

1016-33-123

Willard Miller* (miller@ima.umn.edu), School of Mathematics, University of Minnesota, 120 Vincent Hall, 206 Church St. S.E., Minneapolis, MN 55455. *Second order superintegrable systems in two and three dimensions.*

A classical (or quantum) superintegrable system is an integrable n -dimensional Hamiltonian system with potential that admits $2n-1$ functionally independent constants of the motion, polynomial in the momenta, the maximum possible. If the constants are all quadratic the system is second order superintegrable, the most tractable case. Such systems have remarkable properties: multi-integrability and multi-separability, a quadratic algebra of symmetries whose representation theory yields spectral information about the Schrödinger operator, deep connections with expansion formulas relating classes of special functions and with QES systems. Classically the $2n-1$ constants of the motion enable one to determine trajectories using algebraic methods alone: the Kepler problem is so tractable not because it is integrable, but because it is superintegrable. For $n=2$ (and $n=3$ on conformally flat spaces with nondegenerate potentials) we have worked out the structure and classified the possible spaces and potentials. The quadratic algebra closes at order 6 and there is a 1-1 classical-quantum relationship. All such systems are Stäckel transforms of systems on complex Euclidean space or the complex 3-sphere. Joint work with E.G.Kalnins, J.R.Kress and G.S.Pogosyan. (Received February 07, 2006)