

Cryptographic readiness levels, and the impact of quantum computers

Daniel J. Bernstein

- How is crypto developed?
- How confident are we that crypto is secure?
- How do we know what a quantum computer will do?

Many stages of research
from cryptographic design
to real-world deployment:

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$$(C_0 + o(1))b^2(\lg b)^2.$$

Here $C_0 \approx 0.7418860694$.

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2013 Bernstein–Jeffery–Lange–
Meurer (post-quantum).

2015 May–Ozerov.

Key size needed for 2^b security
vs. best attack known in 1978:
 $(C_0 + o(1))b^2(\lg b)^2$.

Here $C_0 \approx 0.7418860694$.

Key size needed for 2^b security
vs. best pre-quantum attack
known today:

$(C_0 + o(1))b^2(\lg b)^2$.

Key size needed for 2^b security
vs. best quantum attack known
today: $(4C_0 + o(1))b^2(\lg b)^2$.

What is a quantum

Quantum computer
stores many “qubits”
can efficiently perform
“Hadamard gate”,
“controlled NOT gate”

**Making these instructions
is the main goal of a
computer engine**

Combine these instructions
to compute “Toffoli gate”
... “Simon’s algorithm”
... “Shor’s algorithm”
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What is a quantum computer?

Quantum computer type 1 (QC1):
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What is a quantum computer?

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Quantum computer type 2 (QC2):
stores a simulated universe;
efficiently simulates the
laws of quantum physics
with as much accuracy as de
This is the original concept of
quantum computers introduced
by [1982 Feynman](#) “Simulating
physics with computers”.

What is a quantum computer?

Quantum computer type 1 (QC1):
stores many “qubits”;
can efficiently perform
“Hadamard gate”, “ T gate”,
“controlled NOT gate”.

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Quantum computer type 2 (QC2):
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What is a quantum computer?

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General belief: any QC1 is a QC2.
Partial proof: see, e.g.,
[2011 Jordan–Lee–Preskill](#)
“Quantum algorithms for
quantum field theories”.

a quantum computer?

Quantum computer type 1 (QC1):

uses any “qubits”;

efficiently perform

“CNOT gate”, “ T gate”,

and “NOT gate”.

These instructions work

as the main goal of quantum-

computer engineering.

Even these instructions

can compute “Toffoli gate”;

Shor’s algorithm”;

Simon’s algorithm”;

and Grover’s algorithm”; etc.

Quantum computer type 2 (QC2):

stores a simulated universe;

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Quantum

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Quantum computer?

Quantum computer type 1 (QC1):

“CNOT gates”;

Form

“ T gate”,

“ T^2 gate”.

**Quantum instructions work
efficiently for simulation
of quantum-
field theories.**

Quantum instructions

“CNOT gate”;

“Hadamard algorithm”;

“Quantum algorithm”;

“Quantum algorithm”; etc.

Quantum computer type 2 (QC2):
stores a simulated universe;
efficiently simulates the
laws of quantum physics
with as much accuracy as desired.

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Quantum computers
efficiently compute
that any physical
process can compute efficiently.

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Quantum computer type 3 (QC3):
efficiently computes anything
that any physical computer
can compute efficiently.

Quantum computer type 2 (QC2):
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General belief: any QC1 is a QC2.

Partial proof: see, e.g.,
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Quantum computer type 3 (QC3):
efficiently computes anything
that any physical computer
can compute efficiently.

General belief: any QC2 is a QC3.

Argument for belief:

any physical computer must
follow the laws of quantum
physics, so a QC2 can efficiently
simulate any physical computer.

Quantum computer type 2 (QC2):
stores a simulated universe;
efficiently simulates the
laws of quantum physics
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Quantum computer type 3 (QC3):
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General belief: any QC2 is a QC3.

Argument for belief:
any physical computer must
follow the laws of quantum
physics, so a QC2 can efficiently
simulate any physical computer.

General belief: any QC3 is a QC1.

Argument for belief:
look, we’re building a QC1.