# From Graphs to Manifolds - Strong Pointwise Consistency of Graph Laplacians

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In recent years methods for clustering and semi-supervised learning based on graph Laplacians have become increasingly popular in statistical learning theory. These methods are often motivated by properties derived from the Laplace-Beltrami operator. However for the setting where the graph is generated by random samples drawn from a probability measure $P$ on a submanifold $M$ in $\mathbb{R}^d$ no rigorous connection has been established yet. We show under mild assumptions that for every function on $M$ the so called 'normalized' graph Laplacian converges pointwise almost surely in the interior of $M$ towards the weighted Laplace-Beltrami operator of $M$. We only use the knowledge of the Euclidean distance in $\mathbb{R}^d$ for the edge weights of the graph; no knowledge of the intrinsic distance of the submanifold $M$ is required. Based on our convergence results we then argue against using the so called 'unnormalized' graph Laplacian since it converges only up to a function of the density of $P$ towards the weighted Laplace-Beltrami operator of $M$.

The poster will be presented during the second and third period of FOCM 2005.