Complexity of Integer Quasiconvex Polynomial Optimization

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Abstract

We study a particular case of integer polynomial optimization:
Minimize a polynomial $\hat{F}$ on the set of integer points described by an inequality system $F_1 \leq 0, \ldots, F_s \leq 0$, where $\hat{F}, F_1, \ldots, F_s$ are quasiconvex polynomials in n variables with integer coefficients.
We design an algorithm solving this problem that belongs to the time-complexity class $O(s) \cdot l^{O(1)} \cdot d^{O(n)} \cdot 2^{O(n^3)}$, where $d \geq 2$ is an upper bound for the total degree of the polynomials involved and l denotes the maximum binary length of all coefficients. The algorithm is polynomial for a fixed number n of variables and represents a direct generalization of Lenstra's algorithm in integer linear optimization. In the considered case, our complexity-result improves the algorithm given by Khachiyan and Porkolab for integer optimization on convex semialgebraic sets.