for IND-CPA security.

IND-CPA security is adequate in the context of key exchange in TLS, if a new public key is generated for each TLS session. For example, New Hope [2] appears to be safe for use in TLS. New Hope does not aim for IND-CCA2 security, and specifically warns against using a key more than once: “No key caching . . . it is crucial that both parties use fresh secrets for each instantiation”.

We emphasize that our attack does not break the IND-CPA security of HILA5. If HILA5 were clearly labeled as aiming merely for IND-CPA security then our attack would merely be a cautionary note, showing the importance of not reusing keys.

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<sup>5</sup> The  $\varepsilon$  arises from the fact that  $\Psi_{16}$  samples from  $33 > 2^5$  distinct values, but the extremal values are assumed so rarely that  $\varepsilon \approx 2^{-27}$ .

However, HILA5 went beyond claiming IND-CPA security. There are some undefined words in the HILA5 security claims, but the most natural interpretation of the security claims is that the HILA5 PKE provides IND-CCA2 security. There is certainly a high risk of the claims being interpreted in this way by potential users. Our attack shows that the HILA5 PKE does not provide IND-CCA2 security.

There is even a risk of users thinking that the HILA5 KEM is being claimed to provide IND-CCA2 security.<sup>6</sup> The HILA5 submission document does not say that the HILA5 KEM security target is merely IND-CPA. Our attack shows that the HILA5 KEM does not provide IND-CCA2 security.

We give four quotes from [12] to explain why the HILA5 security claims are most naturally interpreted as claiming IND-CCA2 security for the HILA5 PKE. We have not found anything in [12] or [13] indicating a different interpretation.

[12, Section 1]: *The HILA5 KEM can be adopted for public key encryption in straightforward fashion. We recommend using the AES-256-GCM AEAD [FIP01, Dwo07] in conjunction with the KEM when public key encryption functionality is desired.*

The details of this “conjunction” are not formally defined. The most natural interpretation is that this is the HILA5 PKE, using the session key produced by the HILA5 KEM as the AES-GCM key.

[12, Section 4.1]: *NIST requires at least IND-CPA [BDPR98] security from a KEM scheme (Section 1.6). . . . The design also provides IND-CCA secure KEM-DEM [CS03] public key encryption if used in conjunction with an appropriate AEAD [Rog02] such as NIST approved AES256-GCM [FIP01, Dwo07]. These properties are derived from [Pei14].*

This is a claim of IND-CCA security for a PKE. “IND-CCA” in the literature usually means IND-CCA2, although sometimes it means merely IND-CCA1. The PKE is not formally defined, but again the most natural interpretation is simply that the session key produced by the HILA5 KEM is the AES-GCM key used to encrypt a user-supplied plaintext. Our attack shows that this PKE does not even provide IND-CCA1 security, let alone IND-CCA2 security.

Our attack does not work against what we call the HILA5FO PKE (see Section 2.4), a more complicated PKE using the Fujisaki–Okamoto transformation. This transformation is also mentioned in “[Pei14]” as a way to achieve IND-CCA security. It is conceivable that the HILA5 submission was alluding to a PKE of this type. However, this interpretation does not appear to be compatible with the statement “Ciphertext size: 2012 Byte expansion (KEM) + payload + MAC” in [12, Section 6]; the HILA5FO ciphertext size is 32 bytes larger than this.

<sup>6</sup> Adam Langley posted an online table of speeds for announced KEMs submitted to NIST. He wrote “I only want to list CCA-secure KEMs here”. He listed HILA5, and accepted a correction from the HILA5 author regarding the speed of HILA5. After the correction, HILA5 had the fastest decapsulation in the entire table.

[12, Section 4.9]: *For active security we suggest that  $K$  is used as keying material for an AEAD (Authenticated Encryption with Associated Data) [Rog02] scheme such as AES256-GCM [Dwo07, FIP01] or Keyak [BDP<sup>+</sup>16] in order to protect message integrity.*

Here “ $K$ ” is defined as the session key produced by the HILA5 KEM. In the context of KEMs and PKEs, “active security” is normally interpreted as IND-CCA2 security, although it might have other interpretations. The authentication in AES-GCM prevents modifications to the message encrypted by AES-GCM, but this is not enough to stop active attacks, since it does not protect the underlying KEM.

[12, Section 6.1]: *HILA5 is essentially drop-in compatible with current public key encryption applications. There are no practical usage restrictions.*

Security against chosen-ciphertext attacks is essential for a wide range of current PKE applications, so this would appear to include a claim of CCA security for the HILA5 PKE. However, our attack retrieves the secret key from the HILA5 PKE.

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