Invariant computation for linear Hamiltonian vector fields
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Guillem Huguet

Departamento de Matemática e Informática
Universidad Pública de Navarra
E 31006 Pamplona, Spain
guillem.huguet@unavarra.es [1]

We classify all possible Williamson normal forms related to a quadratic polynomial Hamiltonian of n degrees of freedom, with n arbitrary. Then, given a semisimple part of the quadratic Hamiltonian, we compute a fundamental set of invariants as well as a basis of its linearly independent invariants for a given degree.

We consider Hamiltonian functions of the form:

$$\mathcal{H}(\mathbf{x}) = \mathcal{H}_0(\mathbf{x}) + \mathcal{H}_1(\mathbf{x}) + \ldots, \tag{1}$$

where $\mathbf{x}$ is a $2\,n$-dimensional vector in the coordinates $x_1, \ldots, x_n$ and respective momenta $X_1, \ldots, X_n$. Each $\mathcal{H}_i$ is a homogeneous polynomial in $\mathbf{x}$ of degree $i + 2$. We present a combinatorial method to generate all possible normal forms corresponding to any $\mathcal{H}_0$ with n arbitrary. This classification is based on the type and number of indecomposable eigenspaces of the matrix A associated with the linear differential system derived from $\mathcal{H}_0$, and the number of normal forms is closely related to the number of partitions of the dimension of the system.

Once determined all possible normal forms of the Hamiltonian, we compute all polynomials invariant under the action of the uniparametric group associated with $\mathcal{H}_0$. The number of linearly independent polynomial invariants of a certain degree is given by the coefficients of the Hilbert-Poincaré series associated with the action of the group aforementioned. Then, these invarints are found after solving a system of Diophantine equations. In this case we have restricted ourselves to semisimple normal forms.

[1] Advisor: Jesús F. Palacián, Departamento de Matemática e Informática, Universidad Pública de Navarra