Counting Irreducible Components of Algebraic Varieties

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The problem of *counting the connected components* of complex algebraic varieties defined by integer equations is complete in FPSPACE, the class of functional problems that can be solved by a Turing machine in polynomial space.

It is natural to ask the corresponding question about the problem #IC of *counting the irreducible components* of complex algebraic varieties. Our main result states that #IC lies in FPSPACE. Moreover, we also show that #IC can be solved in parallel polynomial time assuming unit cost for the arithmetic operations.

Results of Giusti and Heintz show that one can compute the Chow forms of the equidimensional components of a variety in polynomial space, which gives a polynomial space reduction to the problem #IF of *counting the absolutely irreducible factors* of a multivariate polynomial.

We generalize a new method of Gao for absolute factorization of bivariate polynomials to several variables and use it to show that #IF can be solved in parallel polylogarithmic time, when the polynomial is given in dense representation. Moreover, when the polynomial is given as a straight-line program, we show that one can still solve #IF in polynomial time in the model of unit cost for the arithmetic operations.

We will attend all three periods of the conference.

¹Guiding senior researcher Peter Bürgisser