Introduction to
quantum algorithms
and introduction to
code-based cryptography

Daniel J. Bernstein
University of Illinois at Chicago &
Technische Universiteit Eindhoven

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an element of $\{0, 1\}^n$,
often viewed as representing
an element of $\{0, 1, \ldots, 2^n - 1\}$.

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If n qubits have state $(a_0, a_1, \ldots, a_{2^n-1})$ then **measuring** the qubits produces an element of $\{0,1,\ldots,2^n-1\}$ and destroys the state. Measurement produces element q with probability $|a_q|^2/\sum_r |a_r|^2$.

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is "swapping qubits 0 and 2":
$(q_0, q_1, q_2) \mapsto (q_2, q_1, q_0)$.

Complementing qubit 2
= swapping qubits 0 and 2
$\circ$ complementing qubit 0
$\circ$ swapping qubits 0 and 2.

Similarly: swapping qubits i, j.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7)$
$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$
is a "reversible XOR gate" =
"controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1, q_1,$

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_4, a_5, a_6, a_7, a_0, a_1, a_2, a_3)$
is "complementing qubit 2":
$(q_0, q_1, q_2) \mapsto (q_0, q_1, q_2 \oplus 1)$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_4, a_2, a_6, a_1, a_5, a_3, a_7)$
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Example with more qubits:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$a_6, a_7, a_0, a_1, a_2, a_3)$

plementing qubit 2":

$q_2) \mapsto (q_0, q_1, q_2 \oplus 1)$.

$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$a_2, a_6, a_1, a_5, a_3, a_7)$

pping qubits 0 and 2":

$q_2) \mapsto (q_2, q_1, q_0)$.

menting qubit 2

ping qubits 0 and 2

nplementing qubit 0

apping qubits 0 and 2.

y: swapping qubits i, j.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$

is a "reversible XOR gate" =

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$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1, q_1, q_2)$.

Example with more qubits:

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$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$

$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$

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$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$

$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$

$a_{24}, a_{25}, a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1,$

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is a "To

"control

$(q_0, q_1,$

$a_5, a_6, a_7) \mapsto$

$a_1, a_2, a_3)$

g qubit 2":

$, q_1, q_2 \oplus 1)$.

$a_5, a_6, a_7) \mapsto$

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ts 0 and 2":

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ubit 2

s 0 and 2

ng qubit 0

its 0 and 2.

g qubits i, j.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
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$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a$

$(a_0, a_1, a_2, a_3, a_4, a$

is a "Toffoli gate"

"controlled contro

$(q_0, q_1, q_2) \mapsto (q_0$

$\mapsto$

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1).

$\mapsto$

”:

2.

$, j.$

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$
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$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
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$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
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$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7)$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a “Toffoli gate” =
“controlled controlled NOT
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q$

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6)$
is a "reversible XOR gate" $=$
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$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1, q_1, q_2)$.

Example with more qubits:
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$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
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$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_3, a_2, a_4, a_5, a_7, a_6,$
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$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
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$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$a_3, a_2, a_4, a_5, a_7, a_6)$
versible XOR gate" $=$
lled NOT gate":
$q_2) \mapsto (q_0 \oplus q_1, q_1, q_2)$.

e with more qubits:
$a_2, a_3, a_4, a_5, a_6, a_7,$
$_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$_1, a_3, a_2, a_4, a_5, a_7, a_6,$
$_{11}, a_{10}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{19}, a_{18}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{27}, a_{26}, a_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
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$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
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$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversib

Say p is
of $\{0, 1,$

General
these fas
to obtai
$(a_0, a_1, .$
$(a_{p^{-1}(0)}$

$a_5, a_6, a_7) \mapsto$

$a_5, a_7, a_6)$

OR gate" =

gate":

$\oplus q_1, q_1, q_2)$.

e qubits:

$a_5, a_6, a_7,$

$, a_{13}, a_{14}, a_{15},$

$_{20}, a_{21}, a_{22}, a_{23},$

$_{28}, a_{29}, a_{30}, a_{31})$

$_4, a_5, a_7, a_6,$

$, a_{13}, a_{15}, a_{14},$

$_{20}, a_{21}, a_{23}, a_{22},$

$_{28}, a_{29}, a_{31}, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" =
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible comput

Say p is a permuta
of $\{0, 1, \ldots, 2^n -$

General strategy to
these fast quantum
to obtain index pe
$(a_0, a_1, \ldots, a_{2^n-1})$
$(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots$

$\mapsto$

$=$

$q_2)$.

a_{15},

$2, a_{23}$,

$0, a_{31})$

6,

a_{14},

$3, a_{22}$,

$1, a_{30})$.

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" $=$
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
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$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
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$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation
of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose
these fast quantum operatio
to obtain index permutation
$(a_0, a_1, \ldots, a_{2^n-1}) \mapsto$
$(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n$

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" =
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation
of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose
these fast quantum operations
to obtain index permutation
$(a_0, a_1, \ldots, a_{2^n-1}) \mapsto$
$(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n-1)})$:

$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$
$(a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6)$
is a "Toffoli gate" =
"controlled controlled NOT gate":
$(q_0, q_1, q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

Example with more qubits:
$(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$
$\mapsto (a_0, a_1, a_2, a_3, a_4, a_5, a_7, a_6,$
$a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$
$a_{16}, a_{17}, a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$
$a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation
of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose
these fast quantum operations
to obtain index permutation
$(a_0, a_1, \ldots, a_{2^n-1}) \mapsto$
$(a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n-1)})$:

1. Build a traditional circuit
to compute $j \mapsto p(j)$
using NOT/XOR/AND gates.

2. Convert into reversible gates:
e.g., convert AND into Toffoli.

$a_2, a_3, a_4, a_5, a_6, a_7) \mapsto$

$a_2, a_3, a_4, a_5, a_7, a_6)$

ffoli gate" =

led controlled NOT gate":

$q_2) \mapsto (q_0 \oplus q_1 q_2, q_1, q_2)$.

e with more qubits:

$a_2, a_3, a_4, a_5, a_6, a_7,$

$_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15},$

$a_{18}, a_{19}, a_{20}, a_{21}, a_{22}, a_{23},$

$a_{26}, a_{27}, a_{28}, a_{29}, a_{30}, a_{31})$

$_1, a_2, a_3, a_4, a_5, a_7, a_6,$

$_{10}, a_{11}, a_{12}, a_{13}, a_{15}, a_{14},$

$a_{18}, a_{19}, a_{20}, a_{21}, a_{23}, a_{22},$

$a_{26}, a_{27}, a_{28}, a_{29}, a_{31}, a_{30})$.

Reversible computation

Say p is a permutation of $\{0, 1, \ldots, 2^n - 1\}$.

General strategy to compose these fast quantum operations to obtain index permutation $(a_0, a_1, \ldots, a_{2^n-1}) \mapsto (a_{p^{-1}(0)}, a_{p^{-1}(1)}, \ldots, a_{p^{-1}(2^n-1)})$:

1. Build a traditional circuit to compute $j \mapsto p(j)$ using NOT/XOR/AND gates.

2. Convert into reversible gates: e.g., convert AND into Toffoli.

Example

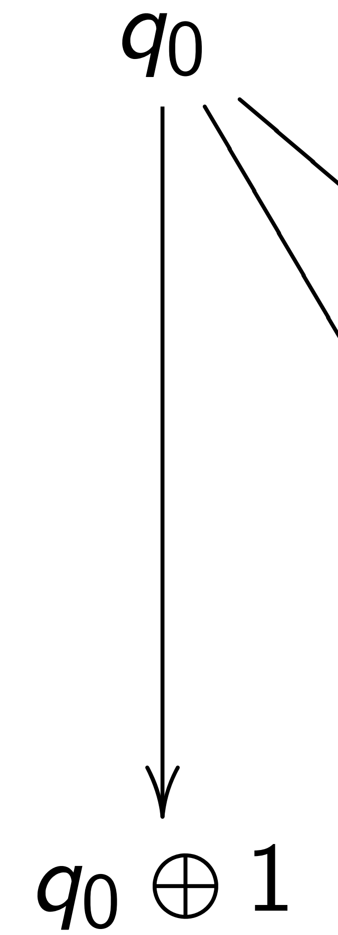
$(a_0, a_1,$

$(a_7, a_0,$

permuta

1. Build

to comp



$a_5, a_6, a_7) \mapsto$

$a_5, a_7, a_6)$

$=$

lled NOT gate":

$\oplus\, q_1 q_2, q_1, q_2)$.

e qubits:

$a_5, a_6, a_7,$

$, a_{13}, a_{14}, a_{15},$

$_{20}, a_{21}, a_{22}, a_{23},$

$_{28}, a_{29}, a_{30}, a_{31})$

$_4, a_5, a_7, a_6,$

$, a_{13}, a_{15}, a_{14},$

$_{20}, a_{21}, a_{23}, a_{22},$

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Example: Let's co

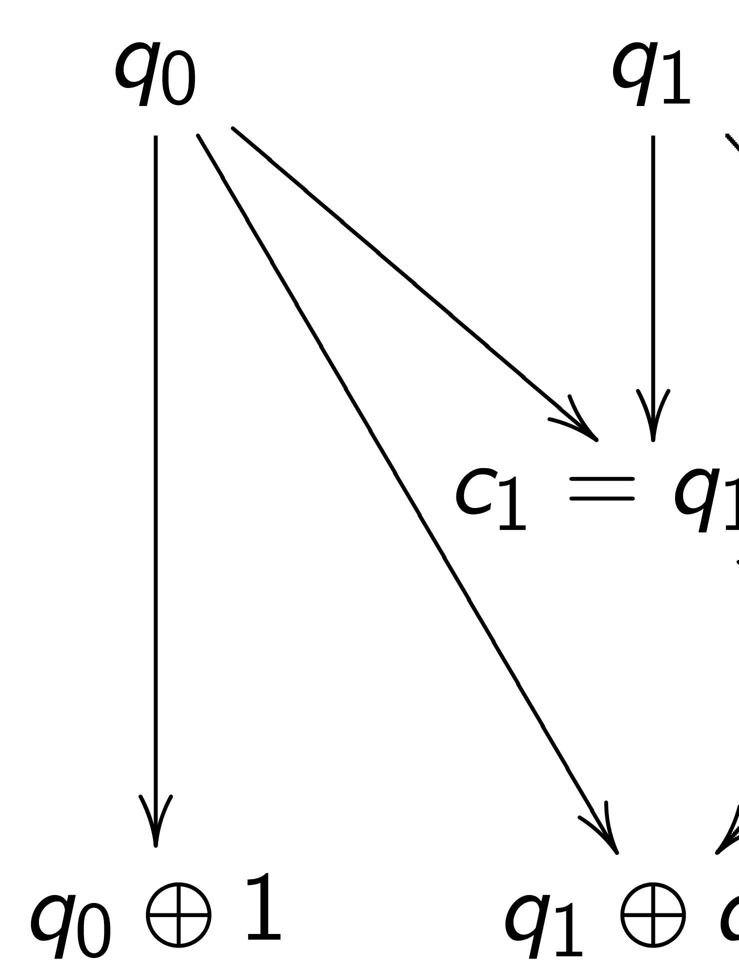
$(a_0, a_1, a_2, a_3, a_4, a$

$(a_7, a_0, a_1, a_2, a_3, a$

permutation $q \mapsto$

1. Build a traditio

to compute $q \mapsto q$



$\mapsto$

gate":

$_1, q_2)$.

a_{15},

$_2, a_{23}$,

$_0, a_{31})$

$_6$,

a_{14},

$_3, a_{22}$,

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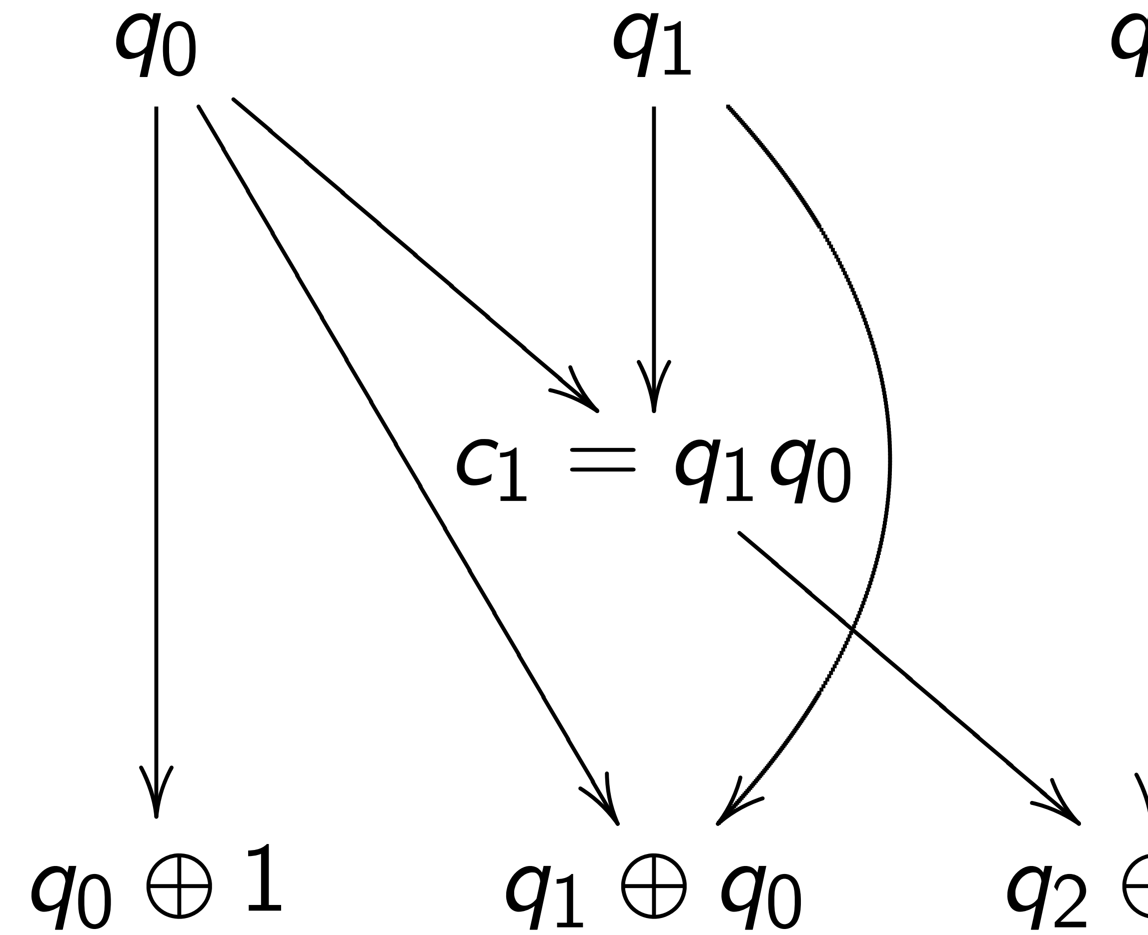
$(a_7, a_0, a_1, a_2, a_3, a_4, a_5, a_6)$;
permutation $q \mapsto q + 1$ mod

1. Build a traditional circuit to compute $q \mapsto q + 1$ mod



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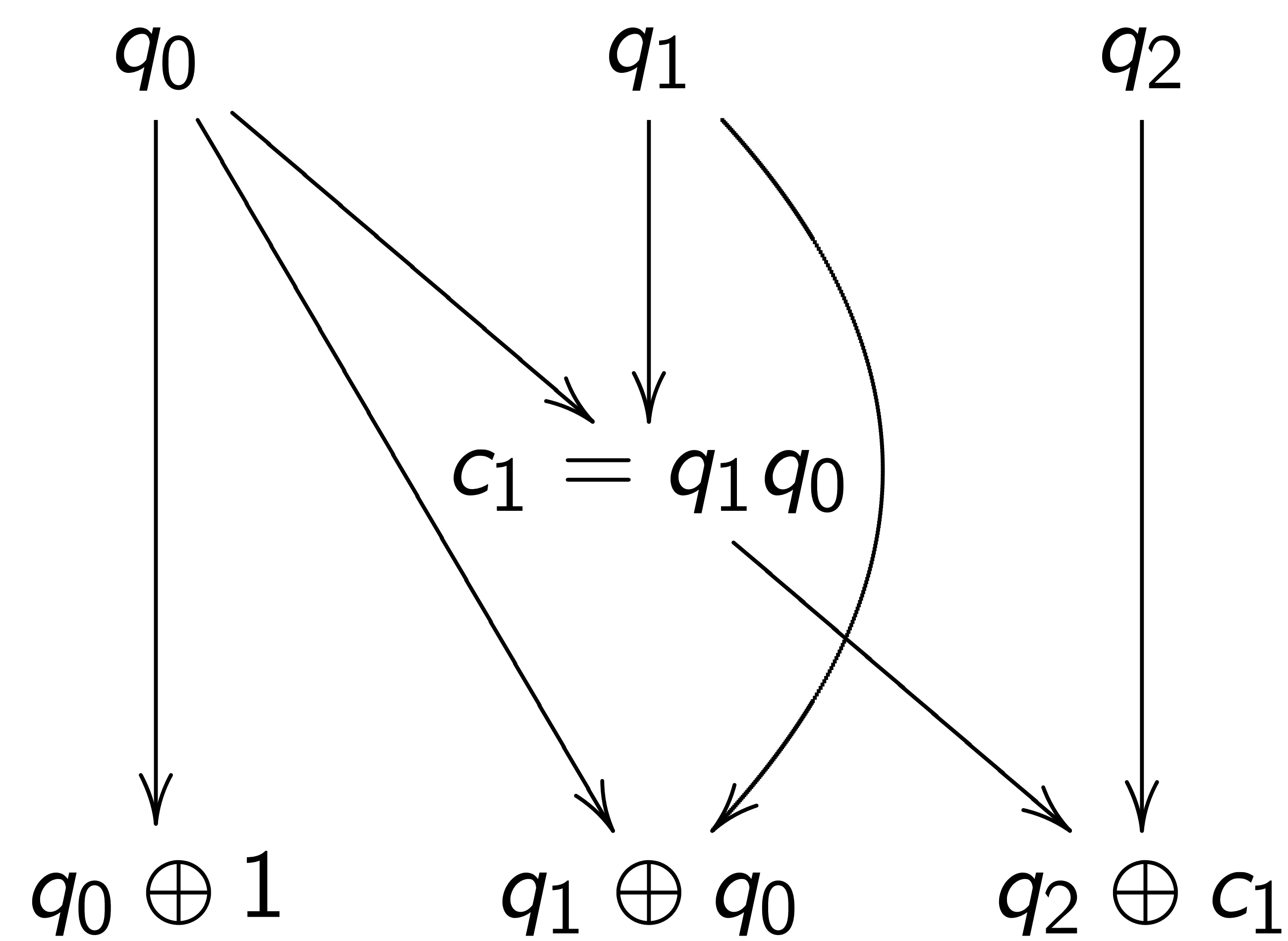
1. Build a traditional circuit to compute $q \mapsto q + 1 \bmod 8$.



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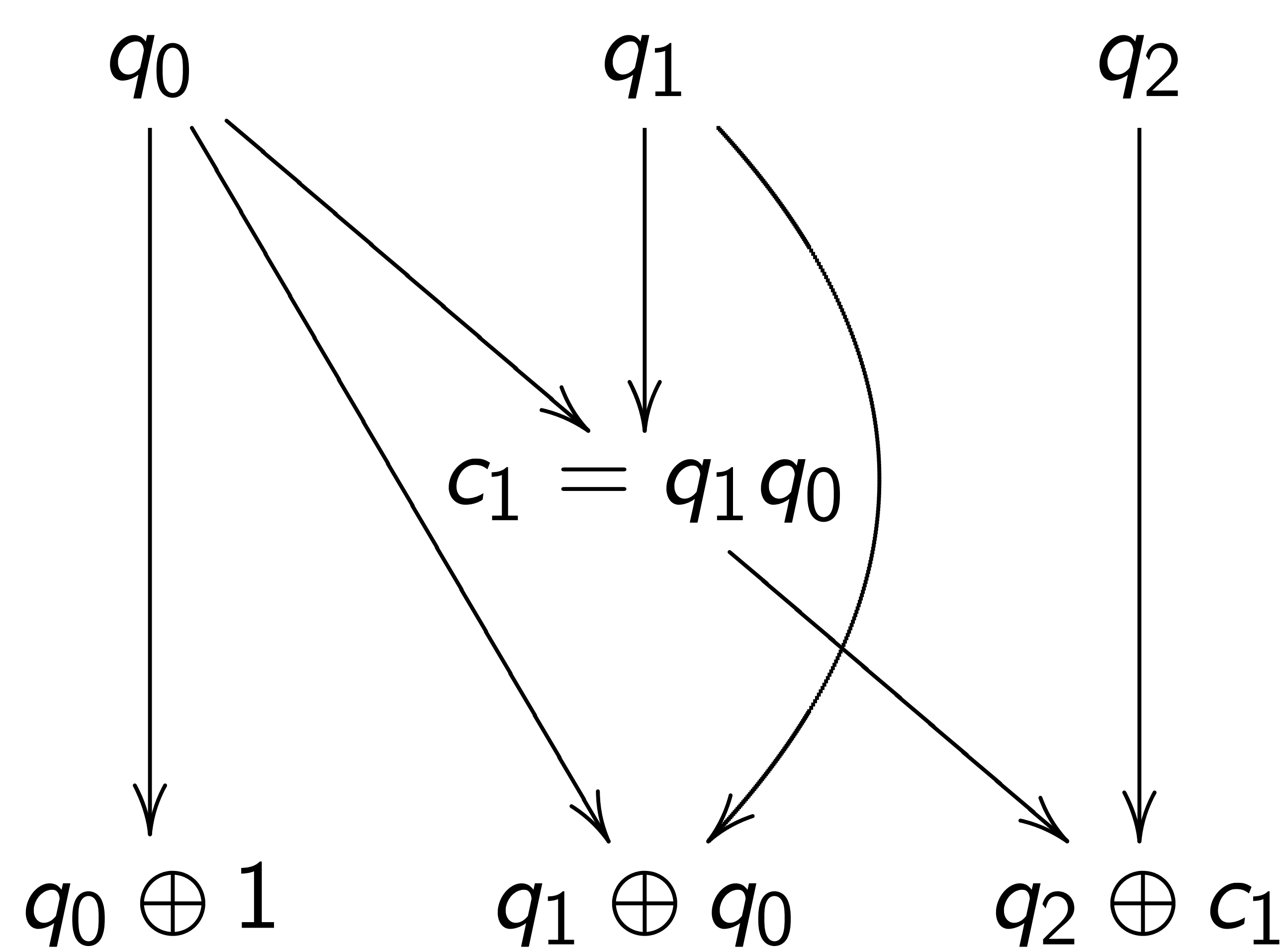

2. Conv

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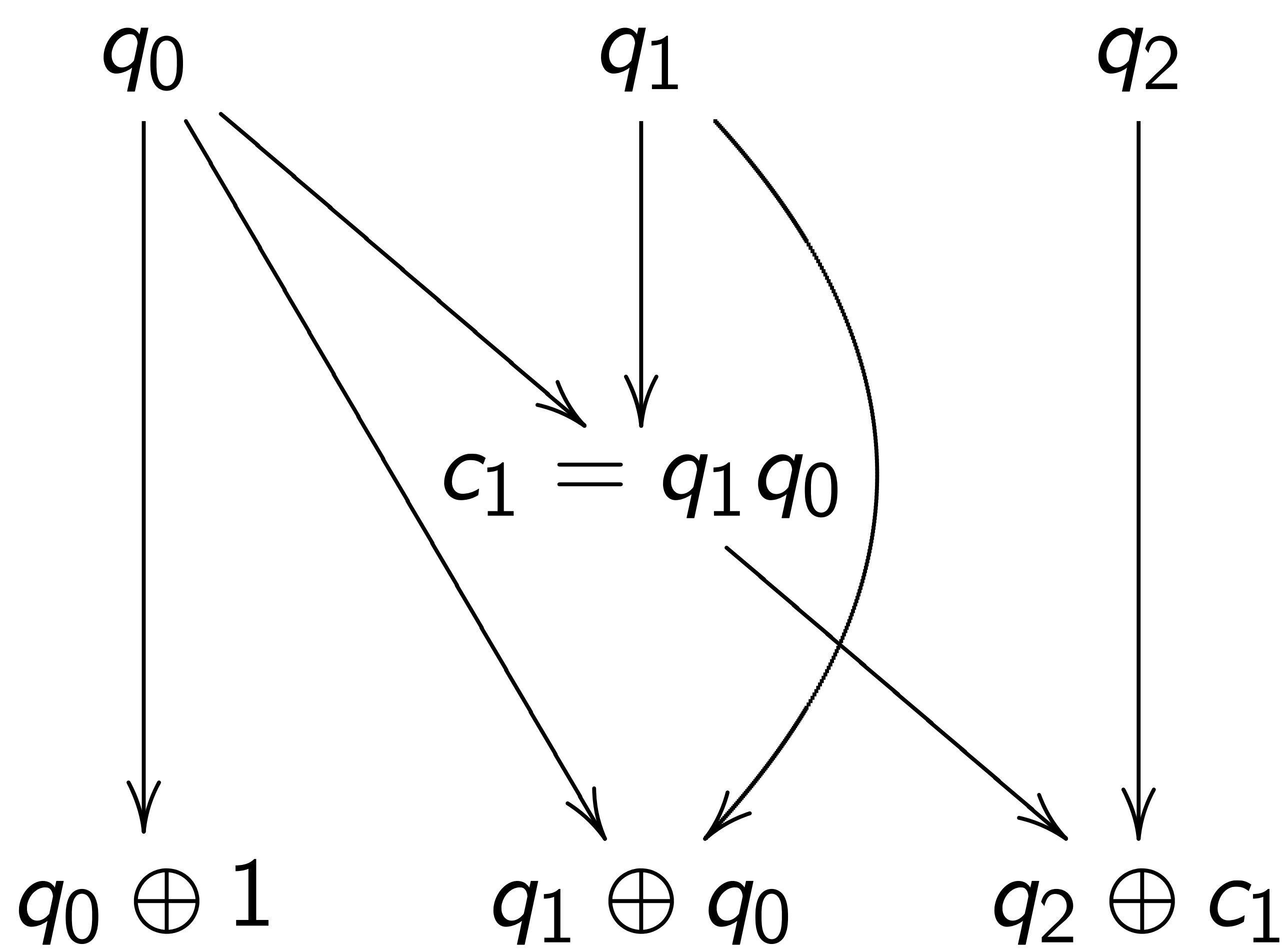

2. Convert into re

Toffoli for $q_2 \leftarrow q$
$(a_0, a_1, a_2, a_3, a_4, a$
$(a_0, a_1, a_2, a_7, a_4, a$

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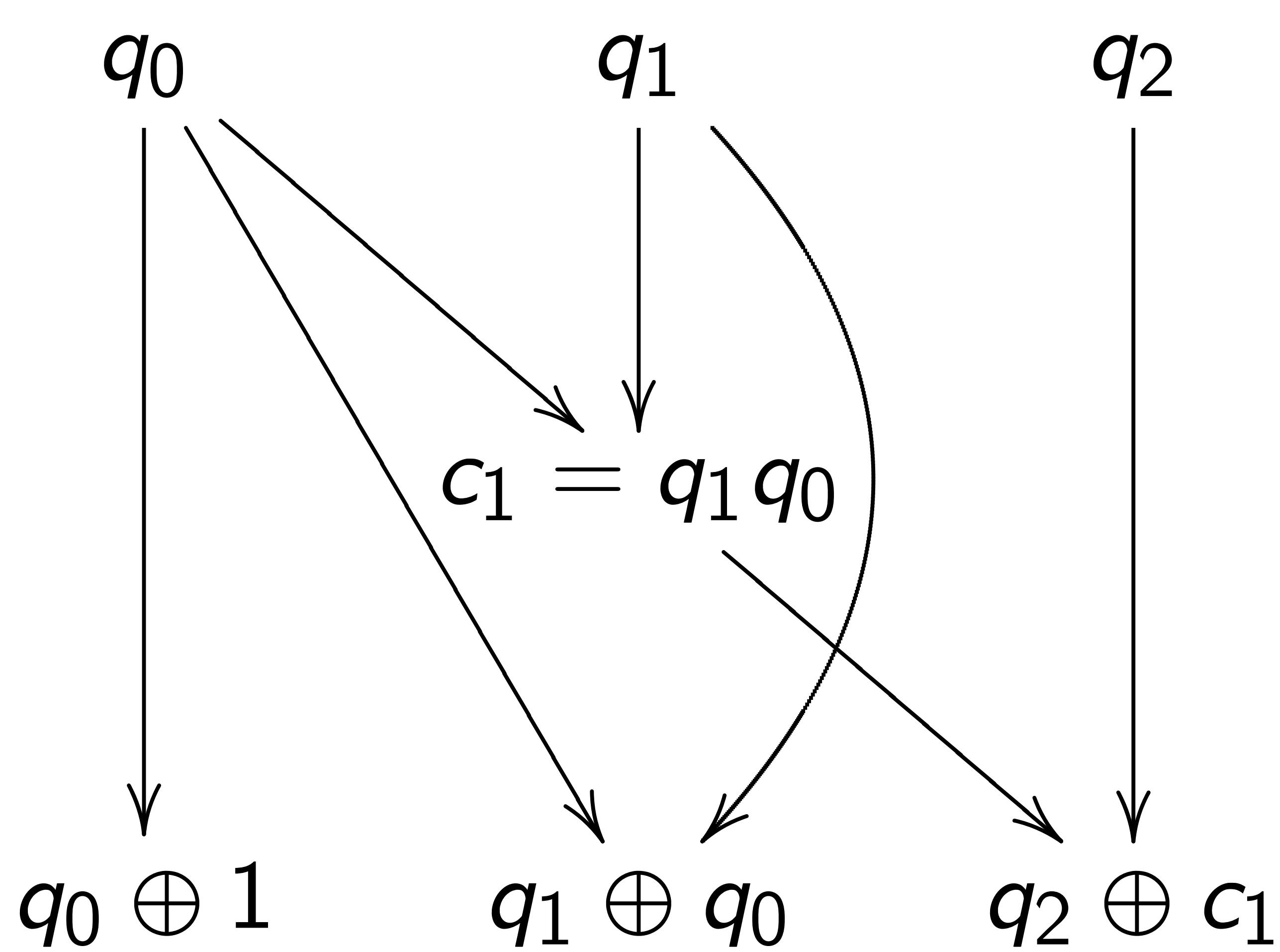

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Toffoli for $q_2 \leftarrow q_2 \oplus q_1 q_0$:
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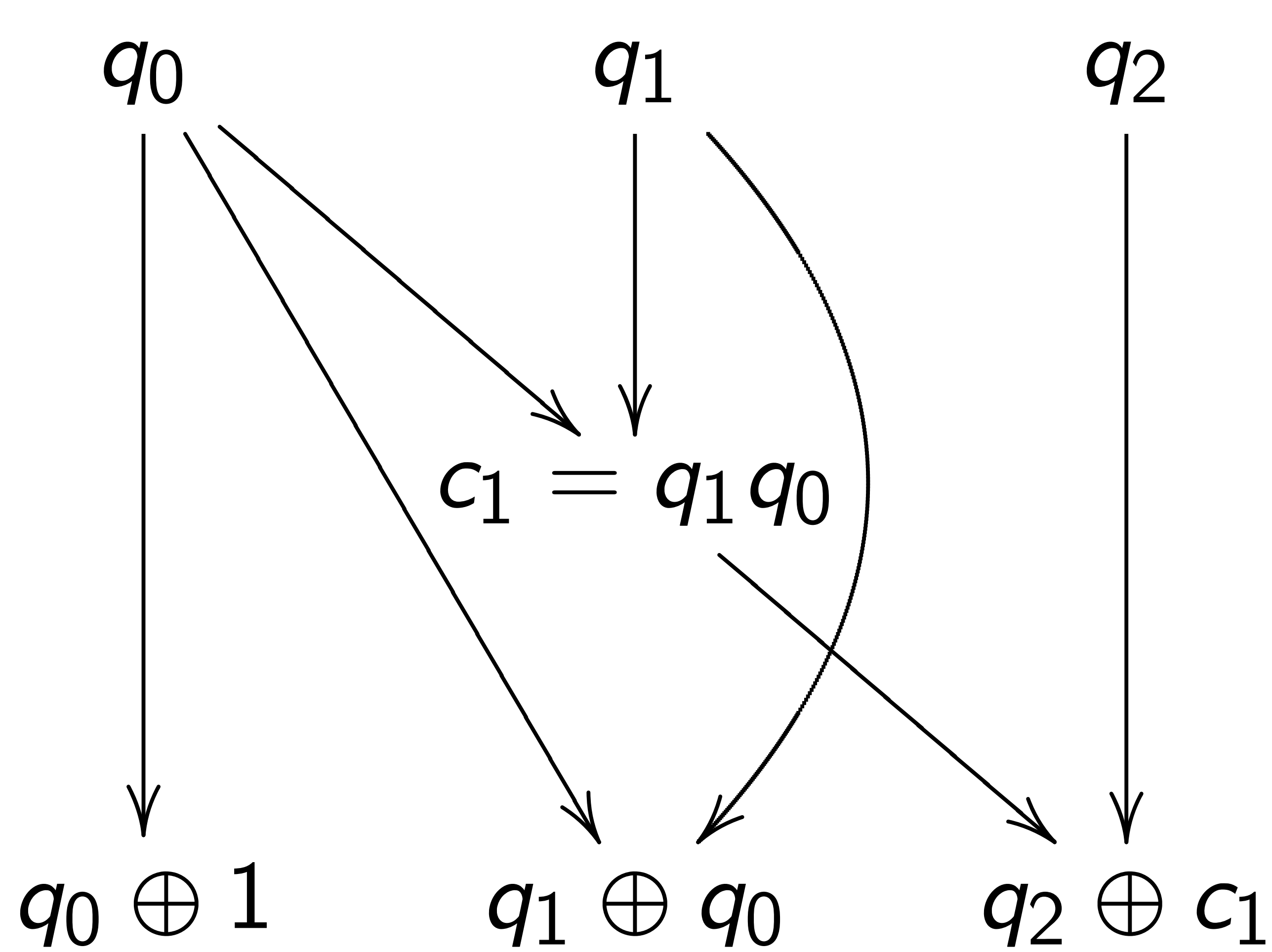

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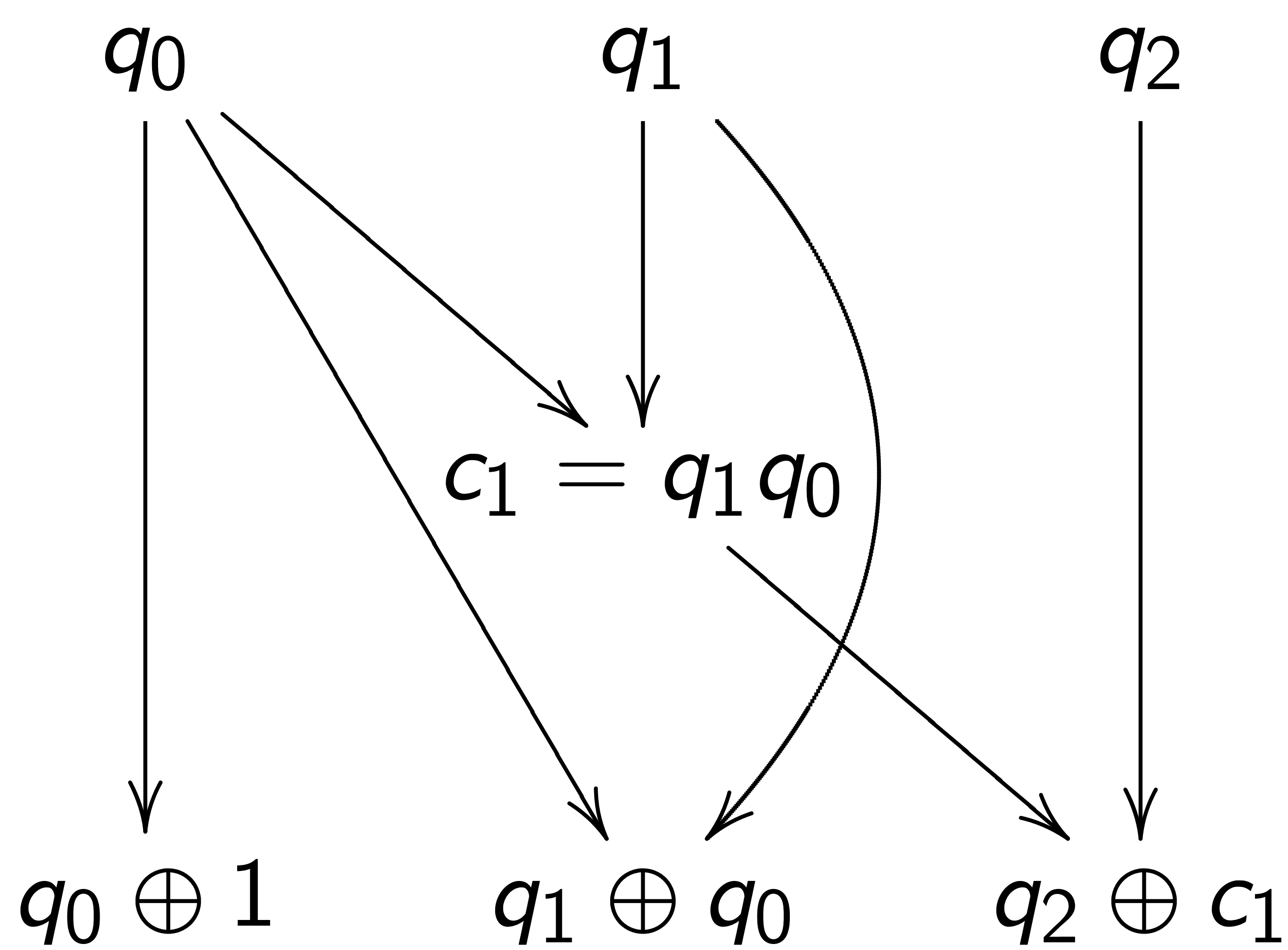

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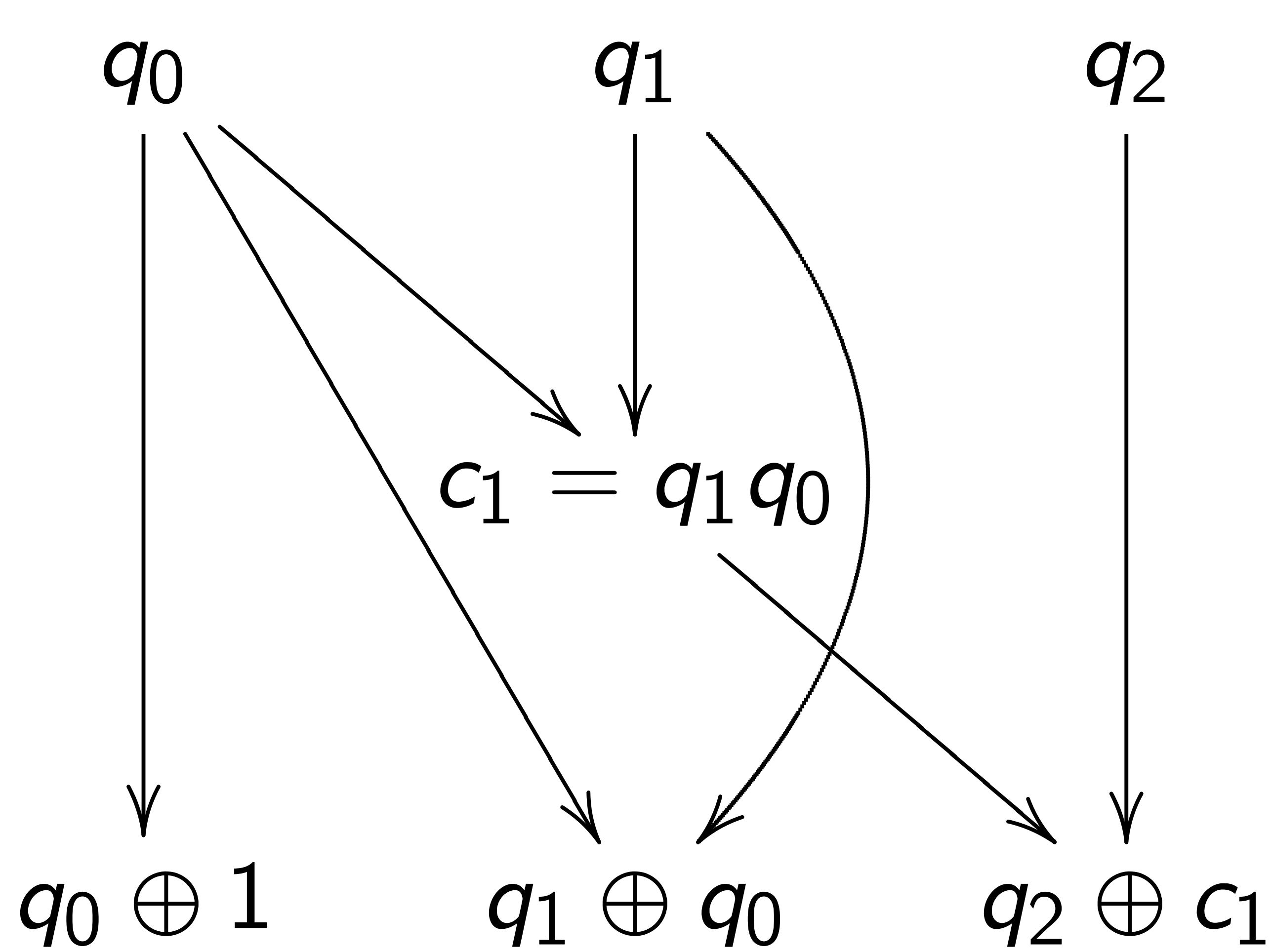

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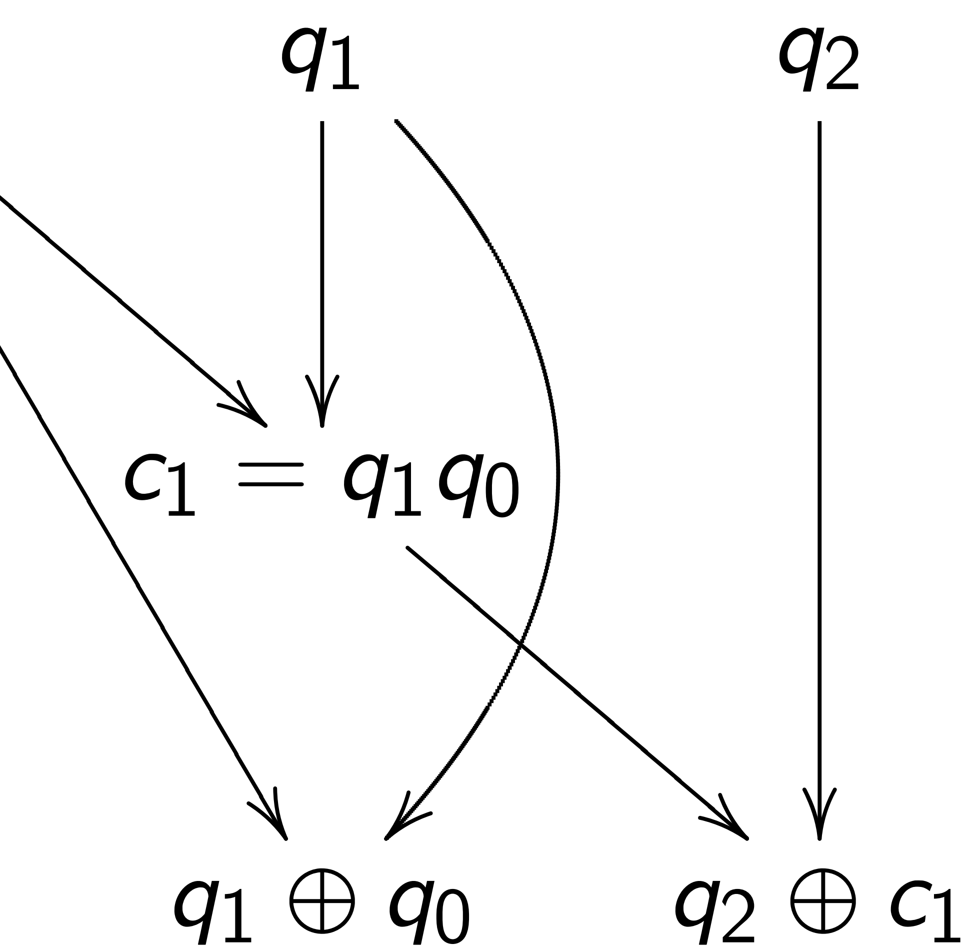

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Typical circuits aren't in-place.

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Start fro

inputs b

$b_{i+1} = 1$

$b_{i+2} = 1$

...

$b_T = 1 \oplus$

specified

versible gates.

$_2 \oplus q_1 q_0$:

$a_5, a_6, a_7) \mapsto$

$a_5, a_6, a_3)$.

or $q_1 \leftarrow q_1 \oplus q_0$:

$a_5, a_6, a_3) \mapsto$

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Start from any cir
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$b_{i+1} = 1 \oplus b_{f(i+1)}$
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...
$b_T = 1 \oplus b_{f(T)} b_{g($
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$b_T = 1 \oplus b_{f(T)} b_{g(T)}$;
specified outputs.

Reversible but dirty:
inputs $b_1, b_2, \ldots, b_T$;
$b_{i+1} \leftarrow 1 \oplus b_{i+1} \oplus b_{f(i+1)} b_{g(i+1)}$;
$b_{i+2} \leftarrow 1 \oplus b_{i+2} \oplus b_{f(i+2)} b_{g(i+2)}$;
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$b_T \leftarrow 1 \oplus b_T \oplus b_{f(T)} b_{g(T)}$.
Same outputs if all of $b_{i+1}, \ldots, b_T$ started as 0.

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Start from any circuit:

inputs $b_1, b_2, \ldots, b_i$;

$b_{i+1} = 1 \oplus b_{f(i+1)} b_{g(i+1)}$;

$b_{i+2} = 1 \oplus b_{f(i+2)} b_{g(i+2)}$;

...

$b_T = 1 \oplus b_{f(T)} b_{g(T)}$;

specified outputs.

Reversible but dirty:

inputs $b_1, b_2, \ldots, b_T$;

$b_{i+1} \leftarrow 1 \oplus b_{i+1} \oplus b_{f(i+1)} b_{g(i+1)}$;

$b_{i+2} \leftarrow 1 \oplus b_{i+2} \oplus b_{f(i+2)} b_{g(i+2)}$;

...

$b_T \leftarrow 1 \oplus b_T \oplus b_{f(T)} b_{g(T)}$.

Same outputs if all of

$b_{i+1}, \ldots, b_T$ started as 0.

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Replace reversible bit operations
with Toffoli gates etc.
permuting $\mathbf{C}^{2^{n+z}} \to \mathbf{C}^{2^{n+z}}$.

Permutation on first 2^n entries is
$(a_0, a_1, \ldots, a_{2^n-1}) \mapsto$
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Typically prepare vectors
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Measuring $(1, 0, 0,$
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Aside from "normalization"
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and then qubit 1:

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$- a_2 - a_3, a_0 - a_1 - a_2 + a_3)$.

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Traditional solution:
Compute f for many inputs,
sort, analyze collisions.
Success probability is very low
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Say f maps n bits to m bits using z "ancilla" bits for reversibility.

Prepare $n + m + z$ qubits in pure zero state:
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Repeat $n + 10$ times. Use Gaussian elimination to (probably) find s.

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Example

$f(0) = 4$
$f(1) = 7$
$f(2) = 2$
$f(3) = 3$
$f(4) = 7$
$f(5) = 4$
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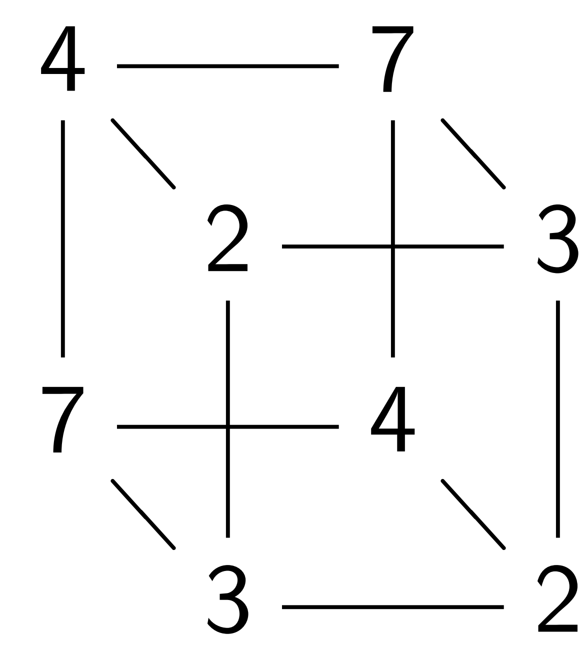

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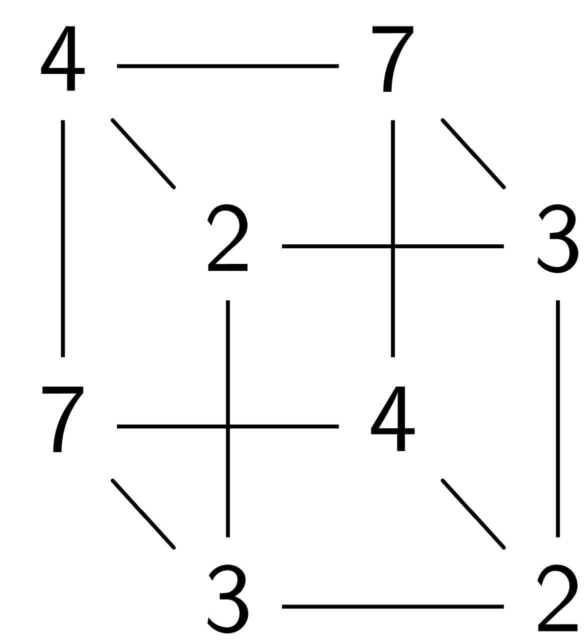

Complete table shows that $f(x) = f(x \oplus 5)$ for all x.

Let's watch Simon's algorithm for f, using 6 qubits.

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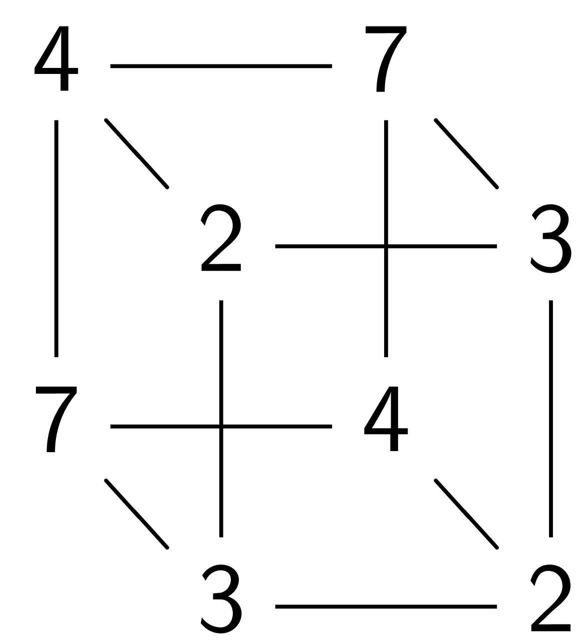

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Step 1.

$1, 0, 0, 0$
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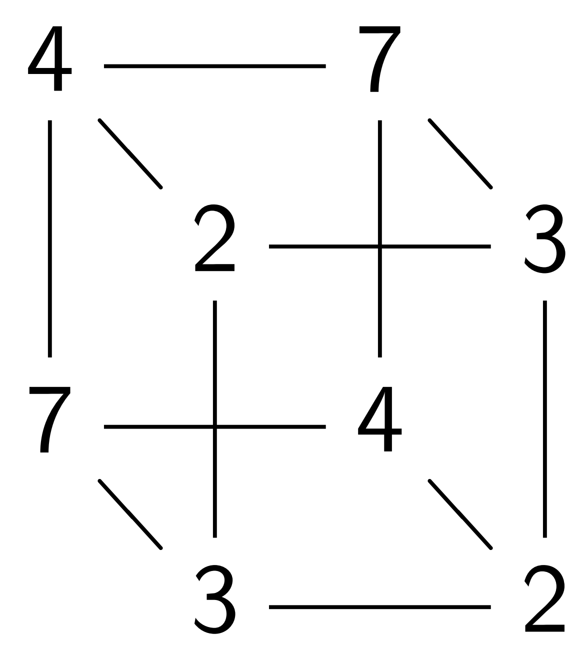

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Step 1. Set up pu

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
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0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
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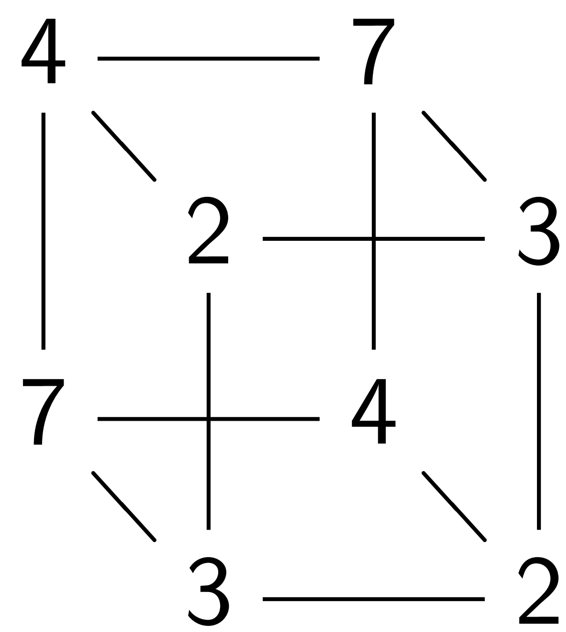

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1, 0, 0, 0, 0, 0, 0, 0,
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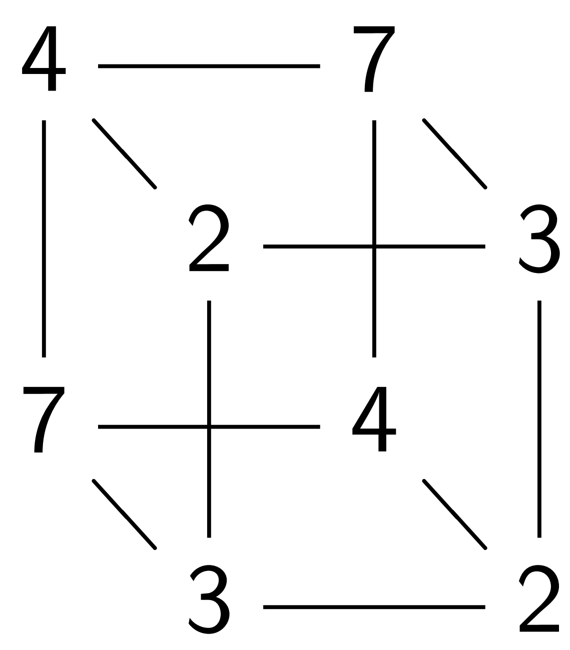

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$f(x) = f(x \oplus 5)$ for all x.

Let's watch Simon's algorithm

for f, using 6 qubits.

Step 1. Set up pure zero state:

1, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0.

Example, 3 bits to 3 bits:

$f(0) = 4.$

$f(1) = 7.$

$f(2) = 2.$

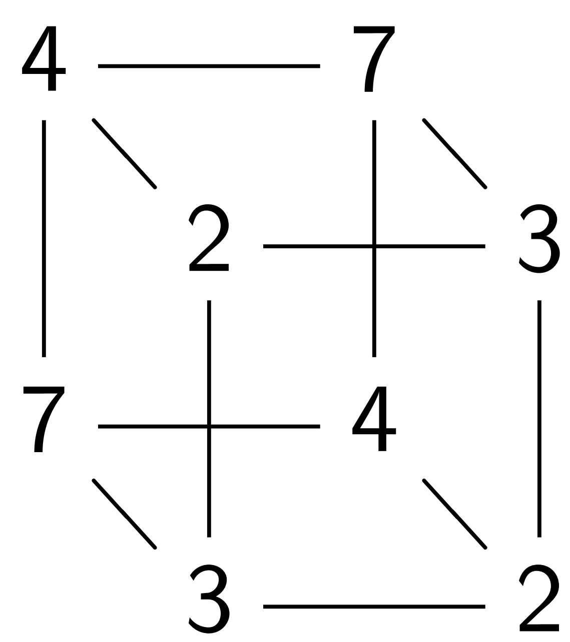
$f(3) = 3.$

$f(4) = 7.$

$f(5) = 4.$

$f(6) = 3.$

$f(7) = 2.$



Complete table shows that

$f(x) = f(x \oplus 5)$ for all x.

Let's watch Simon's algorithm

for f, using 6 qubits.

Step 2. Hadamard on qubit 0:

$\color{blue}{1}, \color{blue}{1}, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0.$

Example, 3 bits to 3 bits:

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$f(2) = 2.$
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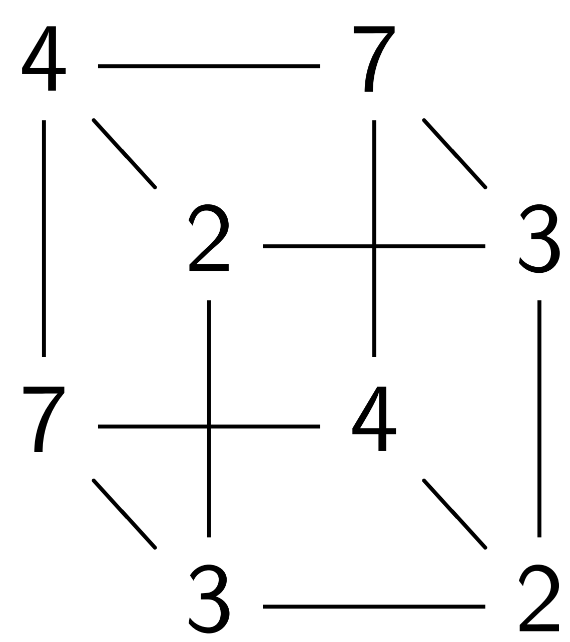
$f(4) = 7.$
$f(5) = 4.$
$f(6) = 3.$
$f(7) = 2.$



Complete table shows that
$f(x) = f(x \oplus 5)$ for all x.

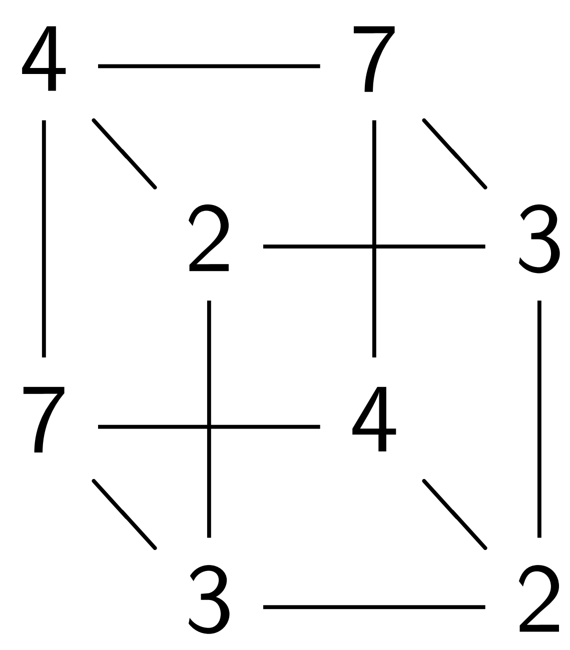
Let's watch Simon's algorithm
for f, using 6 qubits.

Step 3. Hadamard on qubit 1:

1, 1, 1, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Example, 3 bits to 3 bits:

$f(0) = 4.$
$f(1) = 7.$
$f(2) = 2.$
$f(3) = 3.$
$f(4) = 7.$
$f(5) = 4.$
$f(6) = 3.$
$f(7) = 2.$



Complete table shows that
$f(x) = f(x \oplus 5)$ for all x.

Let's watch Simon's algorithm
for f, using 6 qubits.

Step 4. Hadamard on qubit 2:

1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

Example, 3 bits to 3 bits:

$f(0) = 4.$

$f(1) = 7.$

$f(2) = 2.$

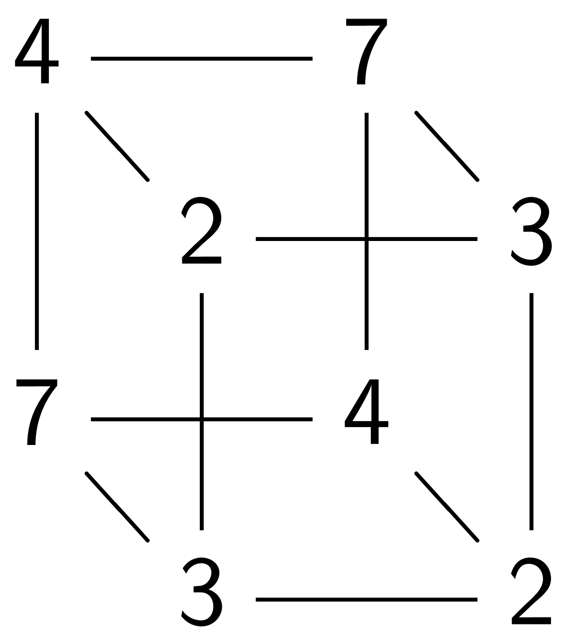
$f(3) = 3.$

$f(4) = 7.$

$f(5) = 4.$

$f(6) = 3.$

$f(7) = 2.$



Complete table shows that

$f(x) = f(x \oplus 5)$ for all x.

Let's watch Simon's algorithm

for f, using 6 qubits.

Step 5. $(q, 0) \mapsto (q, f(q))$:

0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 1, 0, 0, 0, 0, 1,

0, 0, 0, 1, 0, 0, 1, 0,

1, 0, 0, 0, 0, 1, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0,

0, 1, 0, 0, 1, 0, 0, 0.

Example, 3 bits to 3 bits:

$f(0) = 4.$

$f(1) = 7.$

$f(2) = 2.$

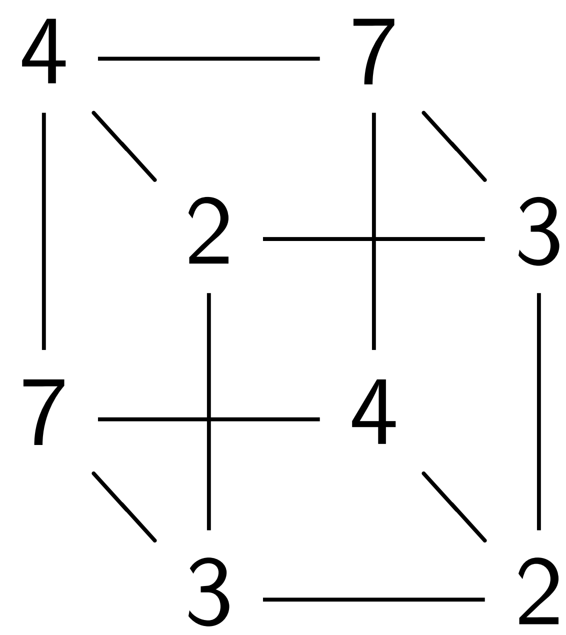
$f(3) = 3.$

$f(4) = 7.$

$f(5) = 4.$

$f(6) = 3.$

$f(7) = 2.$



Complete table shows that

$f(x) = f(x \oplus 5)$ for all x.

Let's watch Simon's algorithm

for f, using 6 qubits.

Step 6. Hadamard on qubit 0:

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 1, 1, 0, 0, 1, \overline{1},$

$0, 0, 1, \overline{1}, 0, 0, 1, 1,$

$1, 1, 0, 0, 1, \overline{1}, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, \overline{1}, 0, 0, 1, 1, 0, 0.$

Notation: $\overline{1} = -1.$

Example, 3 bits to 3 bits:

$f(0) = 4.$

$f(1) = 7.$

$f(2) = 2.$

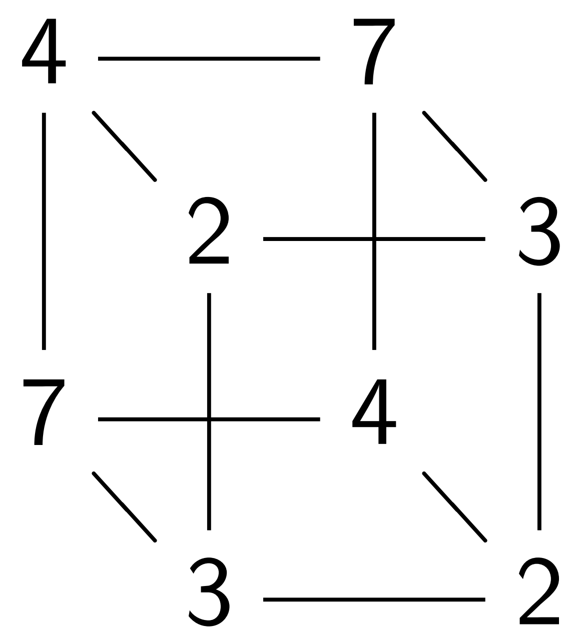
$f(3) = 3.$

$f(4) = 7.$

$f(5) = 4.$

$f(6) = 3.$

$f(7) = 2.$



Complete table shows that $f(x) = f(x \oplus 5)$ for all x.

Let's watch Simon's algorithm for f, using 6 qubits.

Step 7. Hadamard on qubit 1:

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, 1, \overline{1}, \overline{1}, 1, \overline{1}, \overline{1}, 1,$

$1, \overline{1}, \overline{1}, 1, 1, 1, \overline{1}, \overline{1},$

$1, 1, 1, 1, 1, \overline{1}, 1, \overline{1},$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$1, \overline{1}, 1, \overline{1}, 1, 1, 1, 1.$

Example, 3 bits to 3 bits:

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$f(1) = 7.$

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$f(4) = 7.$

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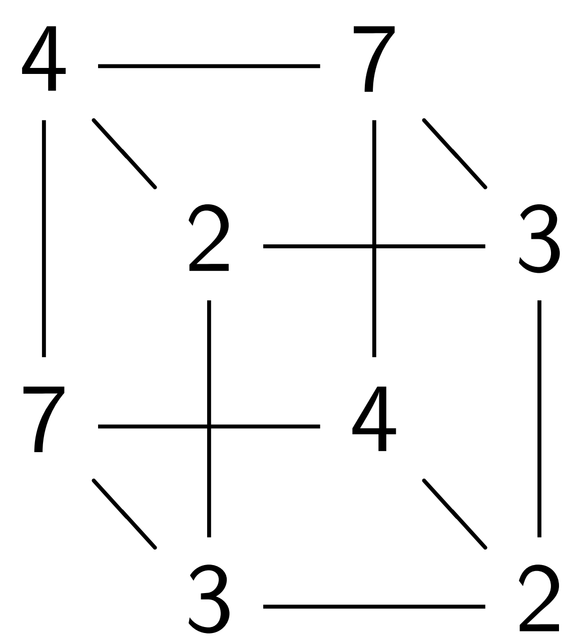
$f(7) = 2.$



Complete table shows that

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Let's watch Simon's algorithm

for f, using 6 qubits.

Step 8. Hadamard on qubit 2:

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$

$2, 0, 2, 0, 0, 2, 0, 2,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2}.$

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$f(1) = 7.$

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$f(4) = 7.$

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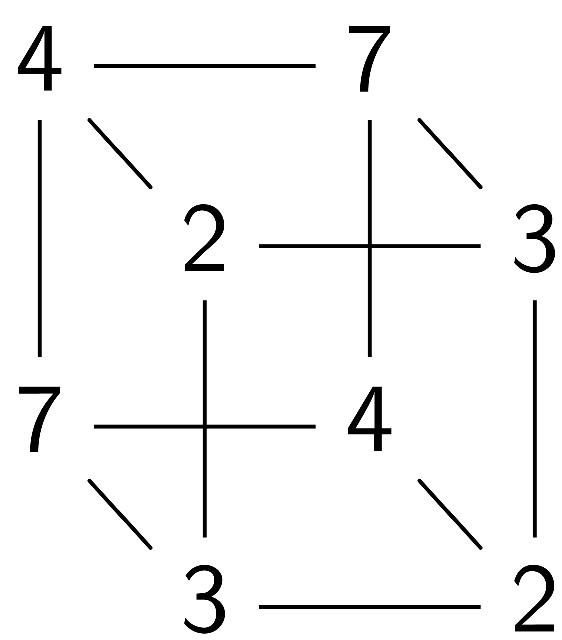

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$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$

$2, 0, 2, 0, 0, 2, 0, 2,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2}.$

Step 9. Measure.

First 3 qubits are uniform random

vector orthogonal to 101: i.e.,

000, 010, 101, or 111.

e, 3 bits to 3 bits:

4.

7.

2.

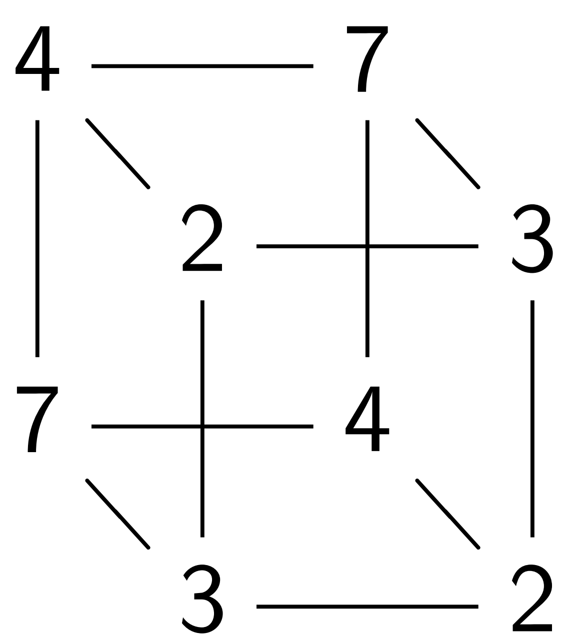
3.

7.

4.

3.

2.



te table shows that

$f(x \oplus 5)$ for all x.

tch Simon's algorithm

sing 6 qubits.

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$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$

$2, 0, 2, 0, 0, 2, 0, 2,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2}.$

Step 9. Measure.
First 3 qubits are uniform random
vector orthogonal to 101: i.e.,
000, 010, 101, or 111.

Grover's

Assume:

has $f(s)$

Traditio

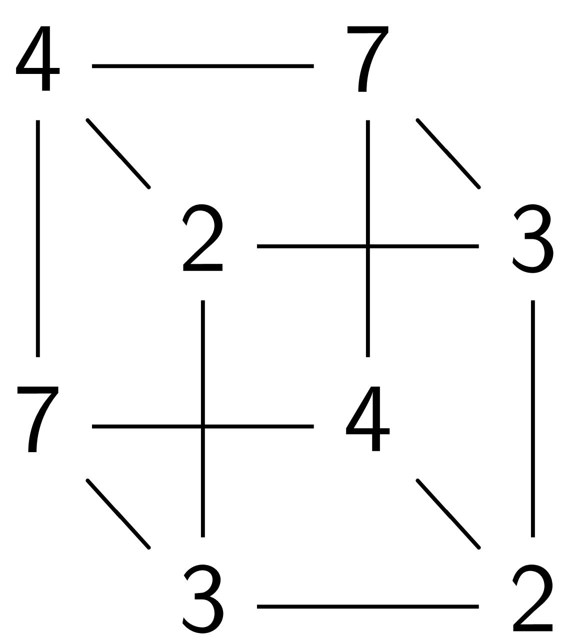
compute

hope to

Success

until #i

3 bits:



ows that

or all x.

's algorithm

its.

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$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$

$2, 0, 2, 0, 0, 2, 0, 2,$

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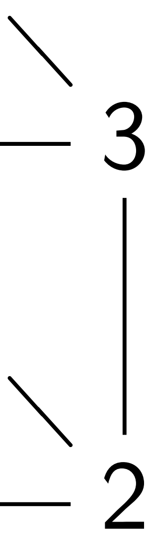
Step 9. Measure.
First 3 qubits are uniform random vector orthogonal to 101: i.e., 000, 010, 101, or 111.

Grover's algorithm

Assume: unique s
has $f(s) = 0$.

Traditional algorit
compute f for ma
hope to find outpu
Success probabilit
until #inputs appr



m

Step 8. Hadamard on qubit 2:

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, 2,$

$2, 0, 2, 0, 0, 2, 0, 2,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

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Step 9. Measure.
First 3 qubits are uniform random vector orthogonal to 101: i.e., 000, 010, 101, or 111.

Grover's algorithm

Assume: unique $s \in \{0, 1\}^n$
has $f(s) = 0$.

Traditional algorithm to find
compute f for many inputs,
hope to find output 0.
Success probability is very lo
until #inputs approaches 2^n

Step 8. Hadamard on qubit 2:

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$

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$2, 0, 2, 0, 0, \overline{2}, 0, \overline{2}.$

Step 9. Measure.
First 3 qubits are uniform random vector orthogonal to 101: i.e., 000, 010, 101, or 111.

Grover's algorithm

Assume: unique $s \in \{0, 1\}^n$ has $f(s) = 0$.

Traditional algorithm to find s: compute f for many inputs, hope to find output 0.
Success probability is very low until #inputs approaches 2^n.

Step 8. Hadamard on qubit 2:

$0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0, 0, 0, 0, 0,$

$2, 0, \overline{2}, 0, 0, \overline{2}, 0, \overline{2},$

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Grover's algorithm takes only $2^{n/2}$ reversible computations of f.
Typically: reversibility overhead is small enough that this easily beats traditional algorithm.

Hadamard on qubit 2:

$, 0, 0, 0, 0,$

$, 0, 0, 0, 0,$

$, 0, \overline{2}, 0, \overline{2},$

$, 0, \overline{2}, 0, 2,$

$, 0, 2, 0, 2,$

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Measure.

qubits are uniform random

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Start fro

over all

Step 1:

$b_q = -a$

$b_q = a_q$

This is f

Step 2:

Negate

This is a

Repeat

about 0.

Measure

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d on qubit 2:

uniform random
to 101: i.e.,
111.

Grover's algorithm

Assume: unique $s \in \{0,1\}^n$
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reversible computations of f.
Typically: reversibility overhead
is small enough that this
easily beats traditional algorithm.

Start from uniform
over all n-bit strin

Step 1: Set $a \leftarrow b$
$b_q = -a_q$ if $f(q)$
$b_q = a_q$ otherwise
This is fast.

Step 2: "Grover d
Negate a around i
This is also fast.

Repeat Step 1 + S
about $0.58 \cdot 2^{0.5n}$

Measure the n qub
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Grover's algorithm

Assume: unique $s \in \{0,1\}^n$ has $f(s) = 0$.

Traditional algorithm to find s: compute f for many inputs, hope to find output 0.
Success probability is very low until #inputs approaches 2^n.

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Start from uniform superpos over all n-bit strings q.

Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this fi

Grover's algorithm

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Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits. With high probability this finds s.

algorithm

unique $s \in \{0, 1\}^n$
$= 0$.

nal algorithm to find s:
e f for many inputs,
find output 0.
probability is very low
nputs approaches 2^n.

algorithm takes only $2^{n/2}$
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Measure the n qubits.
With high probability this finds s.

Normaliz
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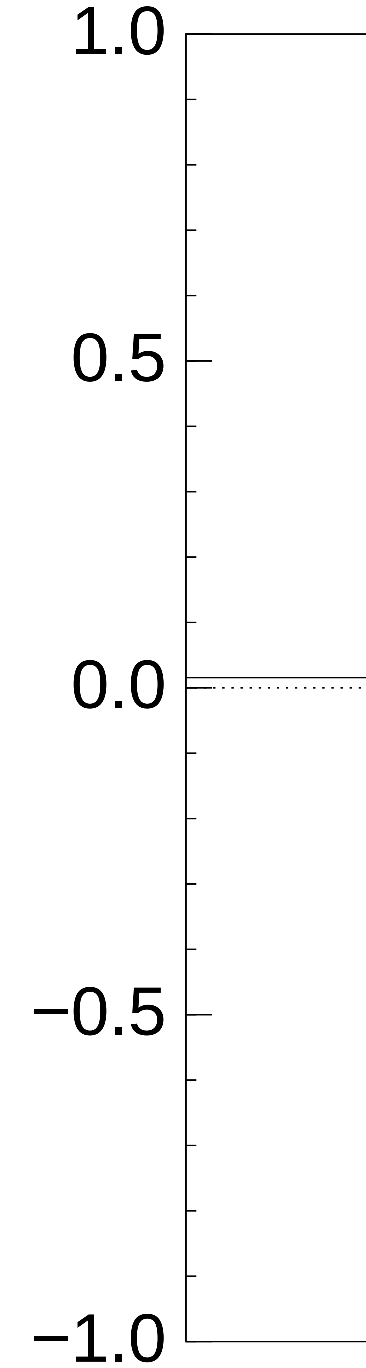

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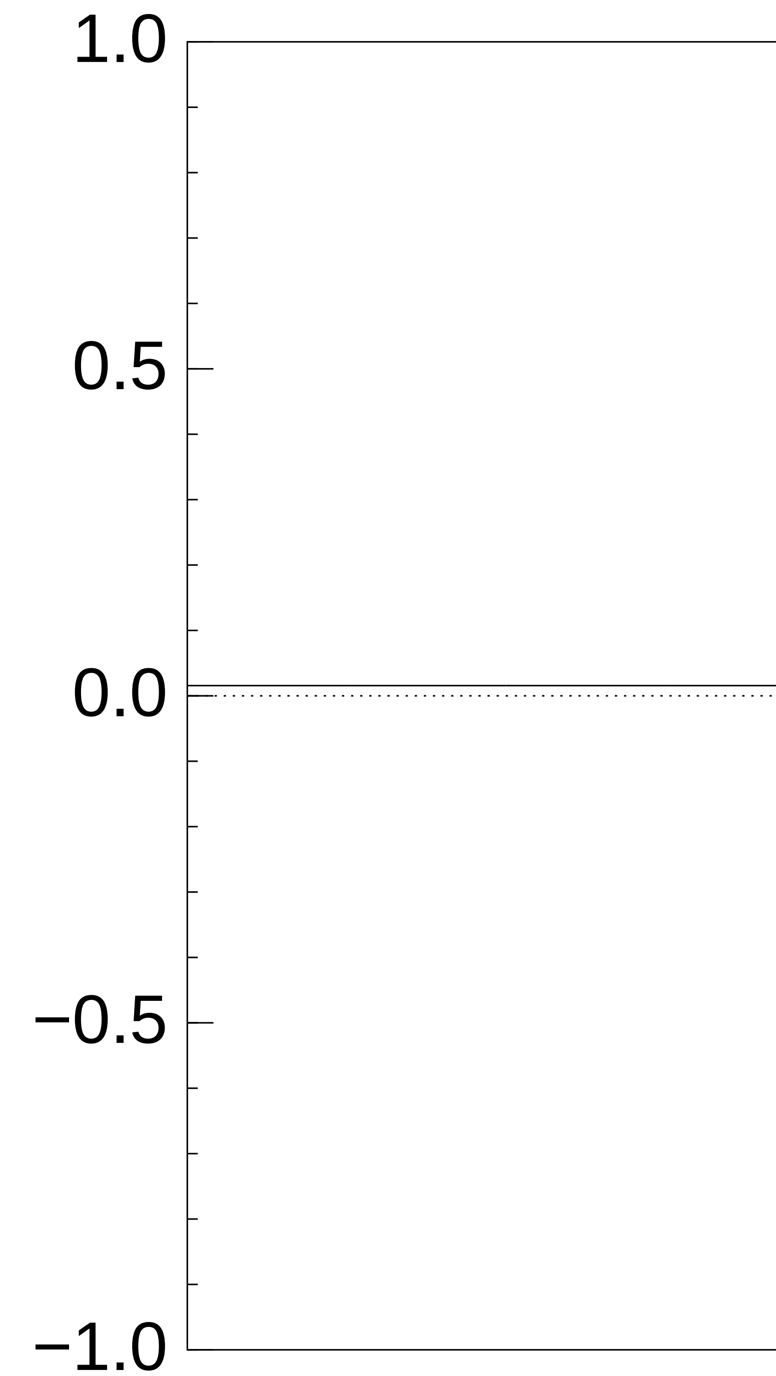
Normalized graph
for an example wit
after 0 steps:



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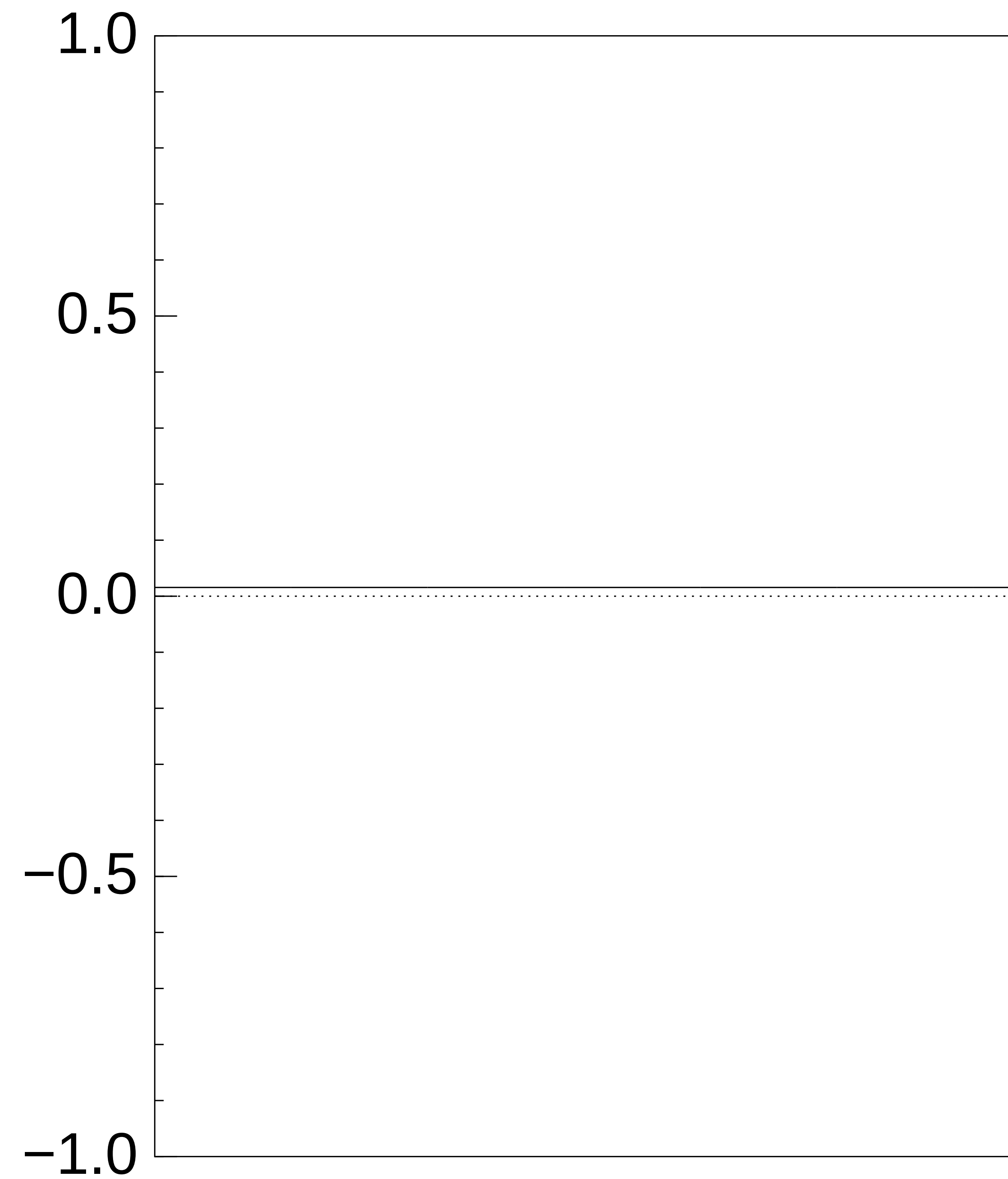
Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$
for an example with $n = 12$
after 0 steps:



Start from uniform superposition over all n-bit strings q.

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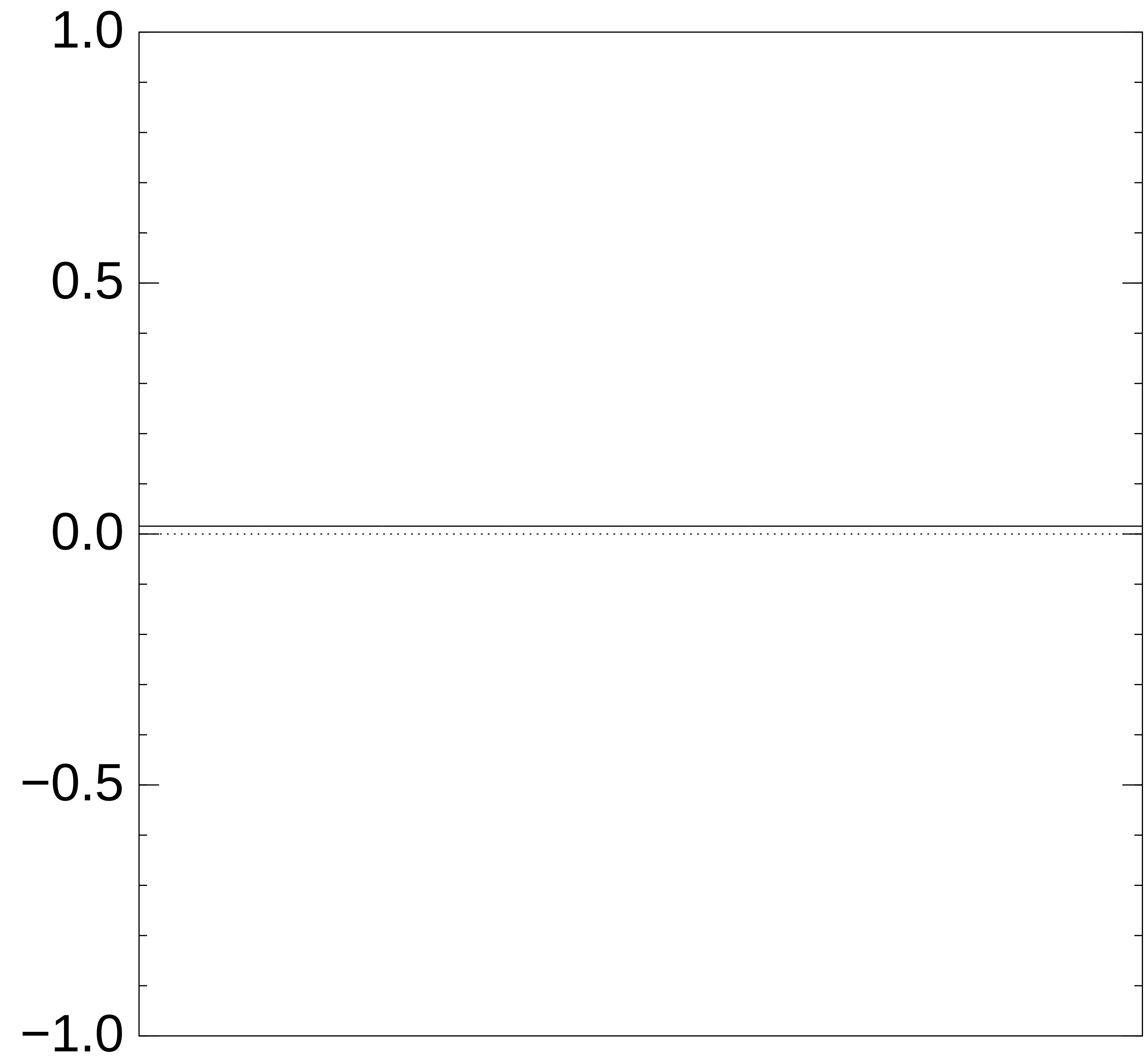
Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after 0 steps:



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This is fast.

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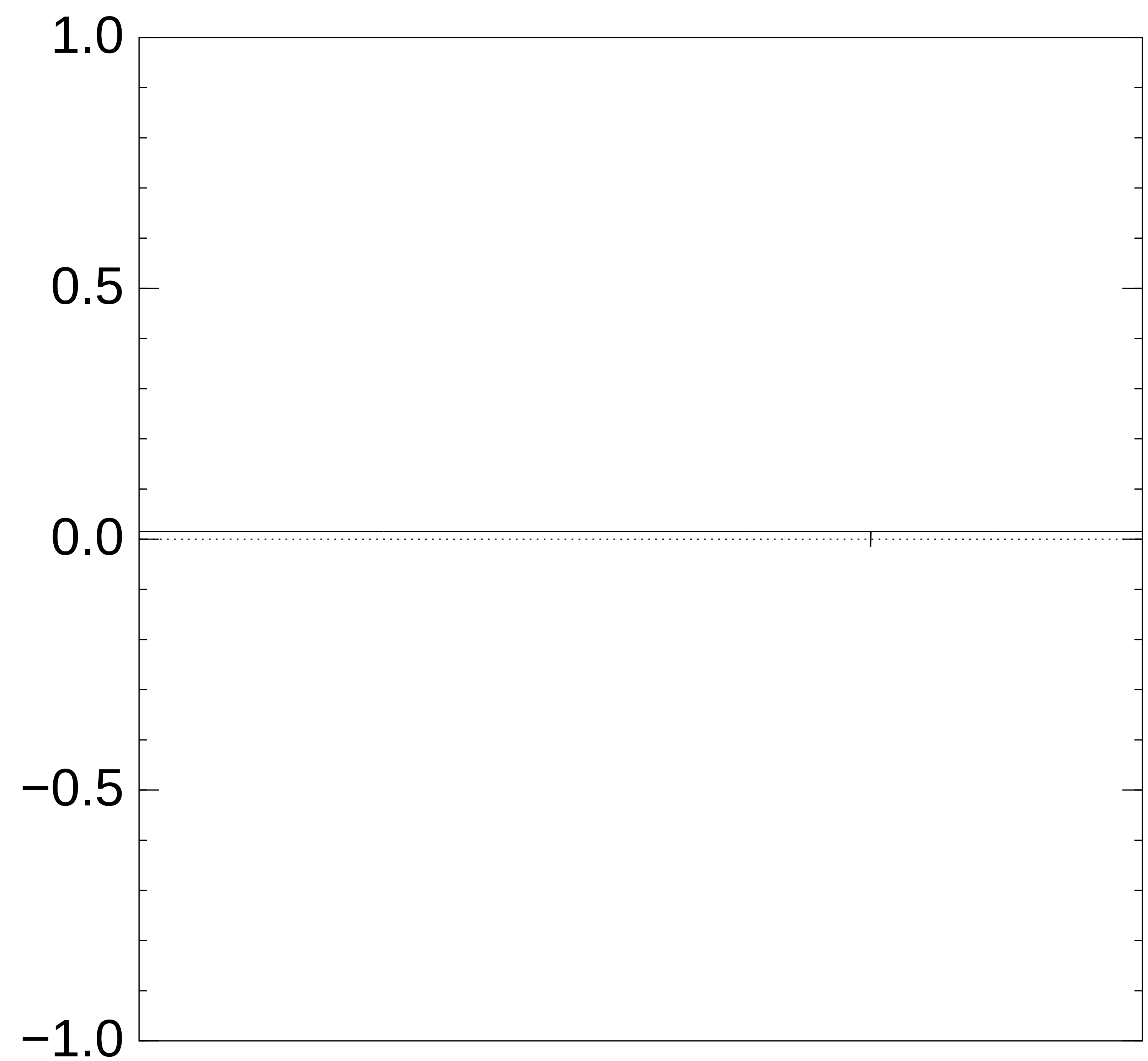
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1:



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Step 2: "Grover diffusion".
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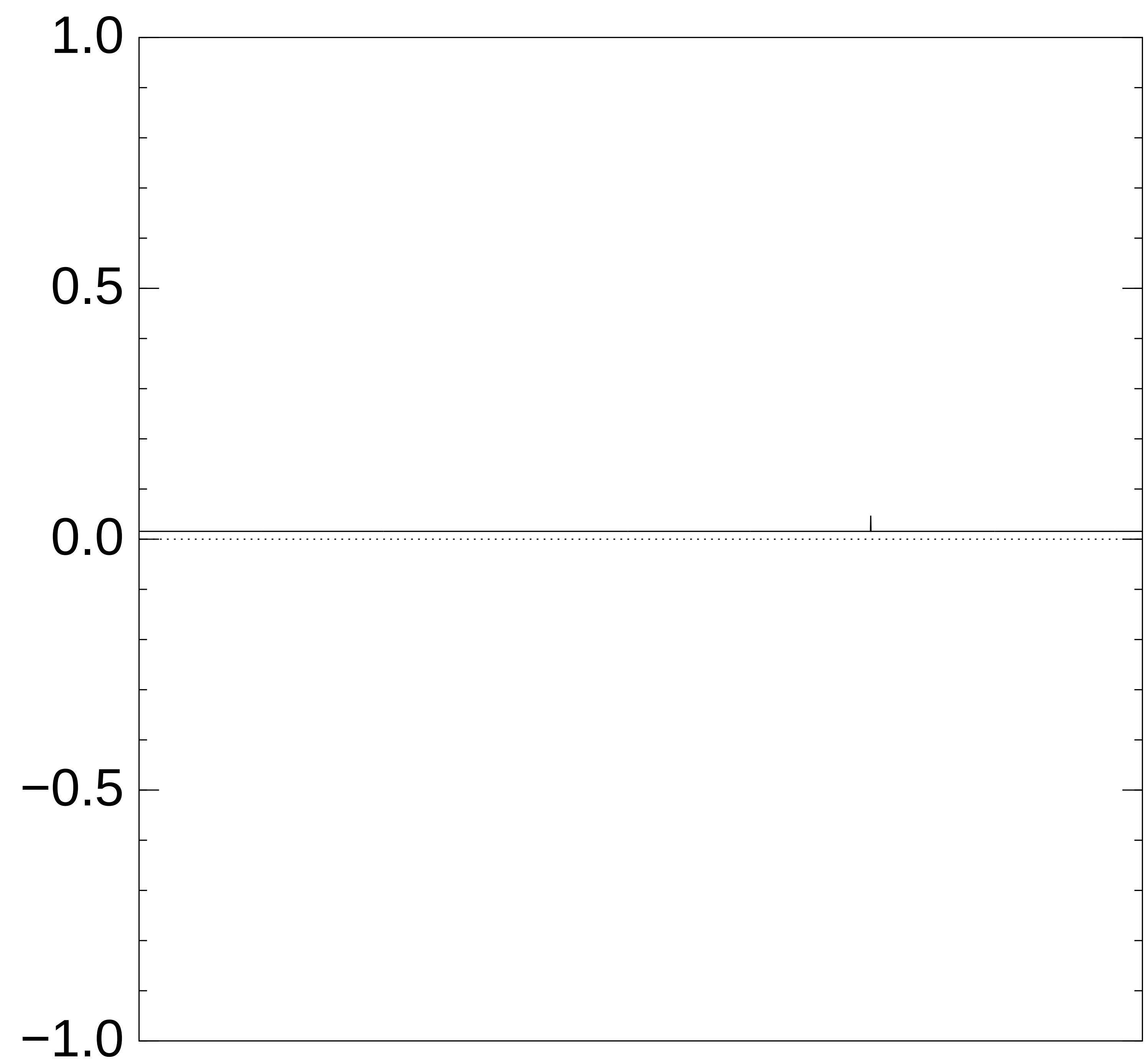
Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1 + Step 2:



Start from uniform superposition over all n-bit strings q.

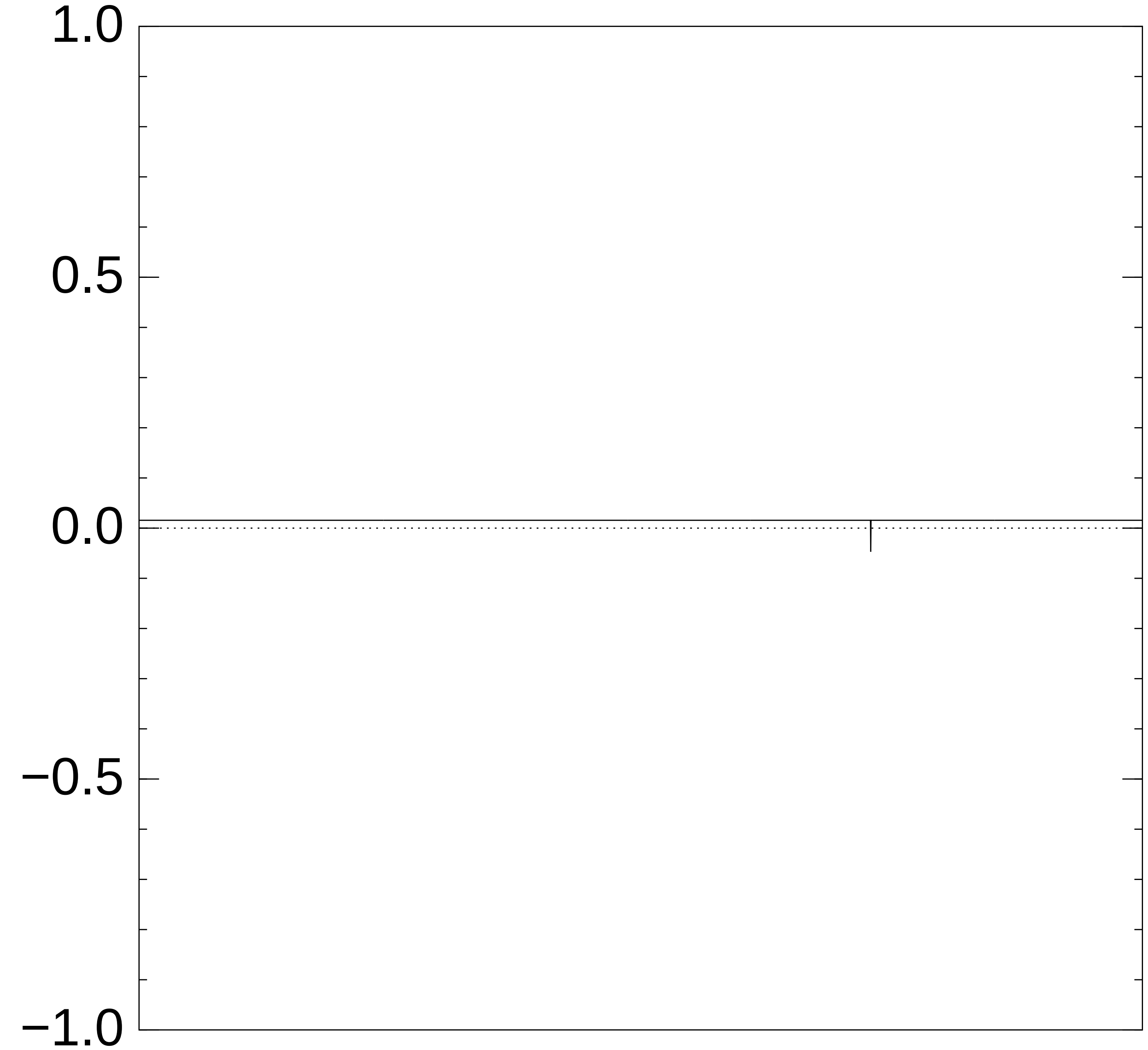
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after Step 1 + Step 2 + Step 1:



Start from uniform superposition over all n-bit strings q.

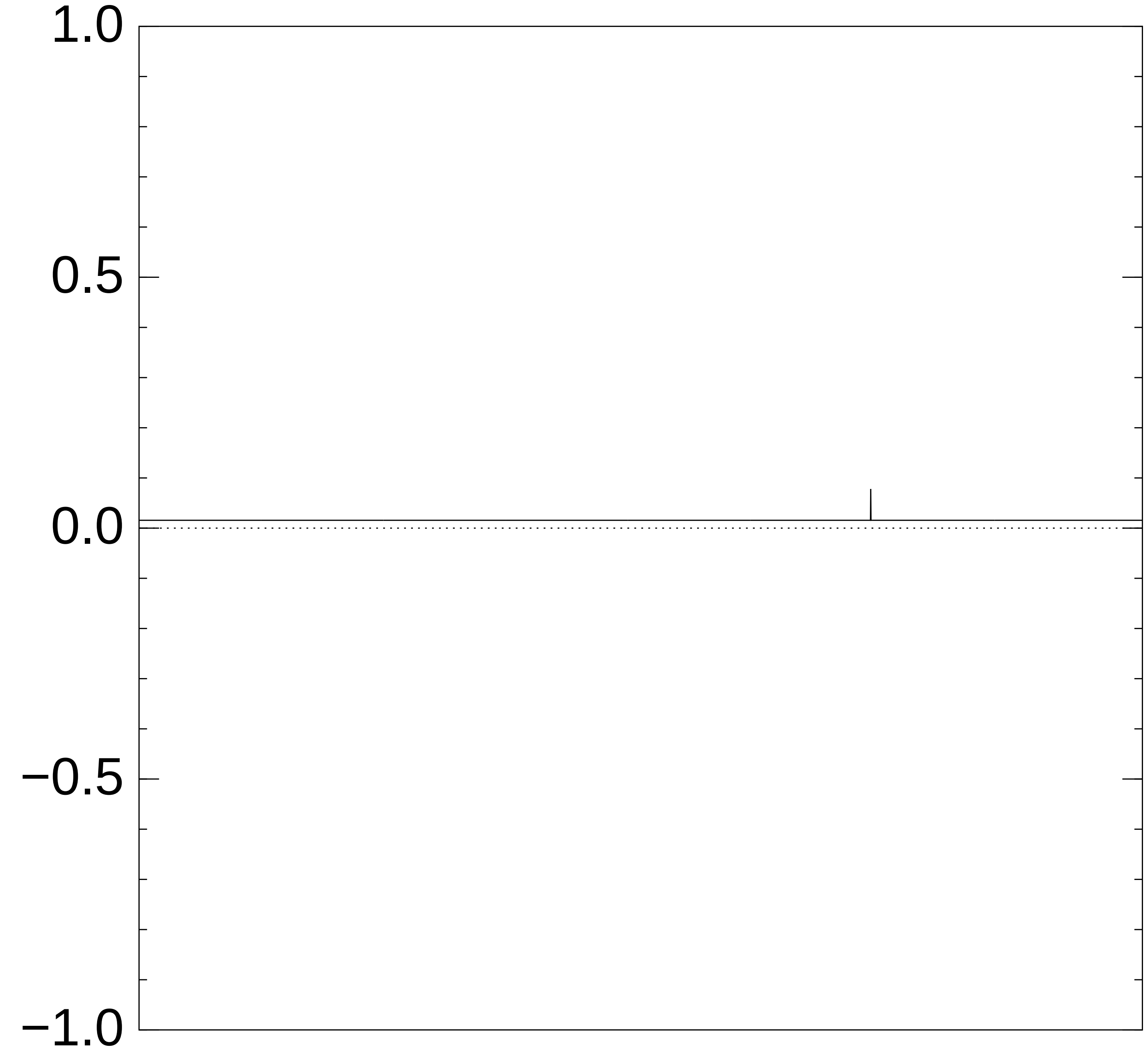
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $2 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

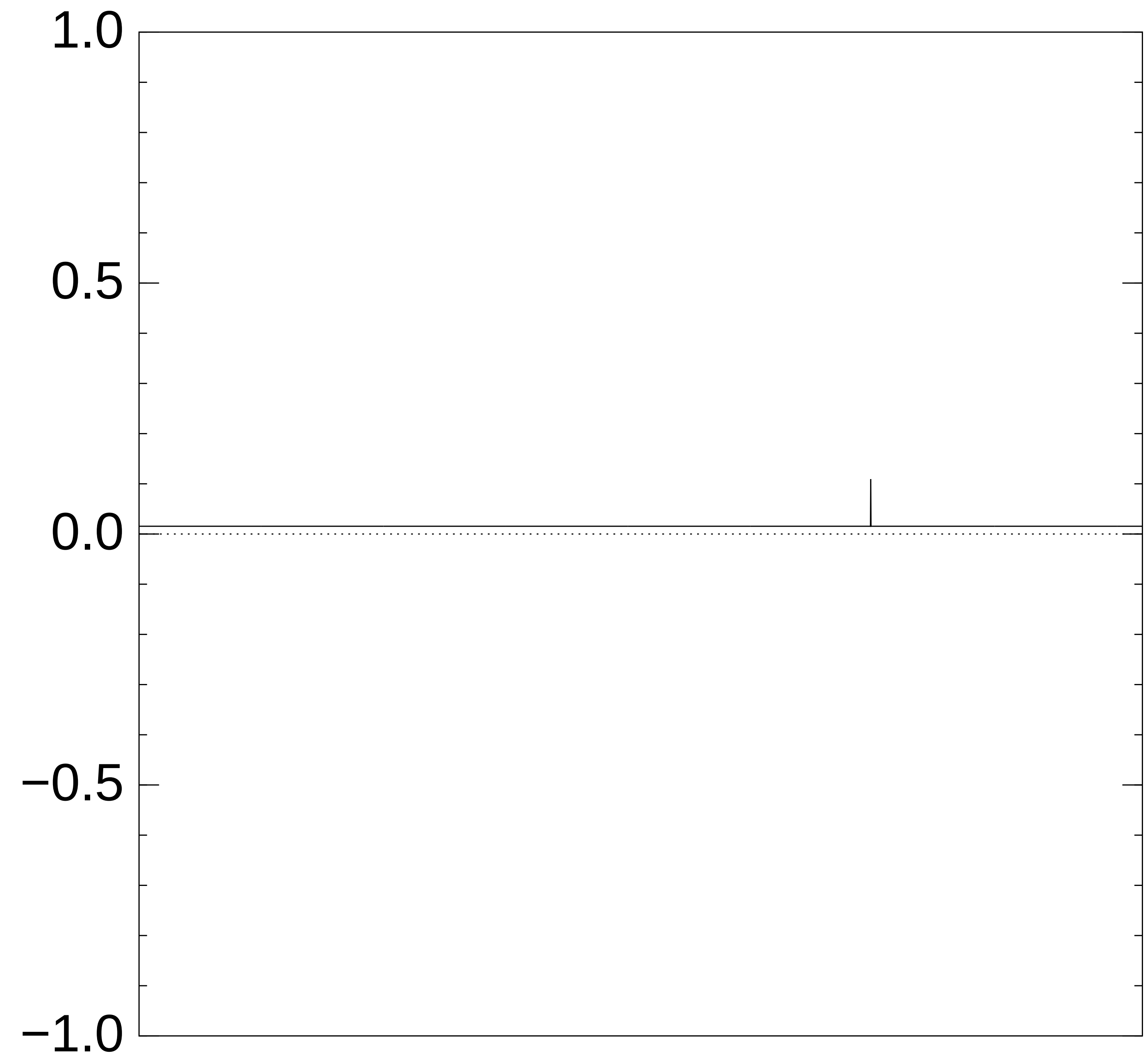
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $3 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

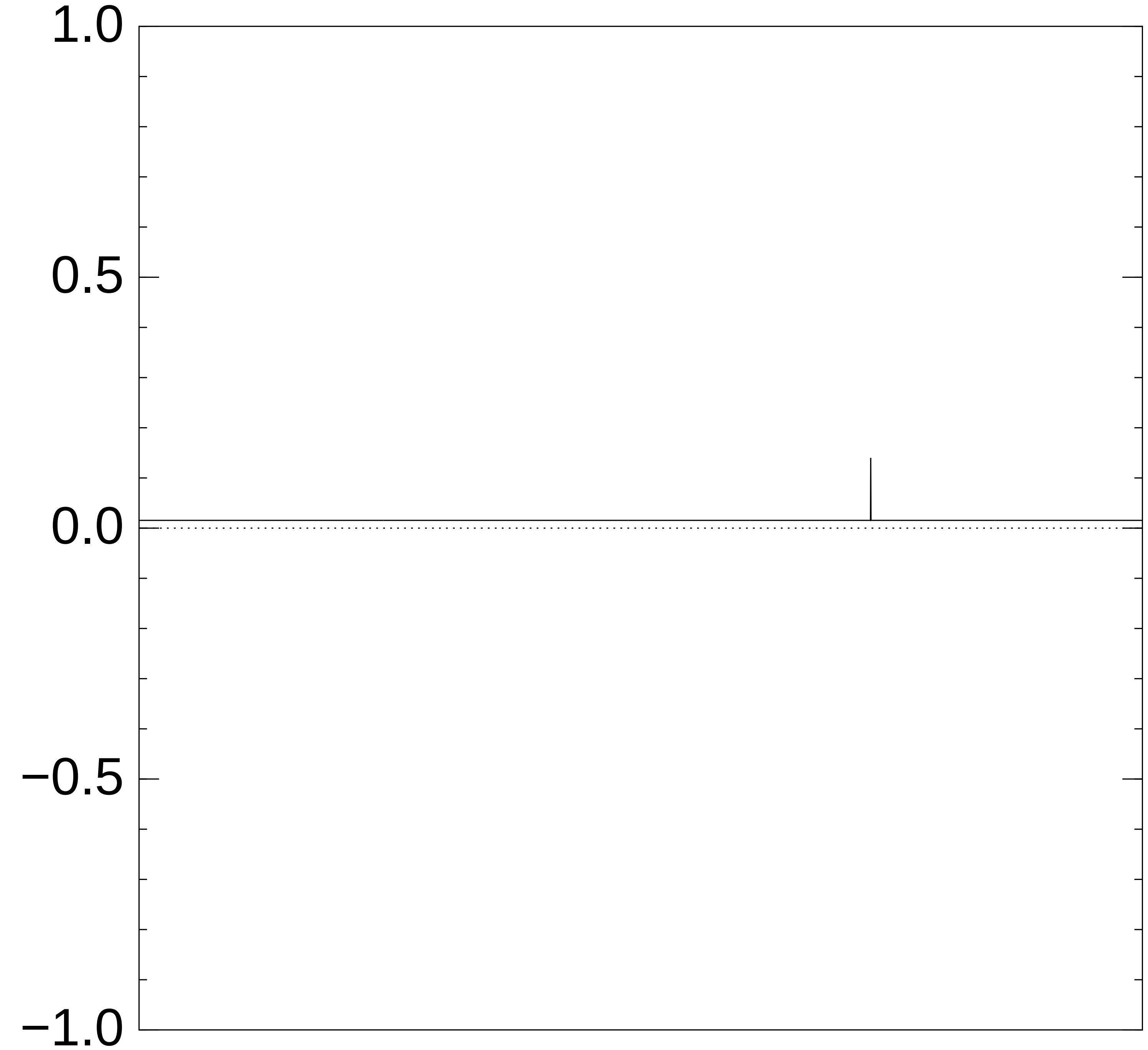
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $4 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

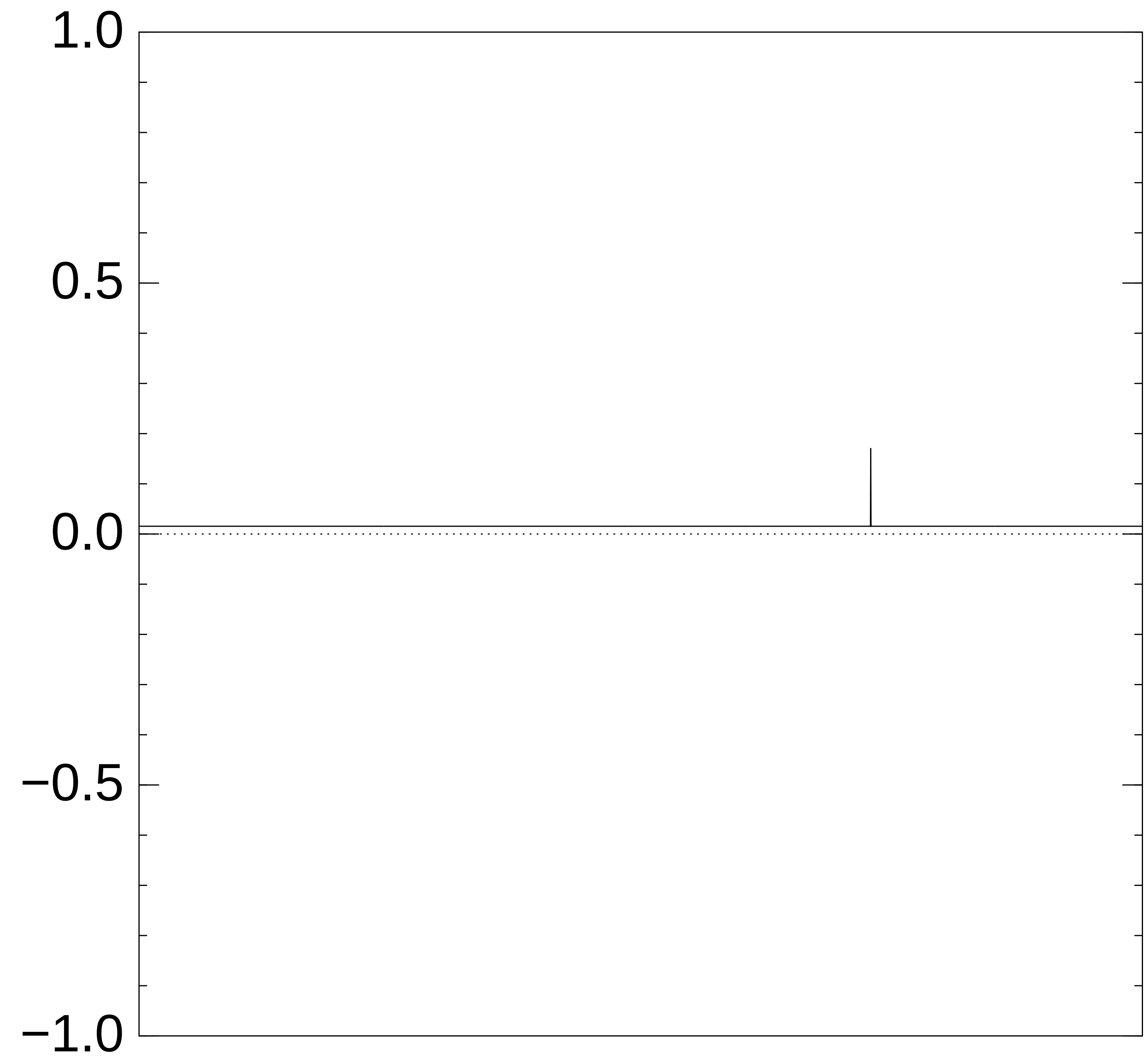
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $5 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

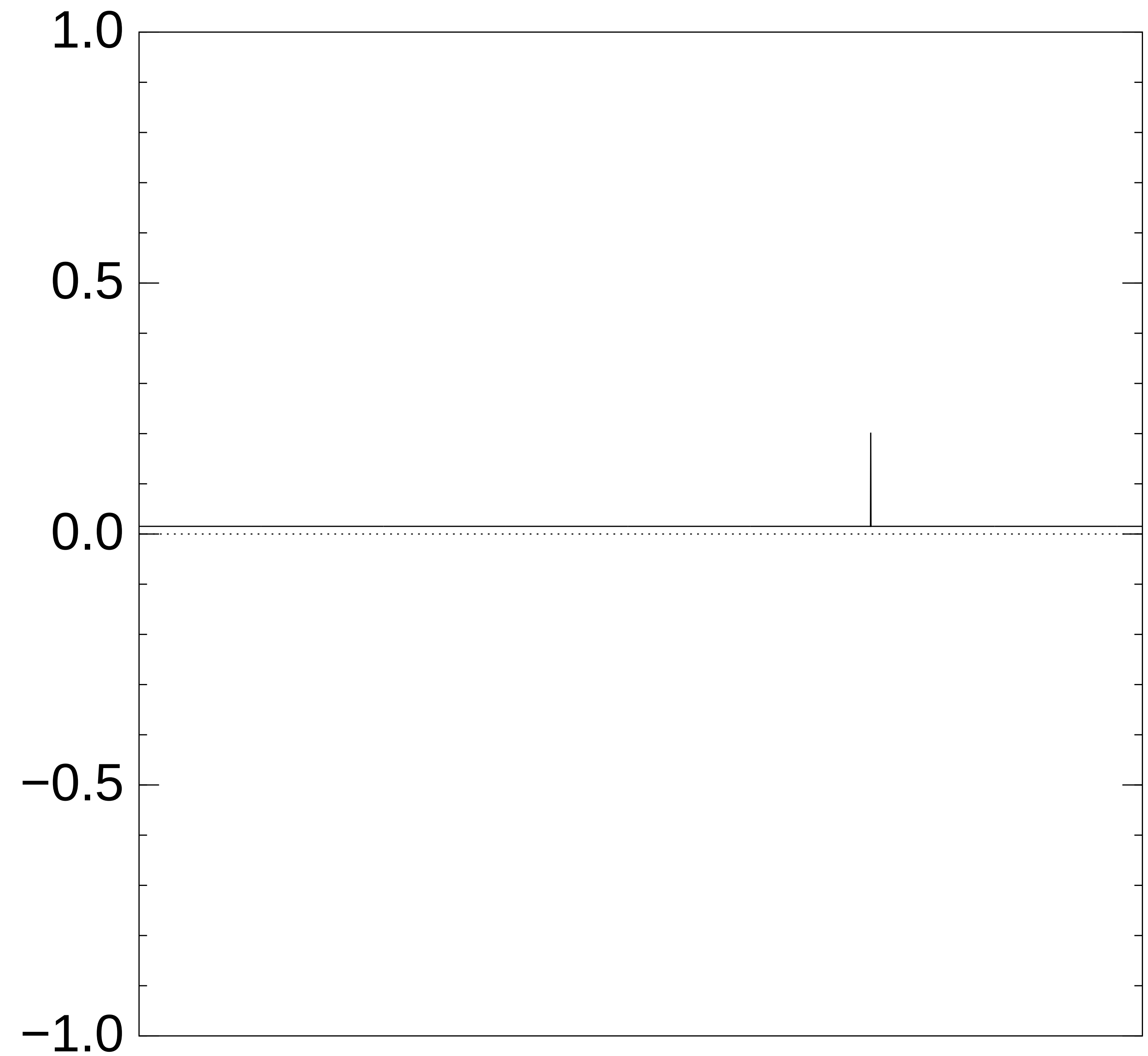
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $6 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

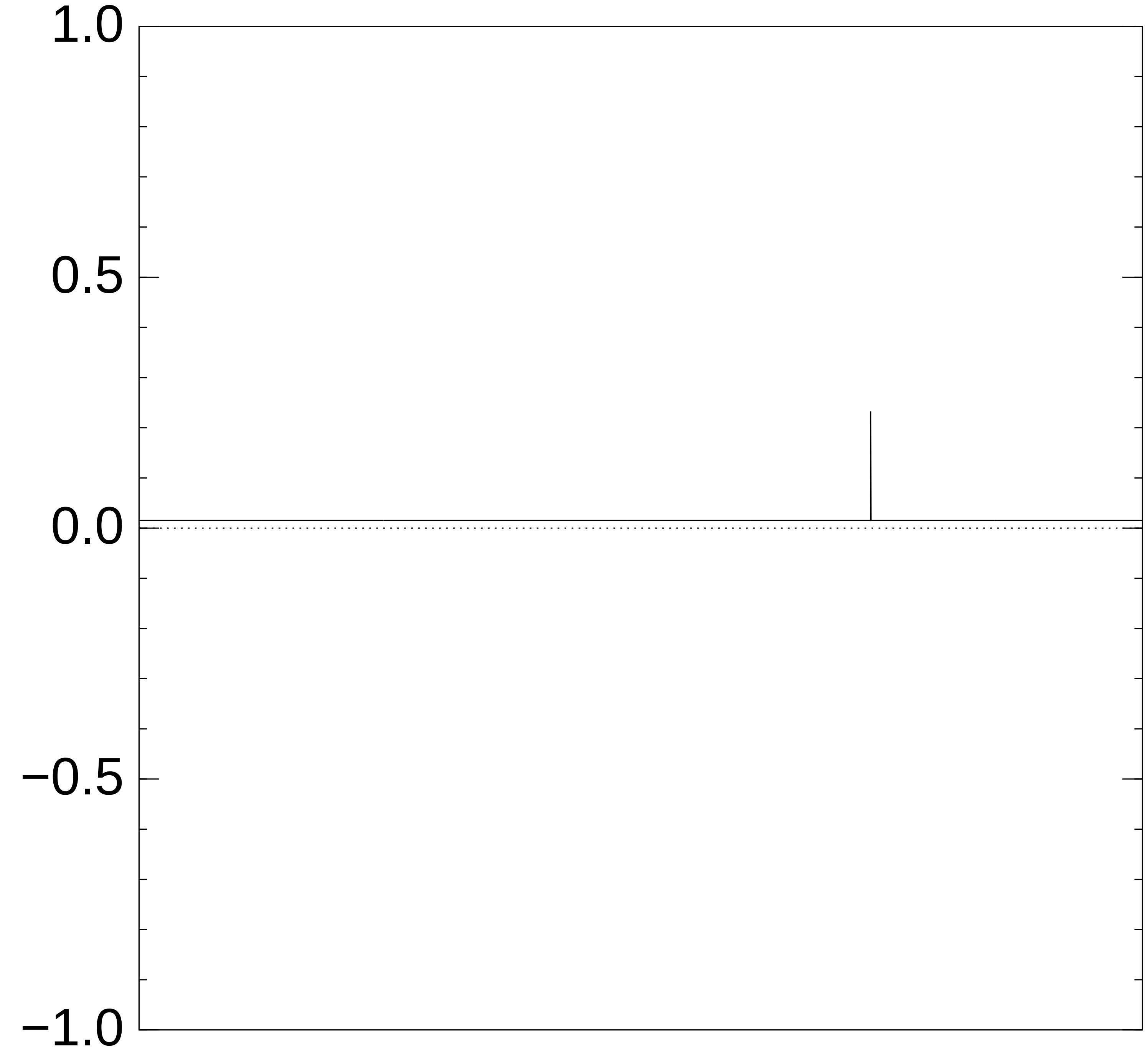
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $7 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

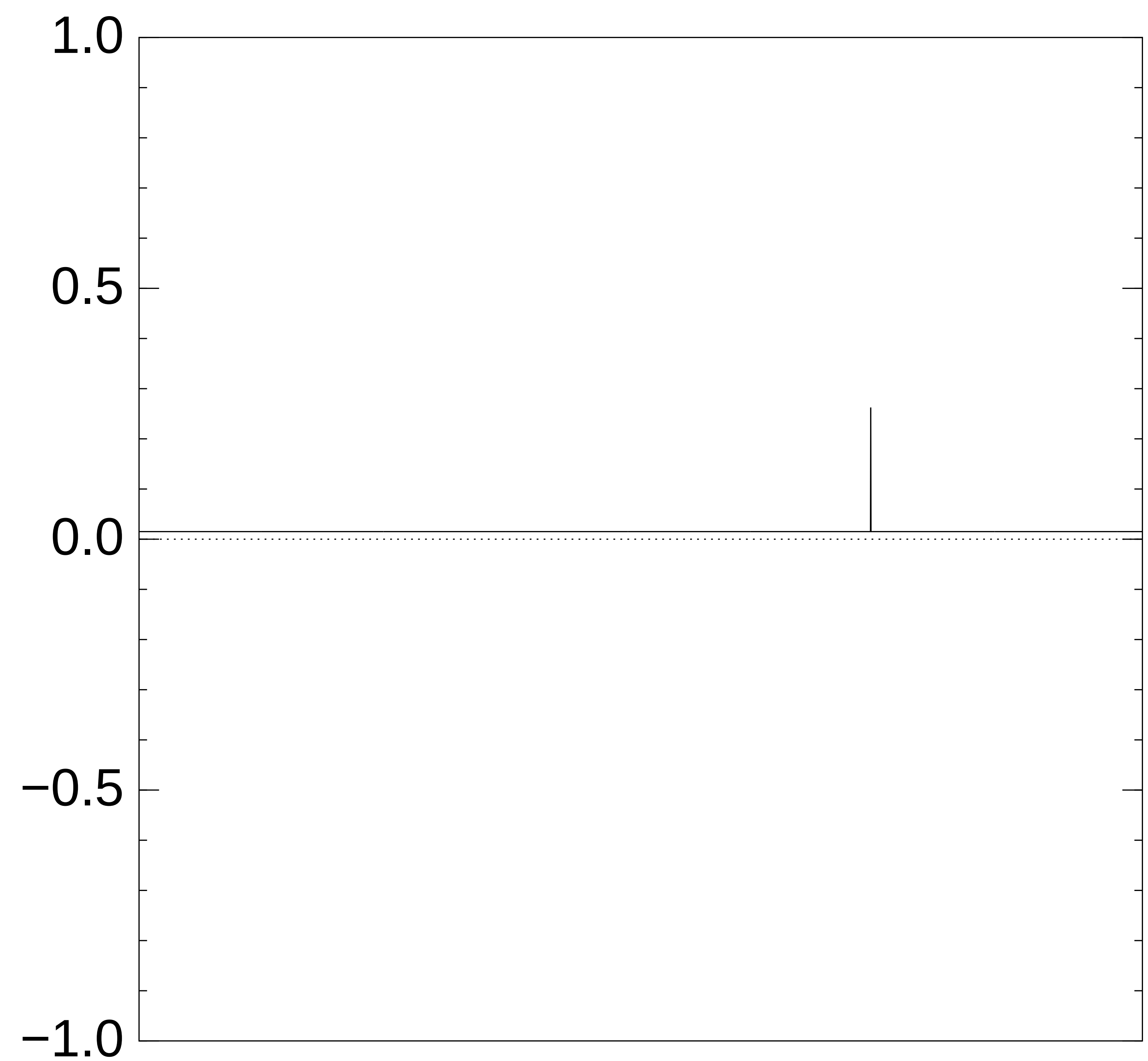
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $8 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

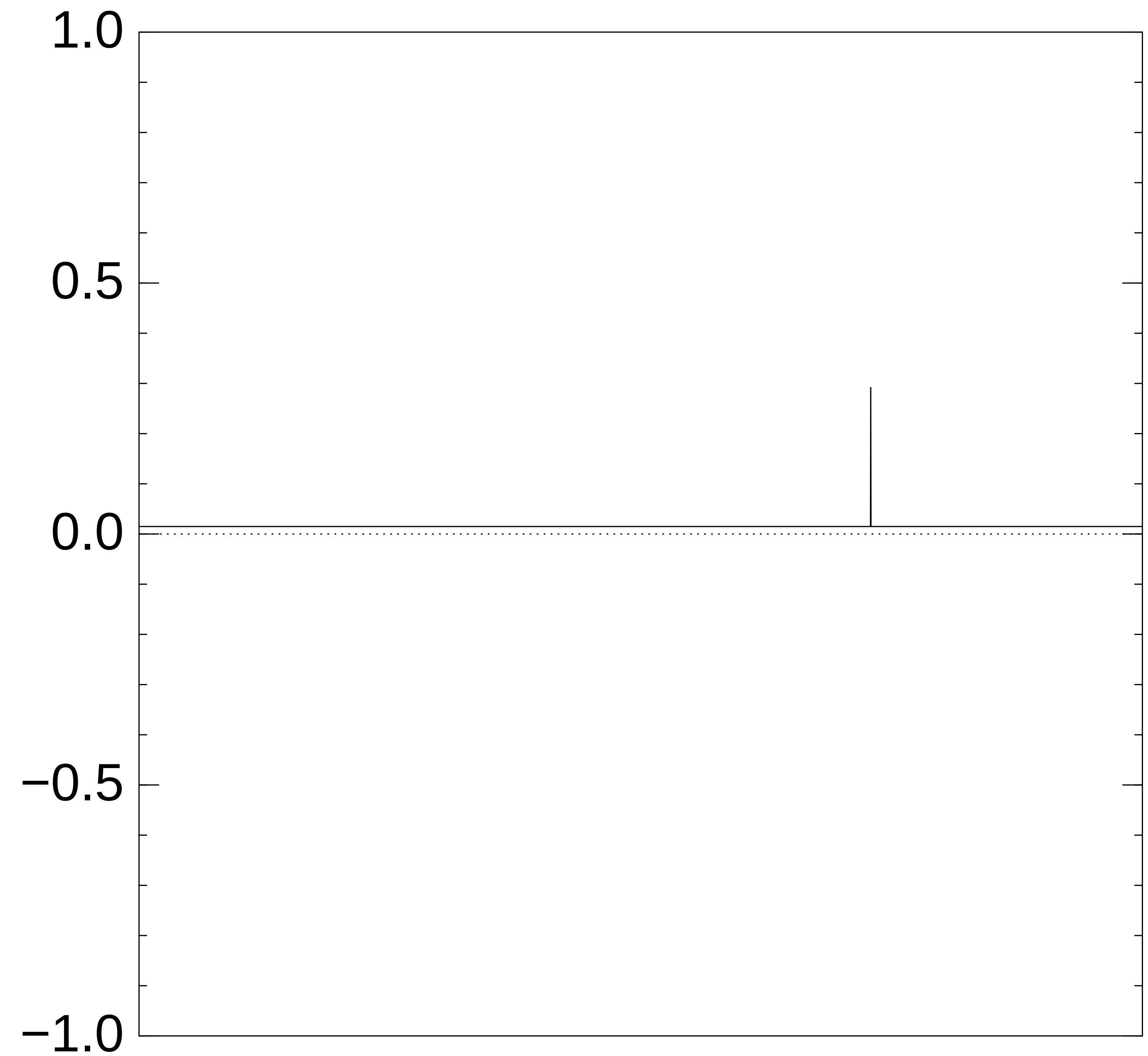
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $9 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

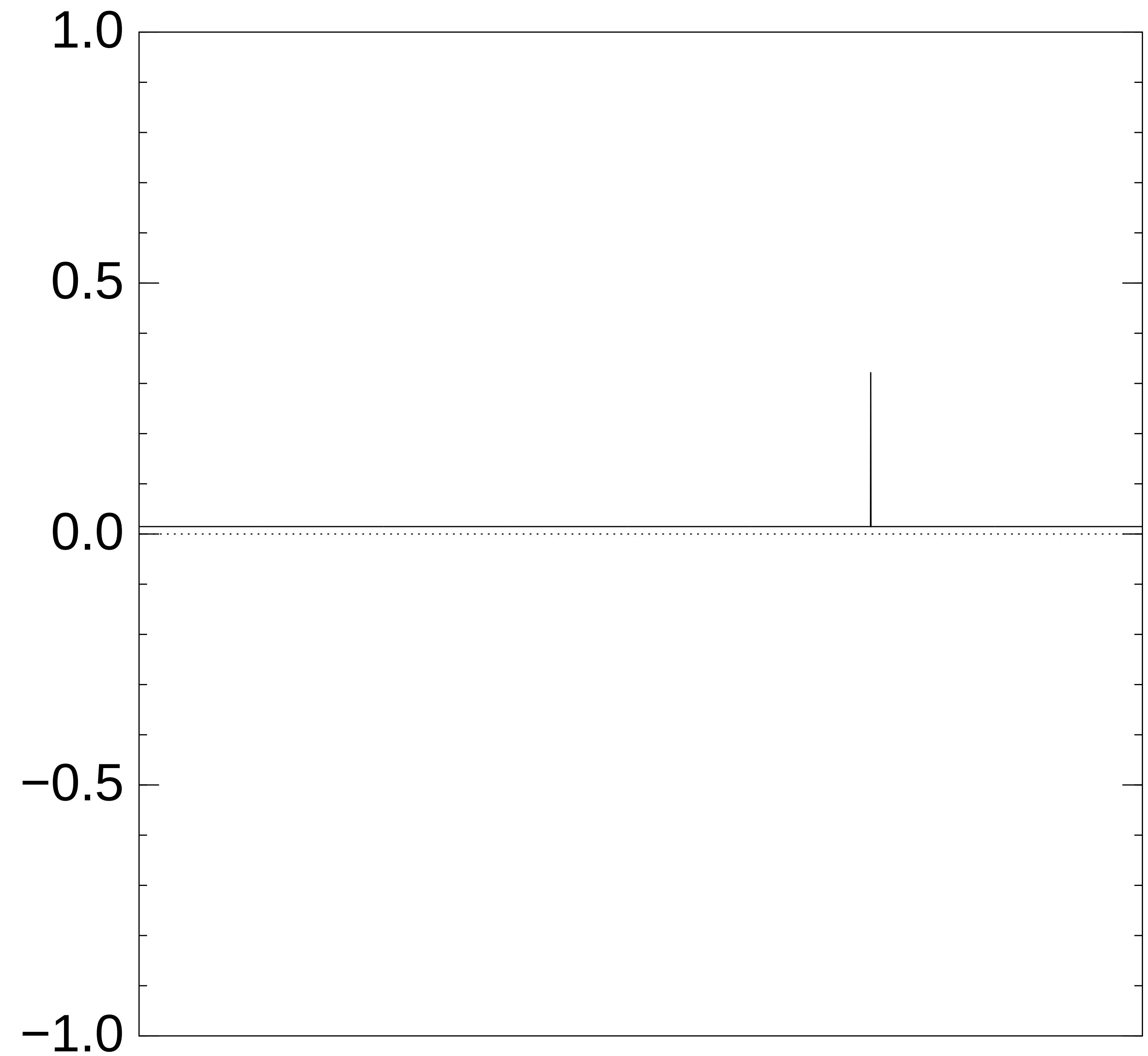
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $10 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

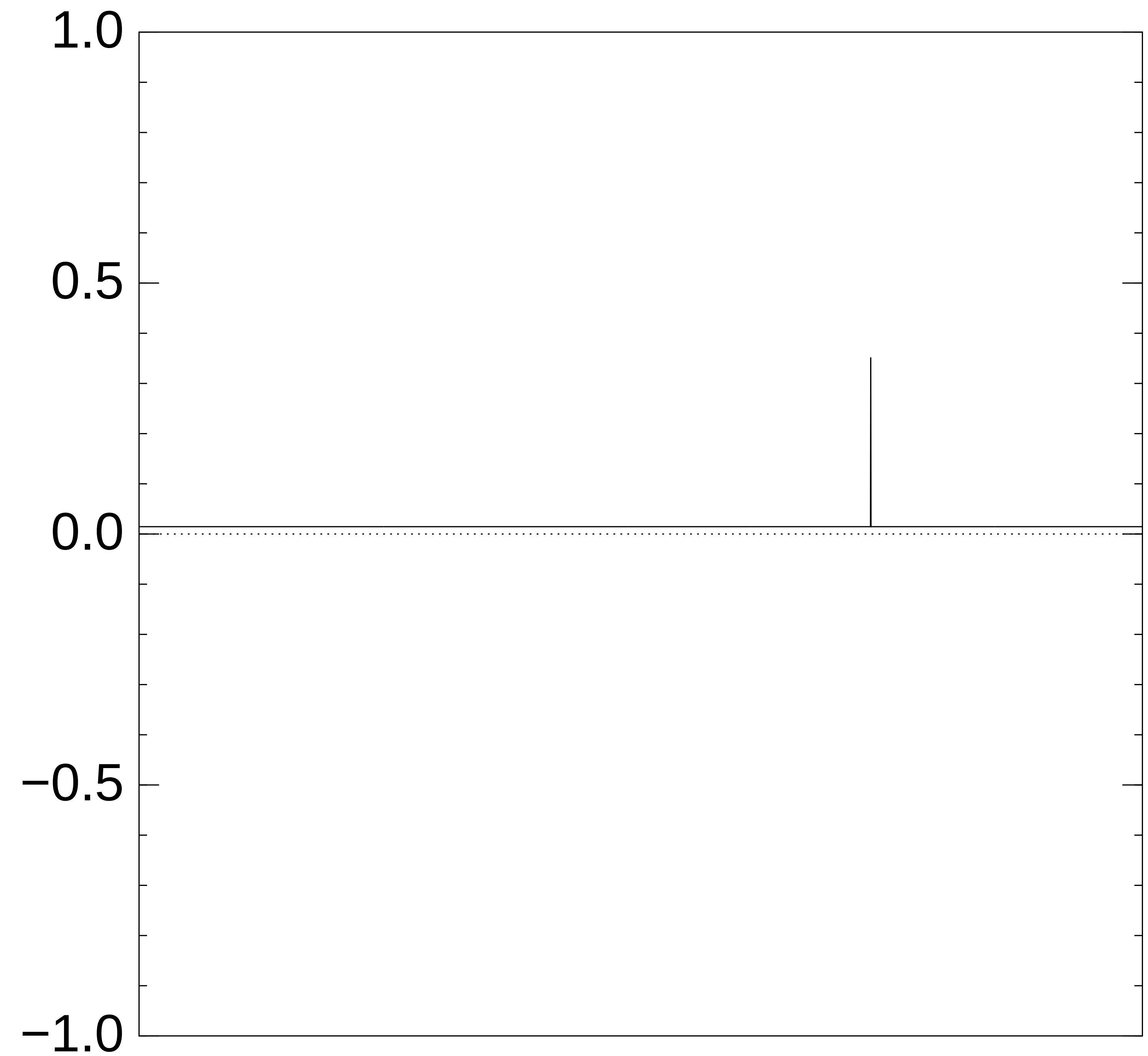
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $11 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

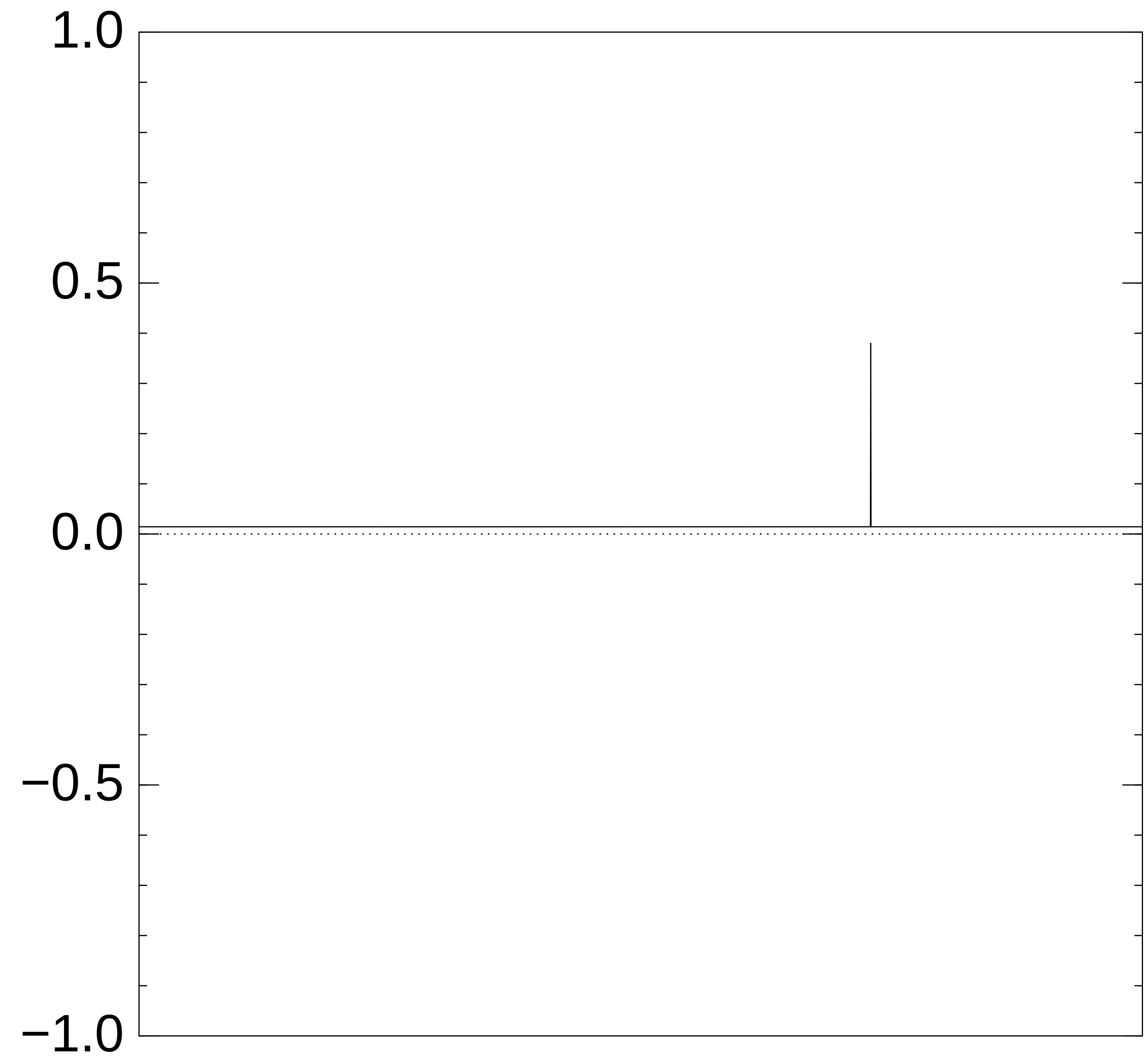
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $12 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

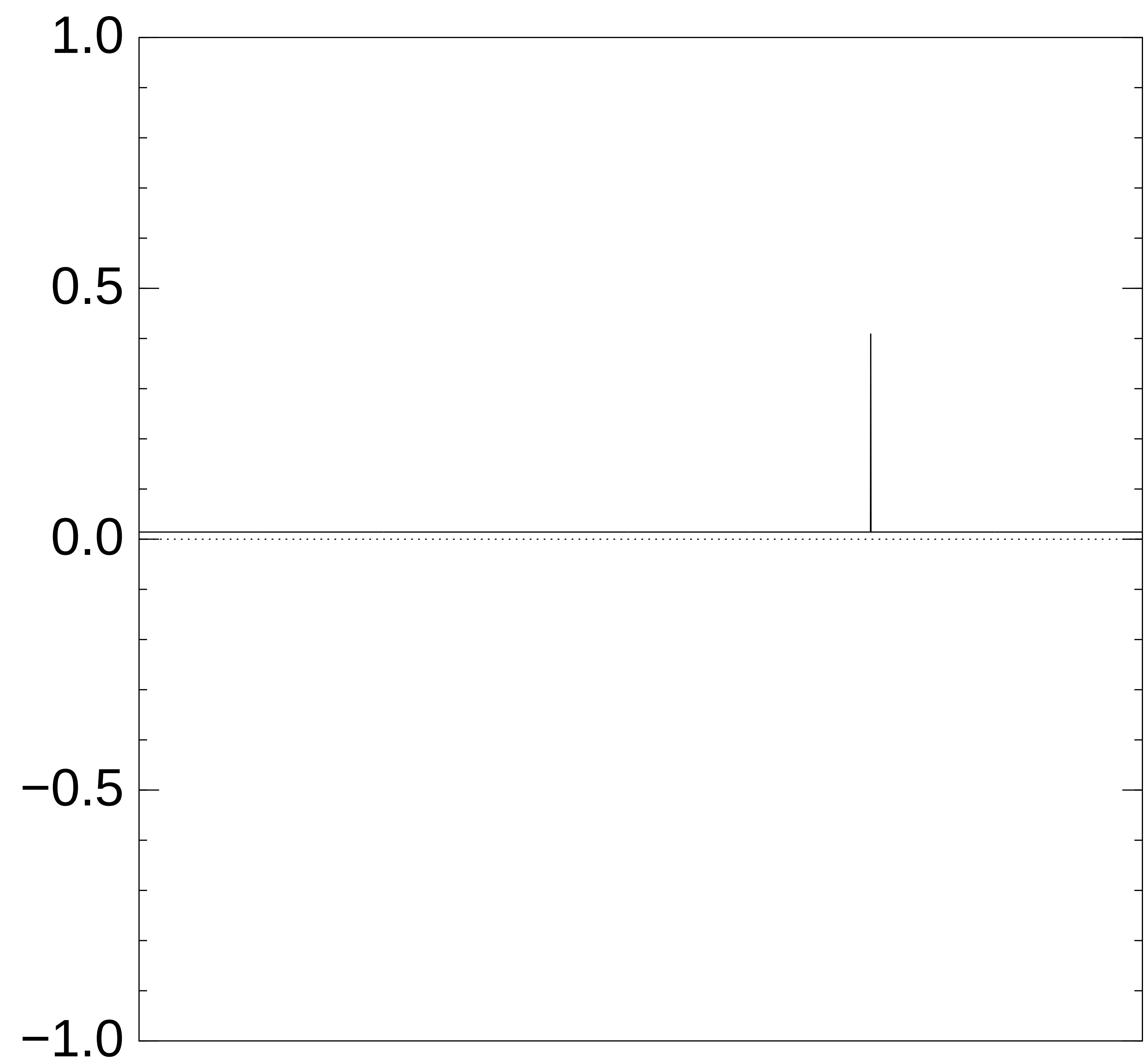
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $13 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

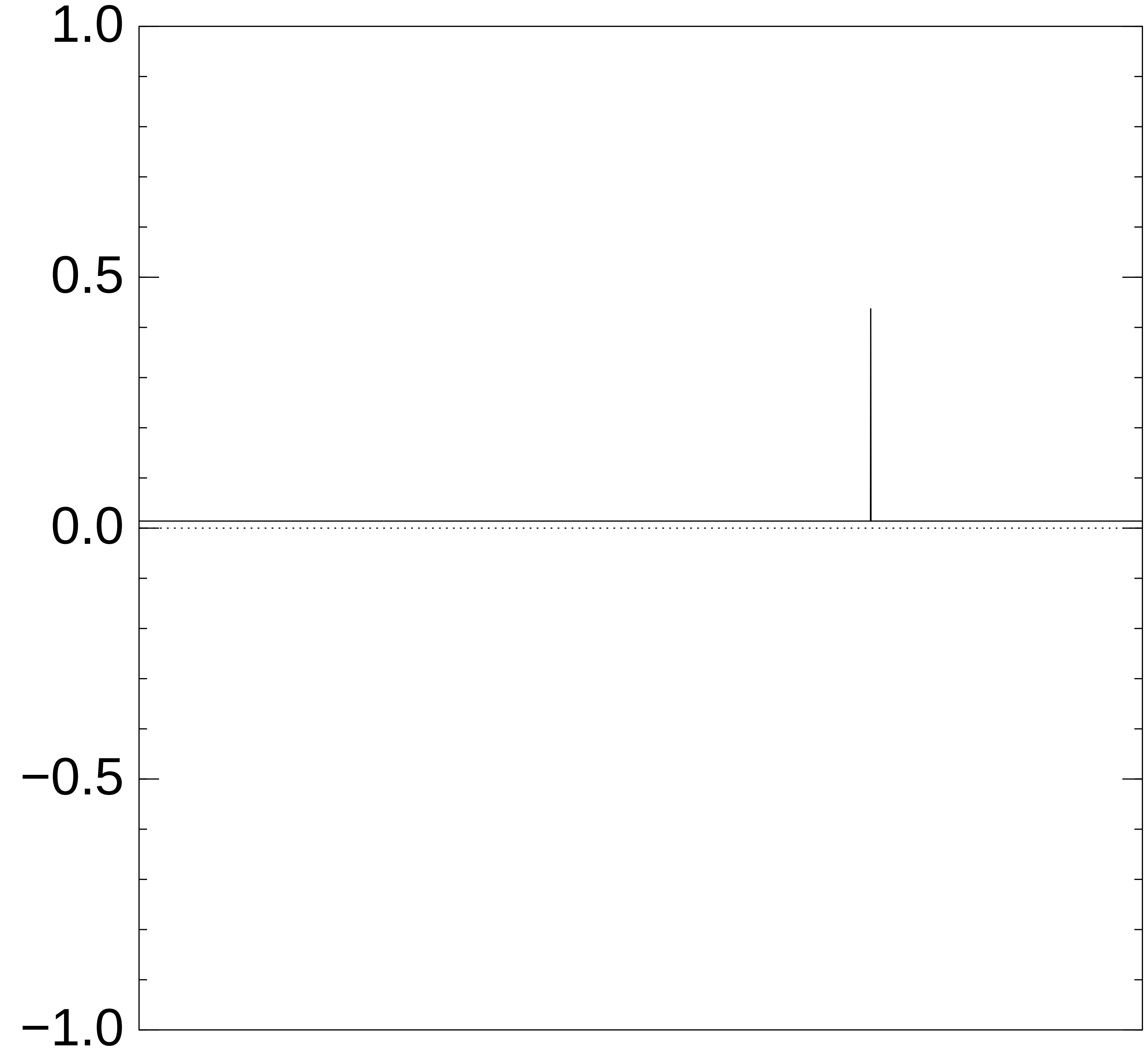
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $14 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

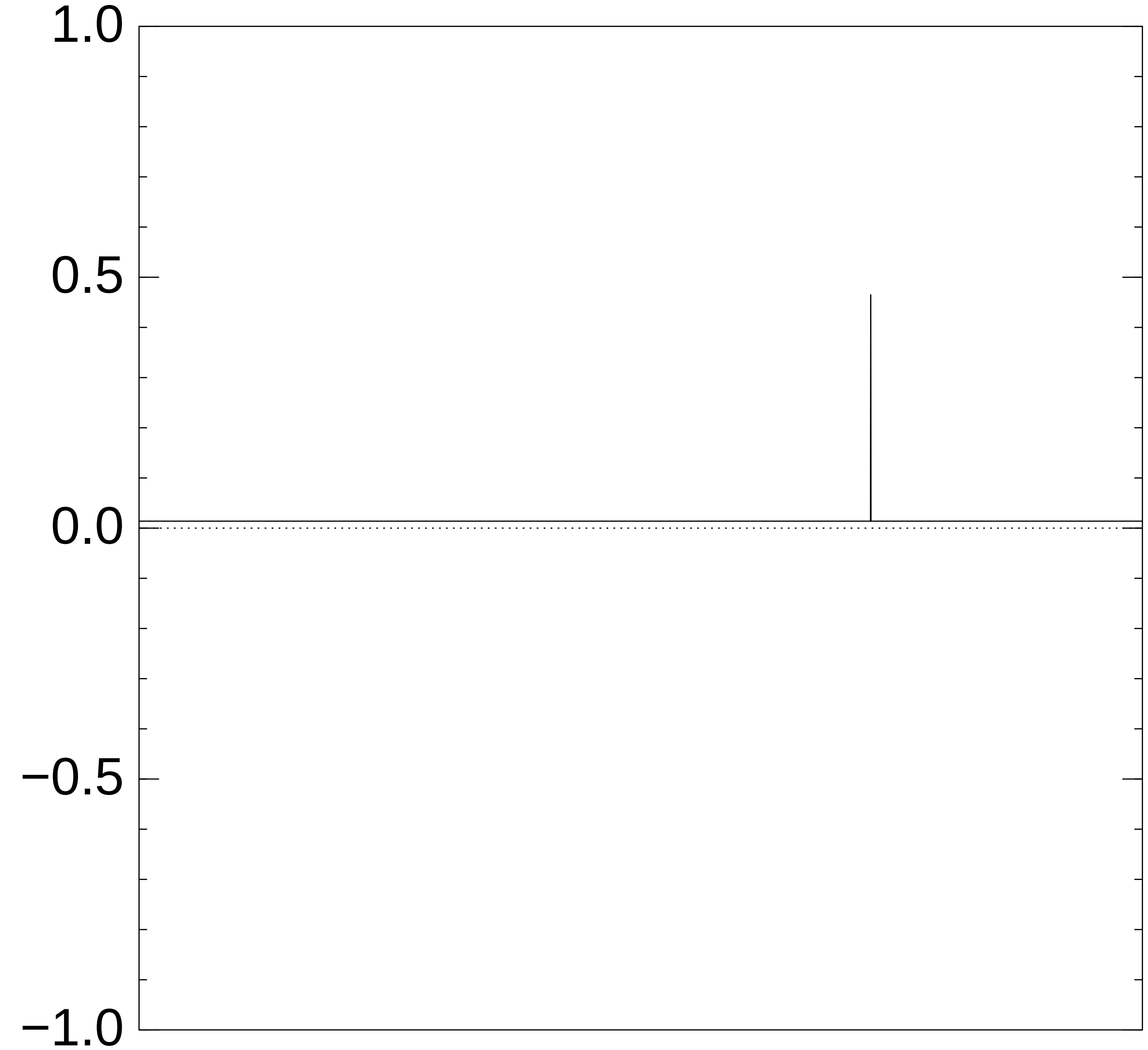
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $15 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

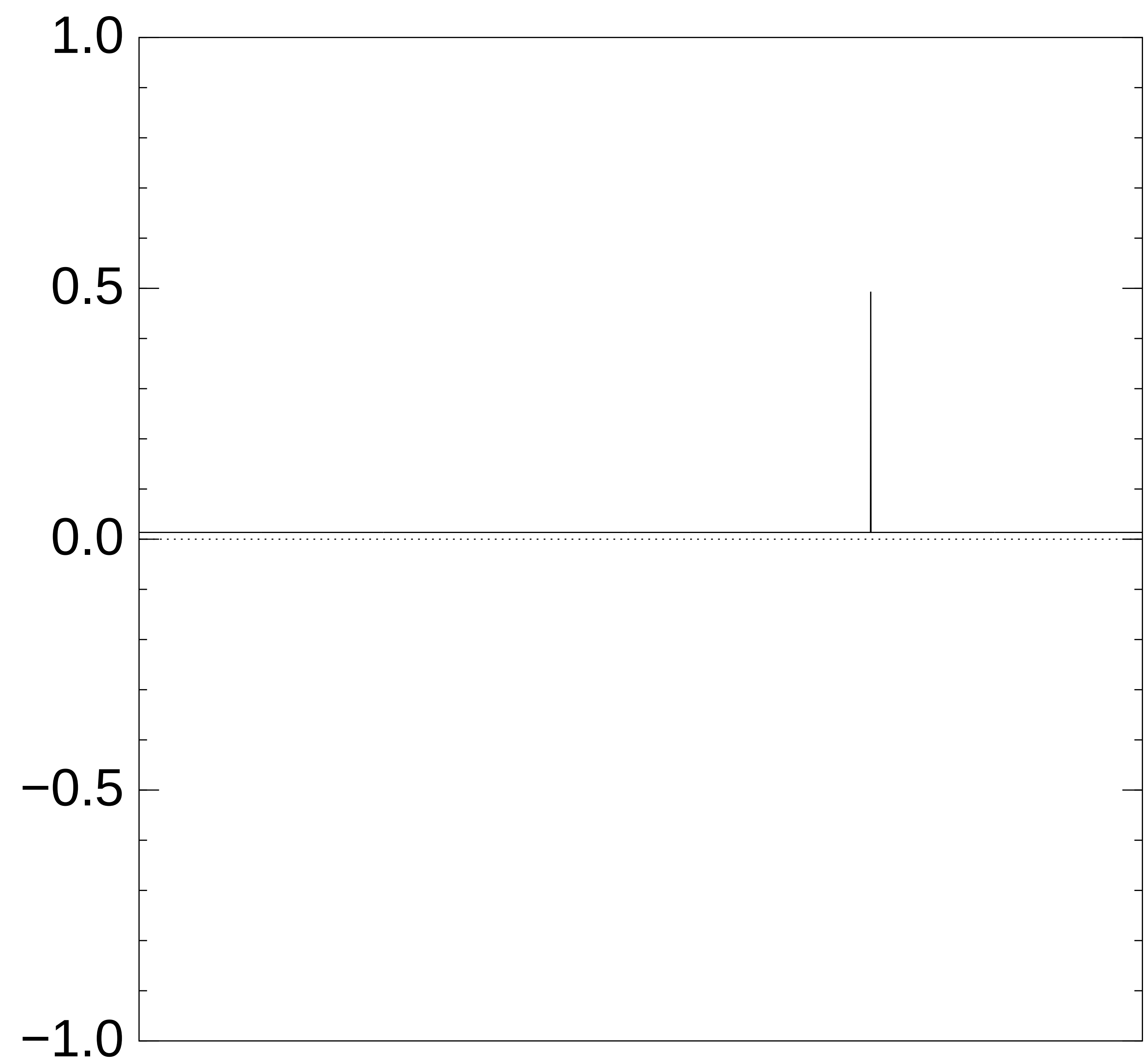
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $16 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

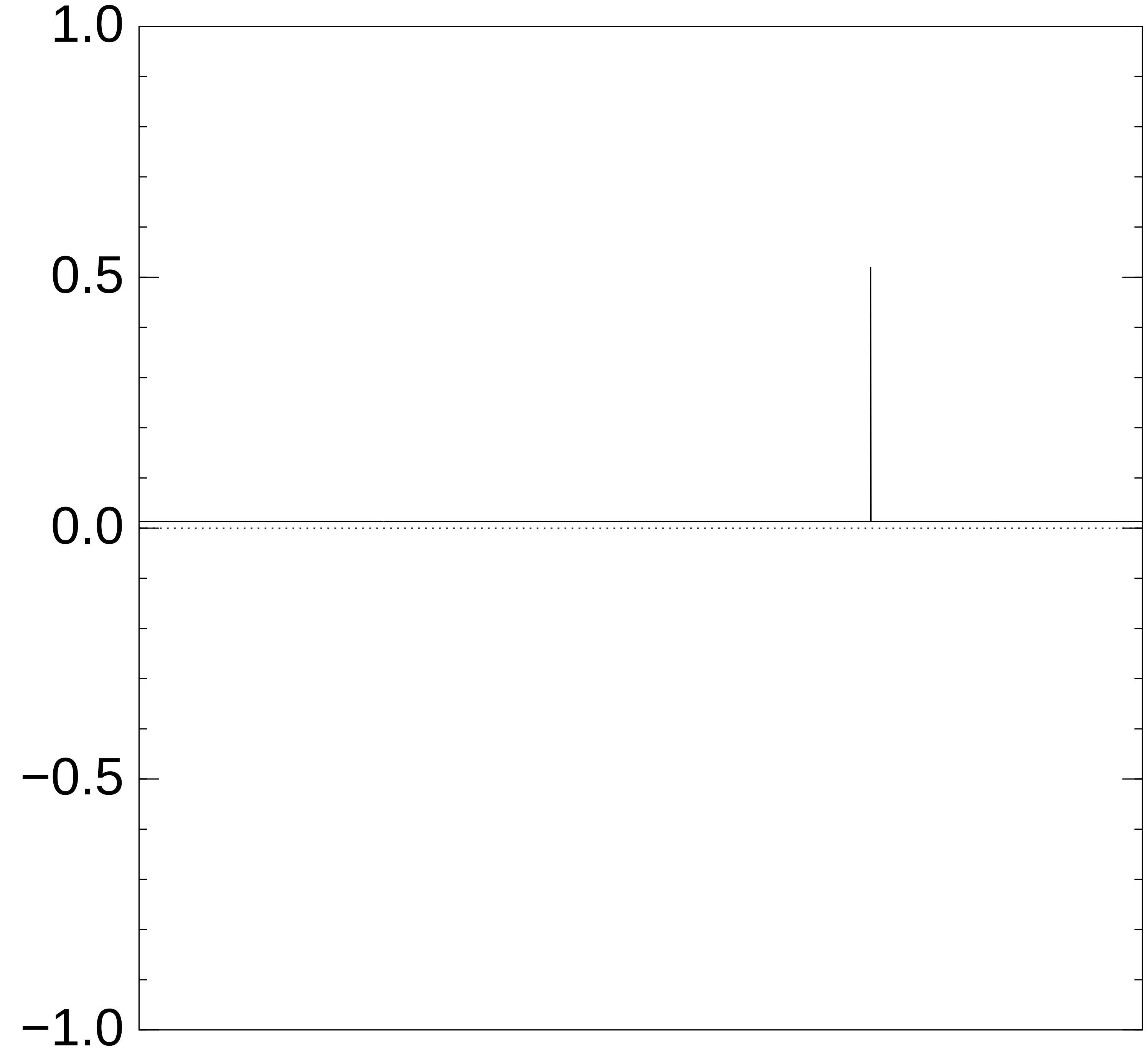
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $17 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

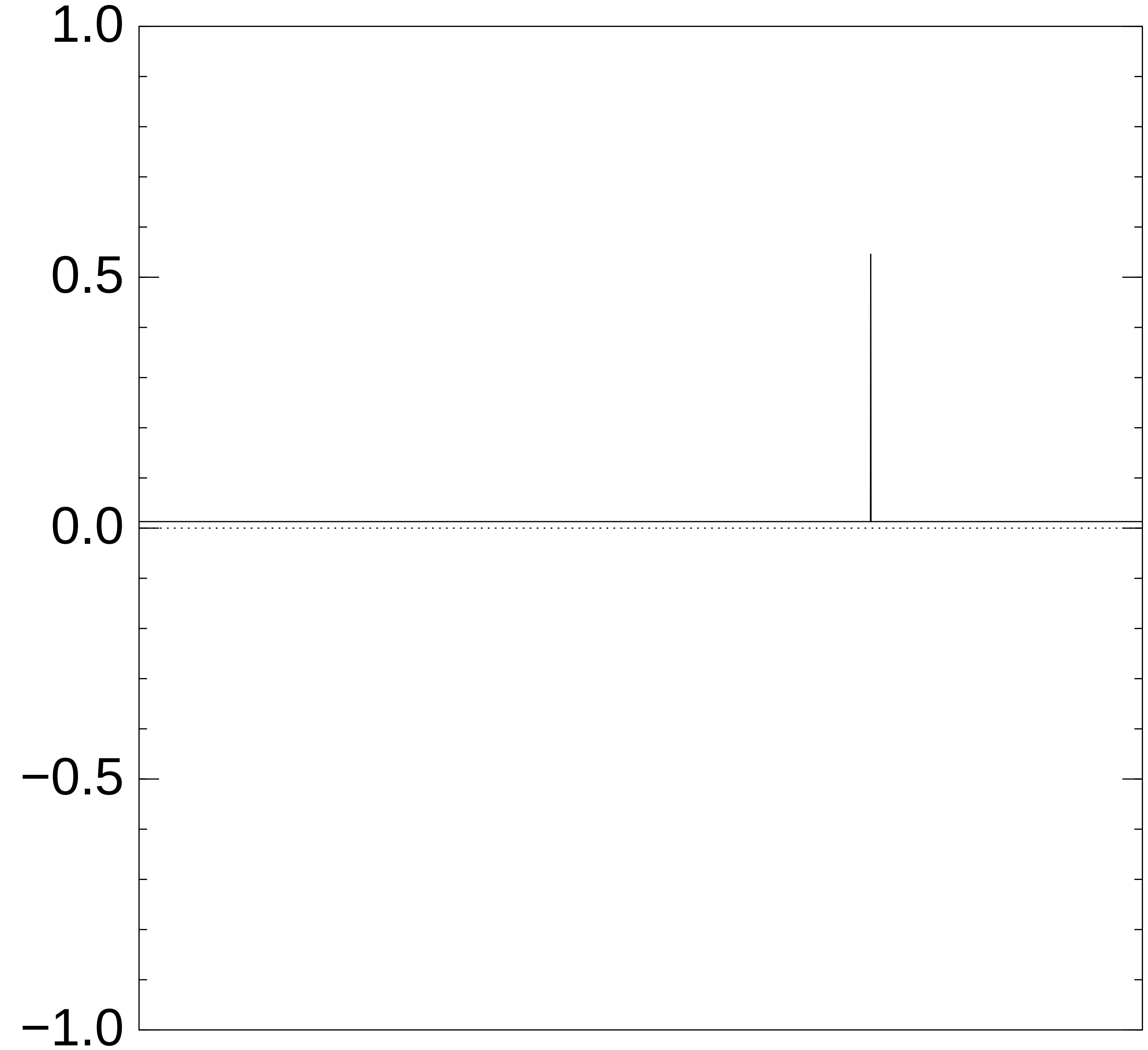
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $18 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

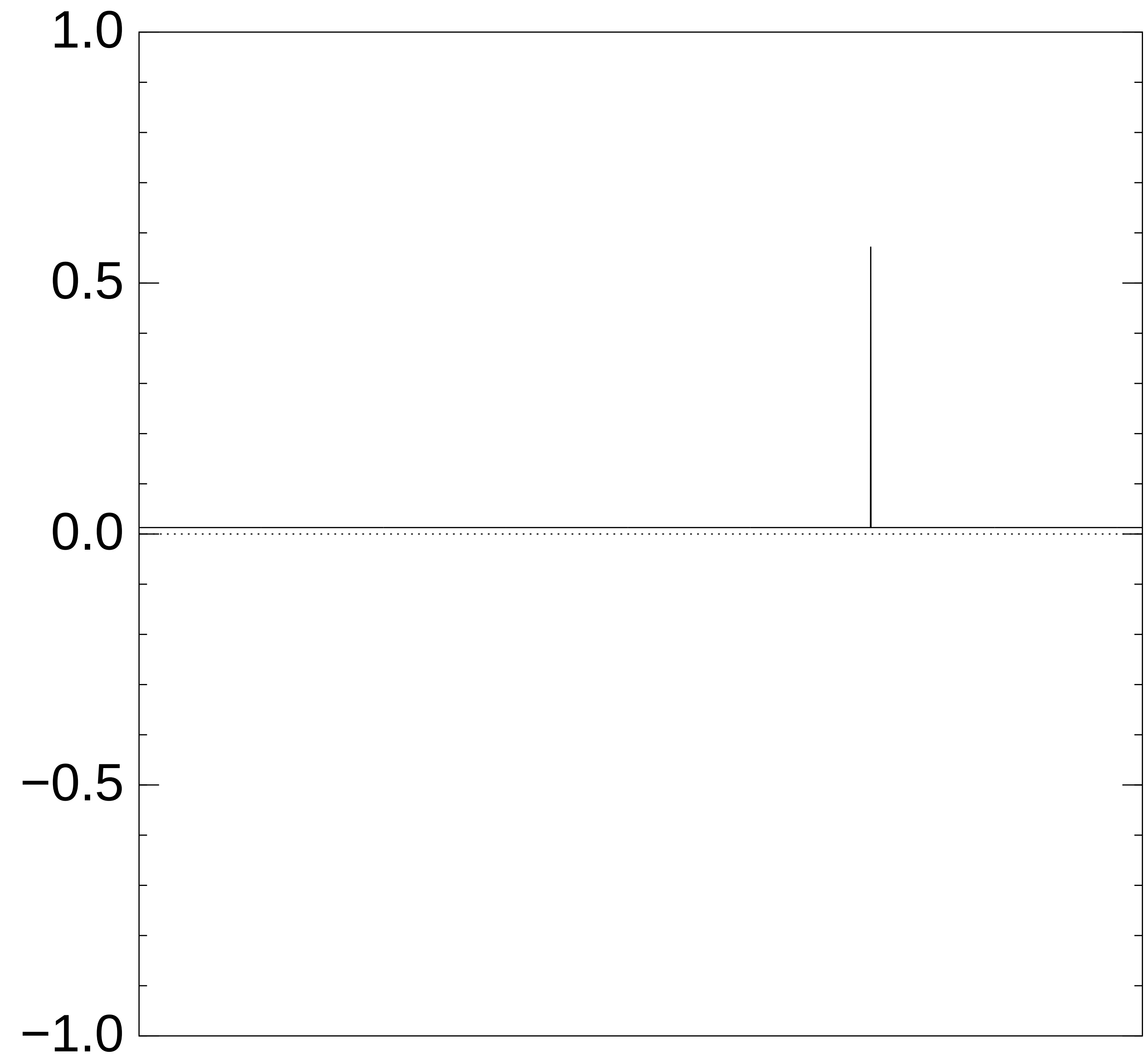
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $19 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

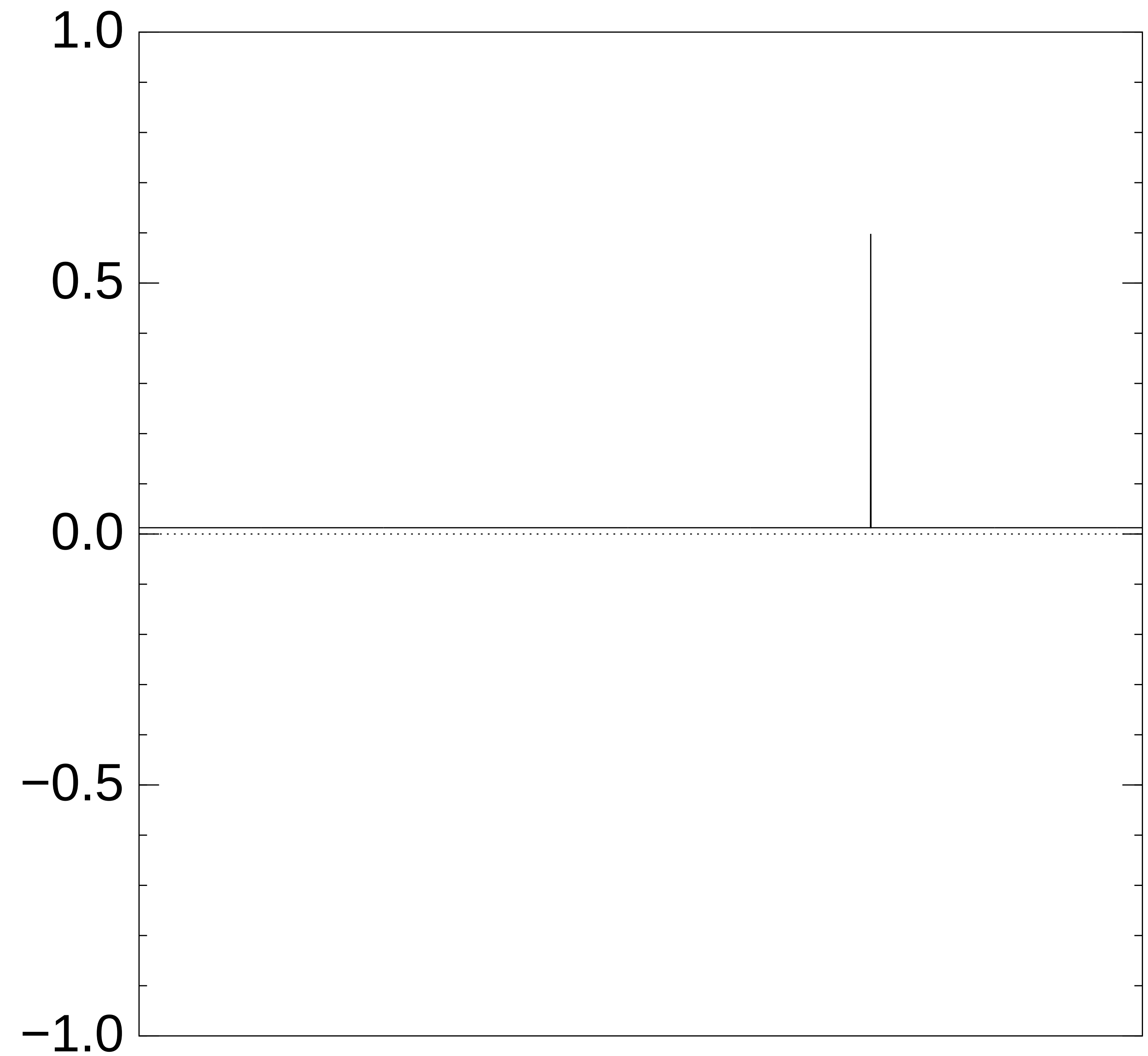
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $20 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

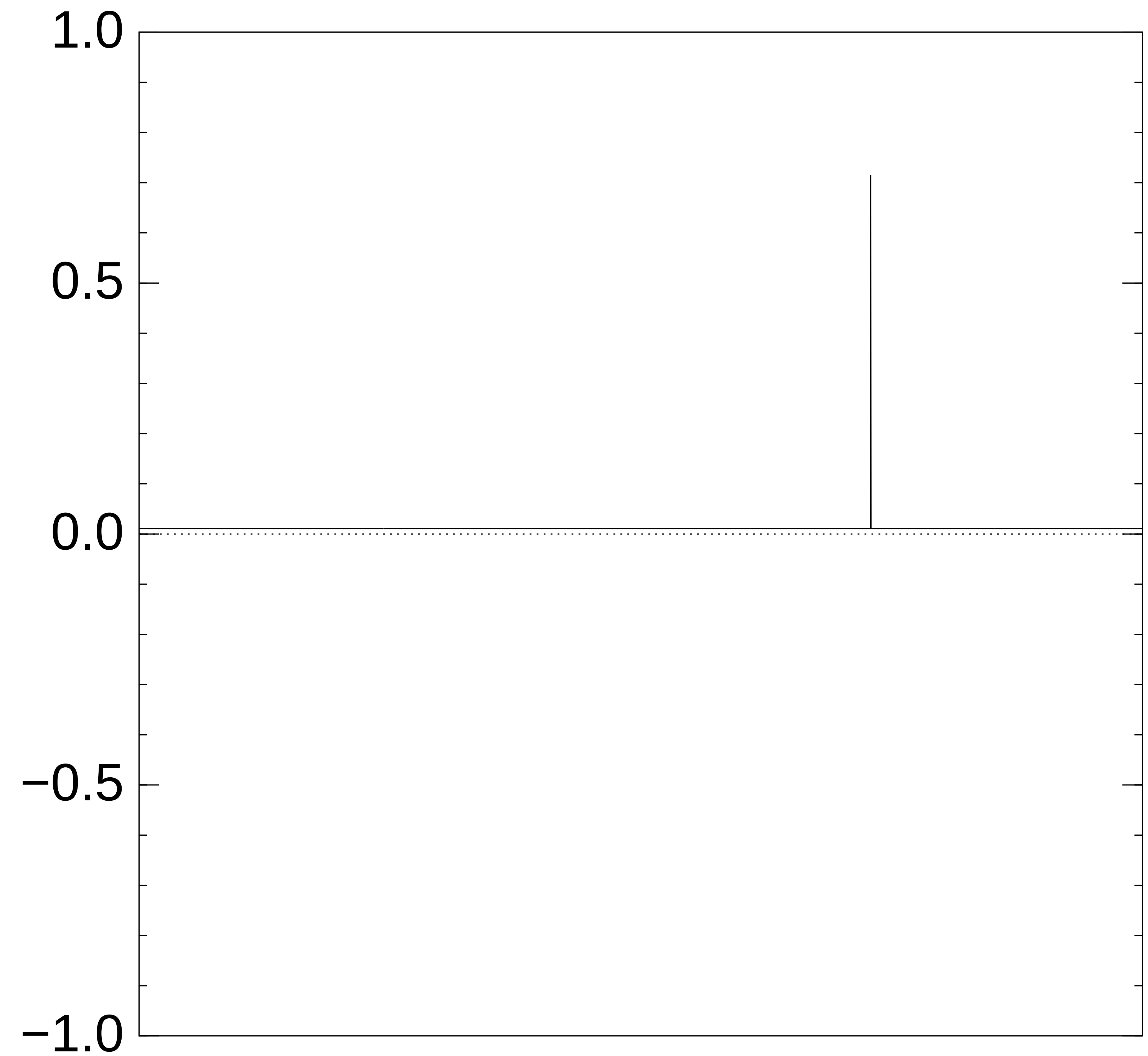
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $25 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

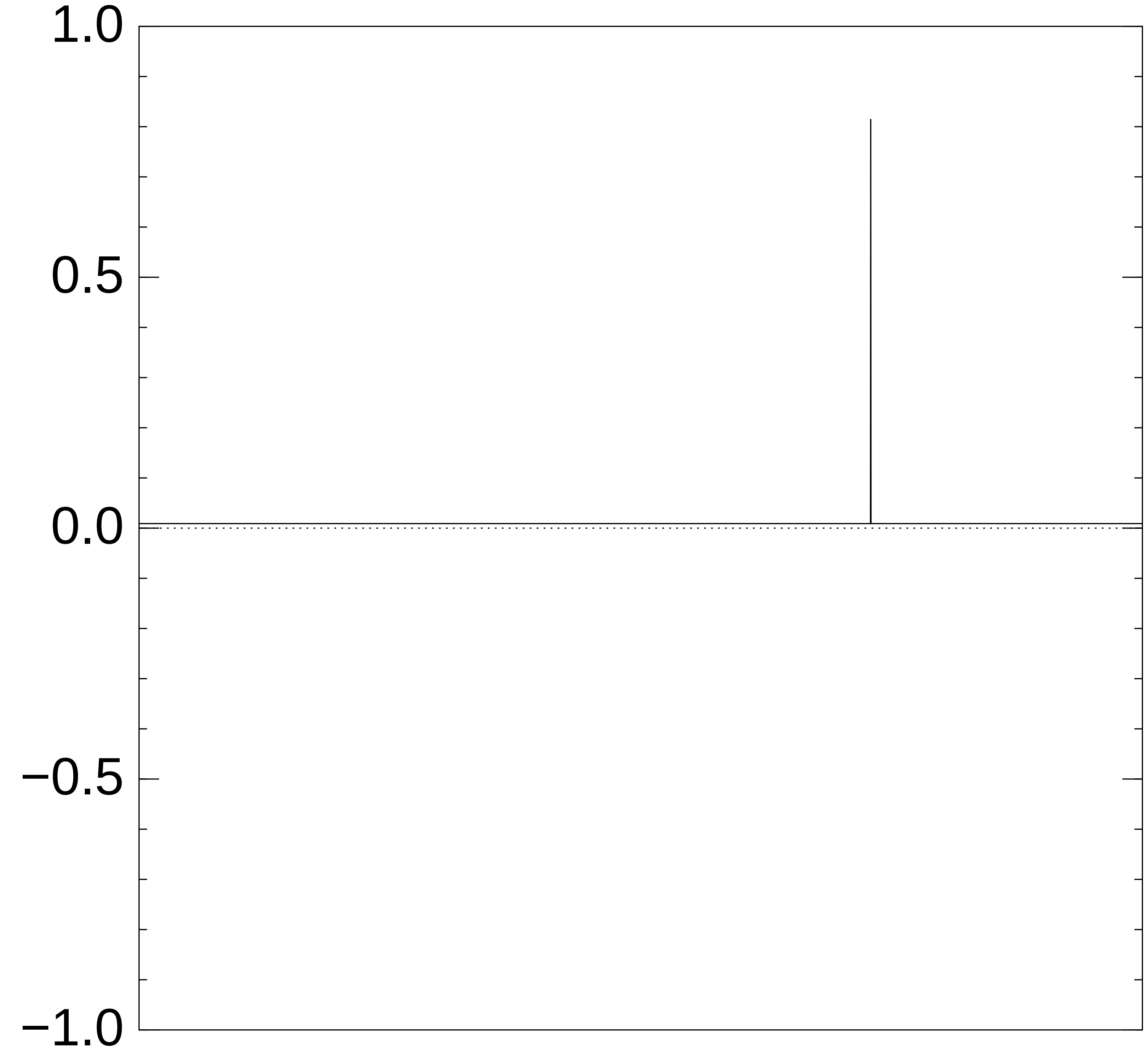
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $30 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

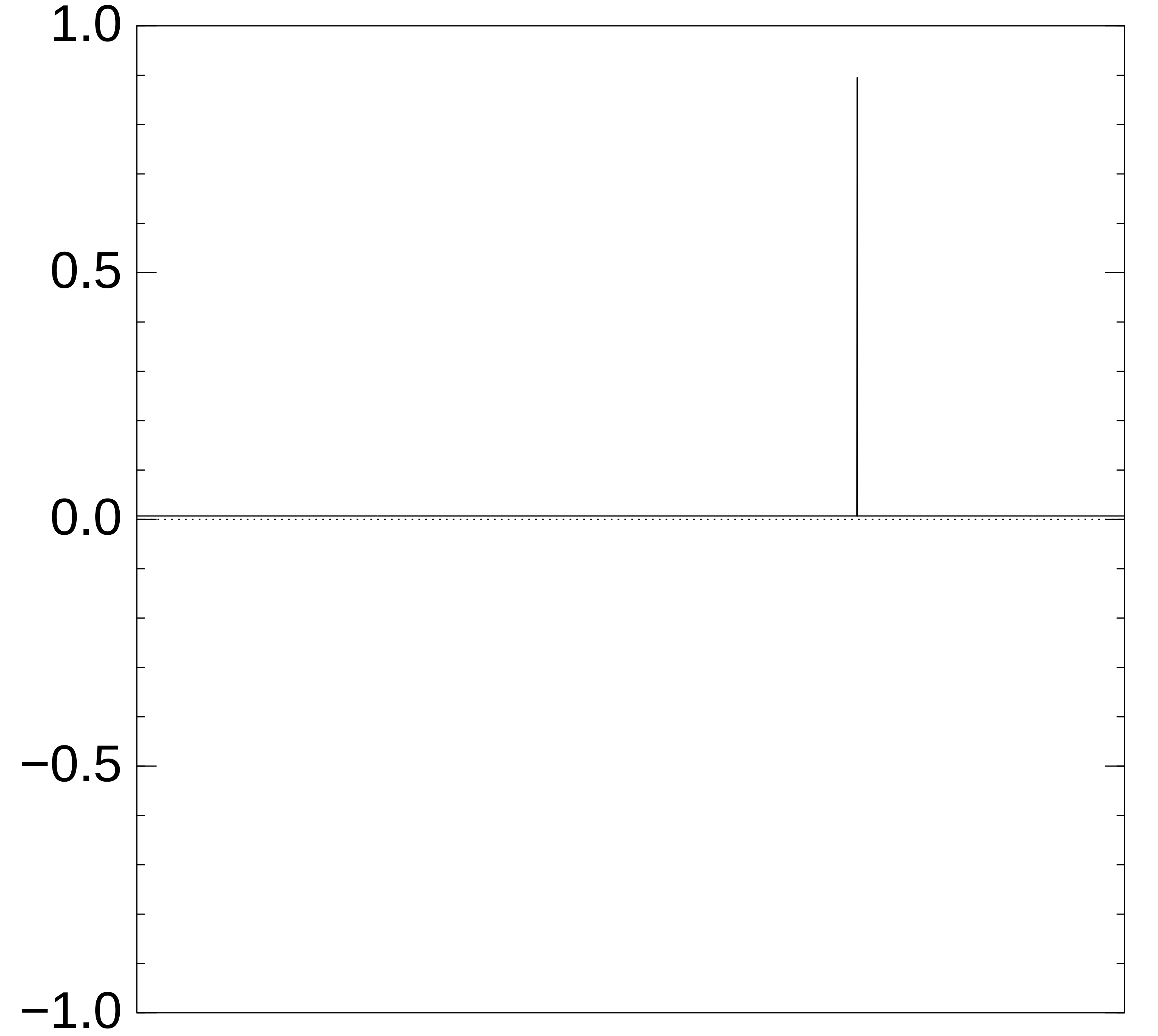
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $35 \times$ (Step 1 + Step 2):



Good moment to stop, measure.

Start from uniform superposition over all n-bit strings q.

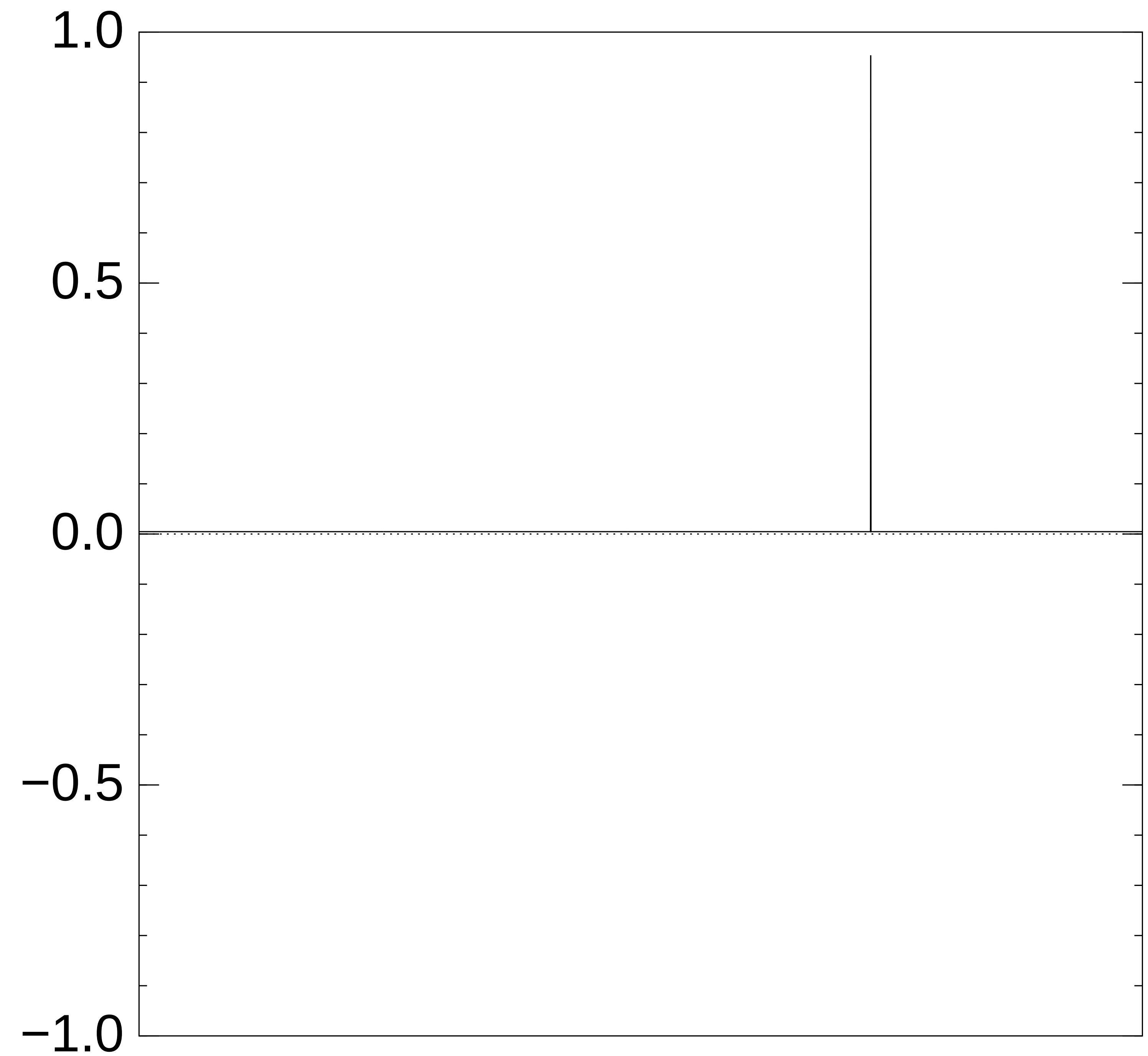
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $40 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

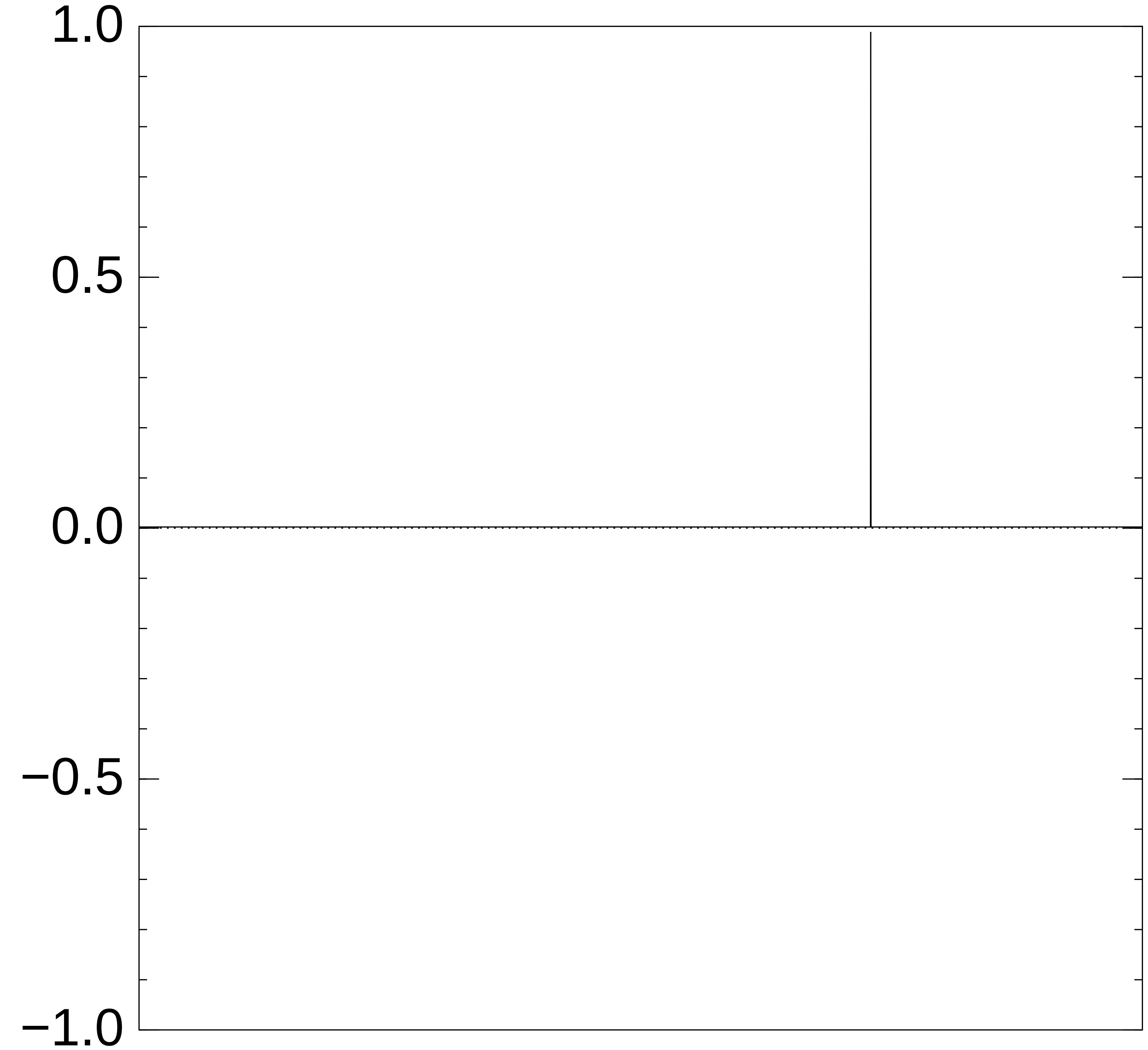
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $45 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

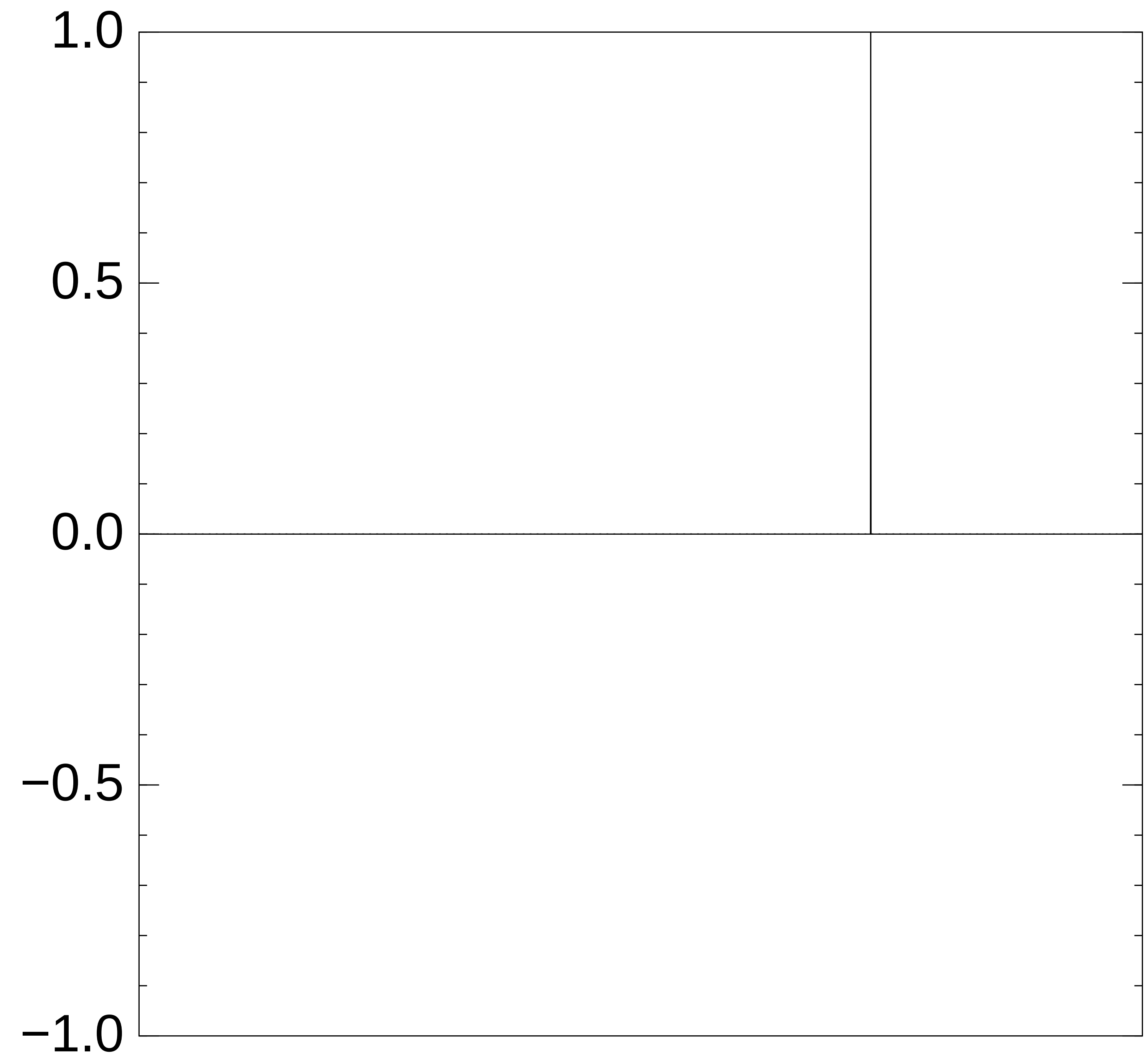
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $50 \times$ (Step 1 + Step 2):



Traditional stopping point.

Start from uniform superposition over all n-bit strings q.

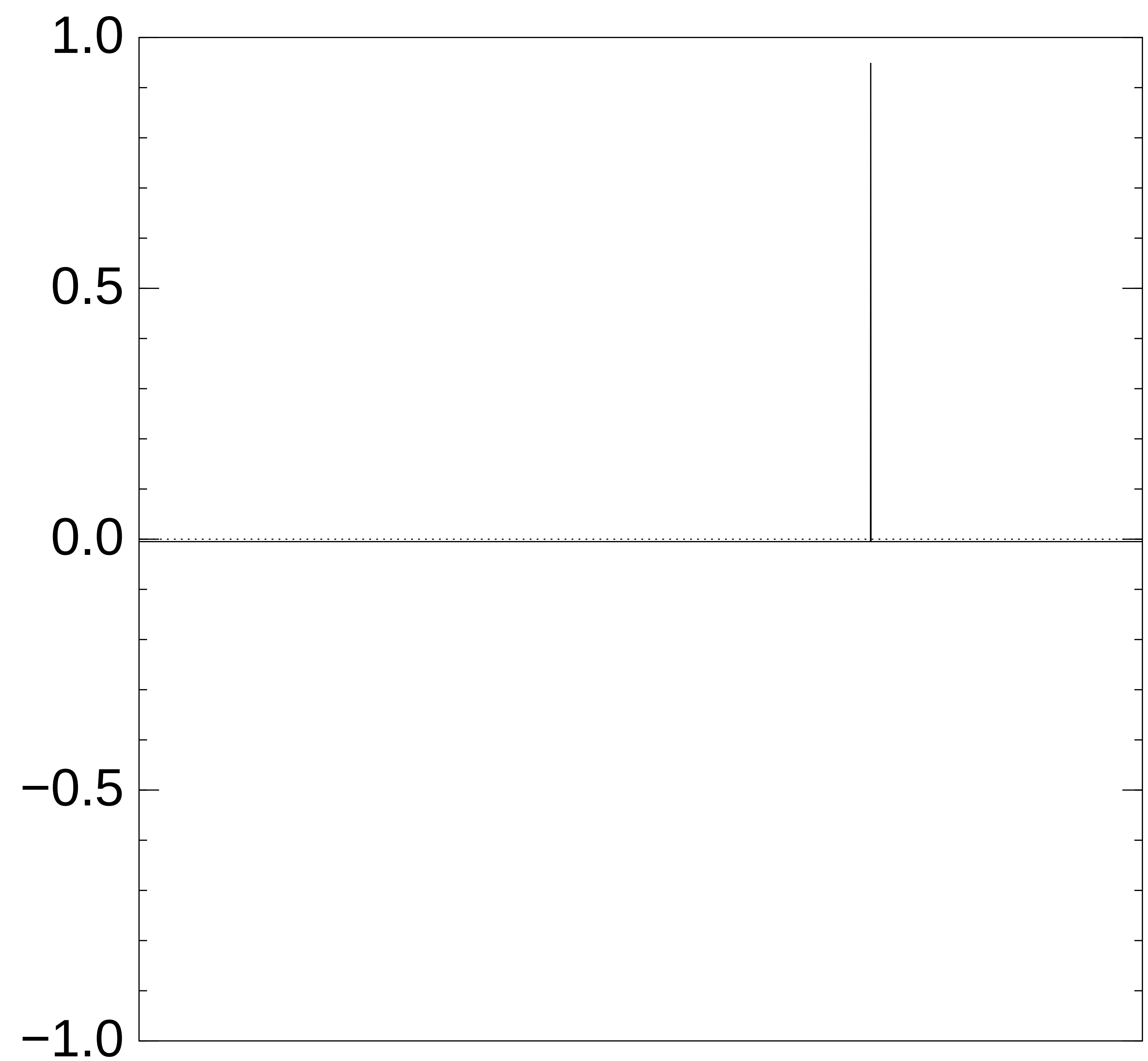
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $60 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

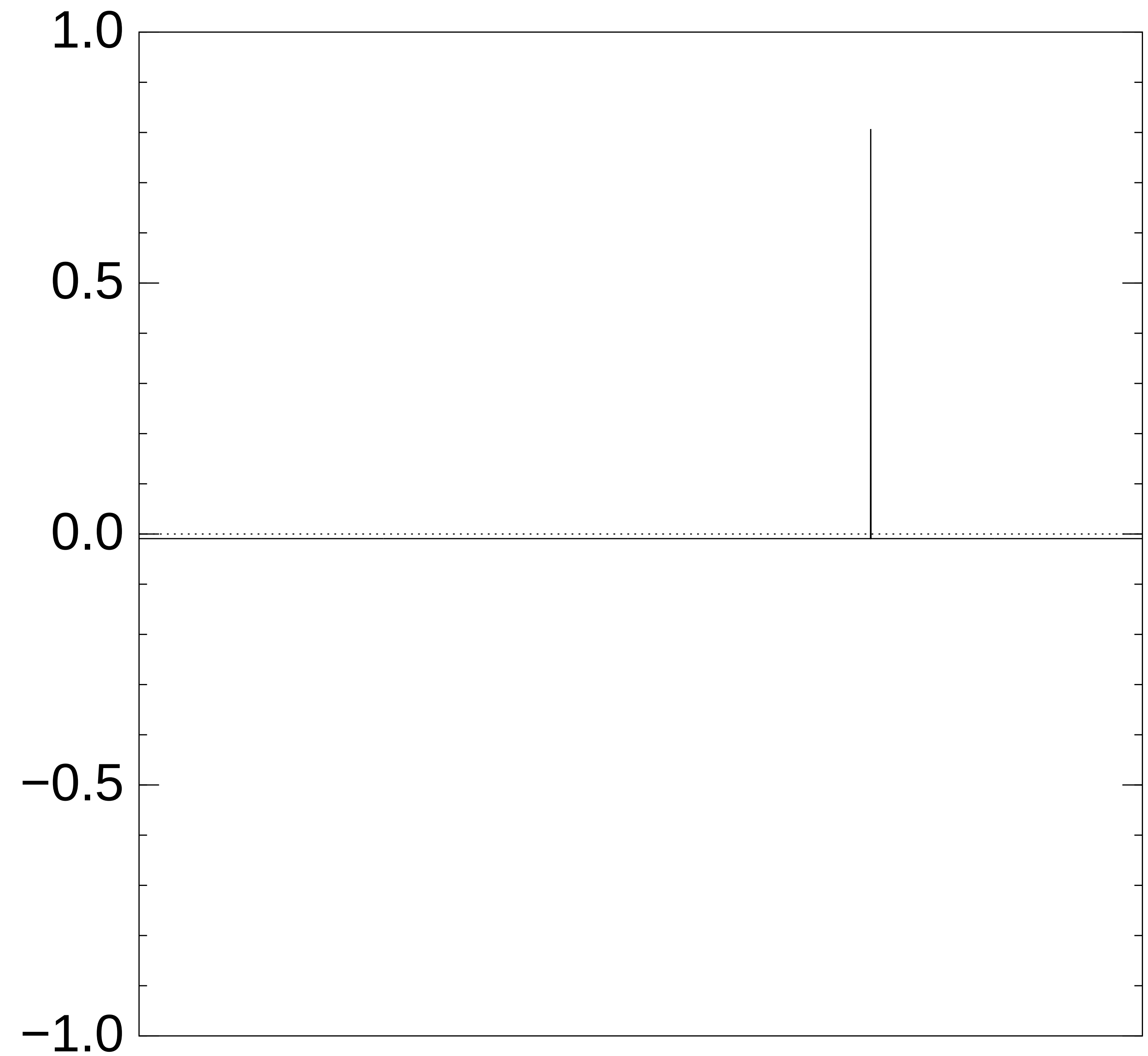
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $70 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

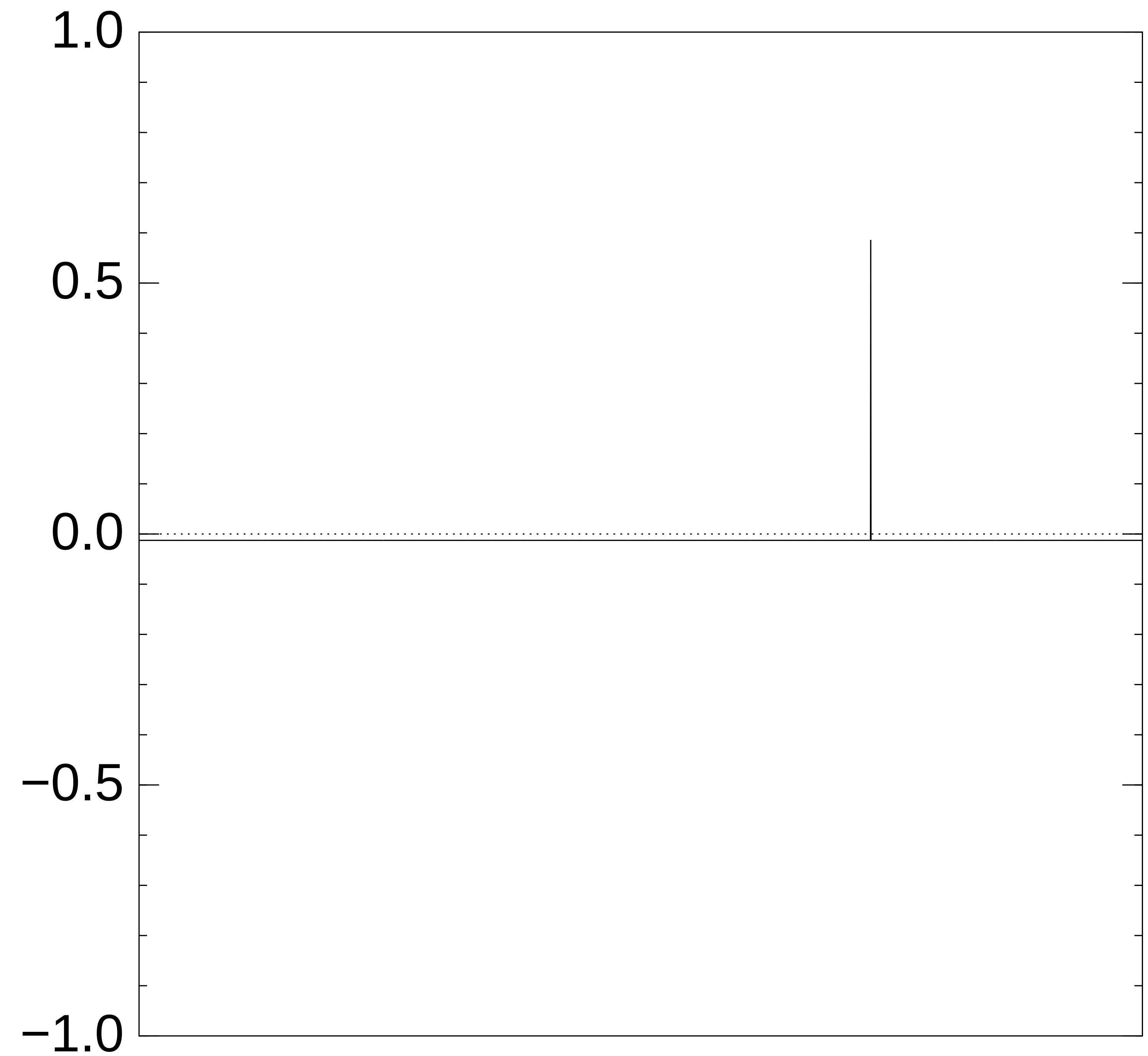
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $80 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

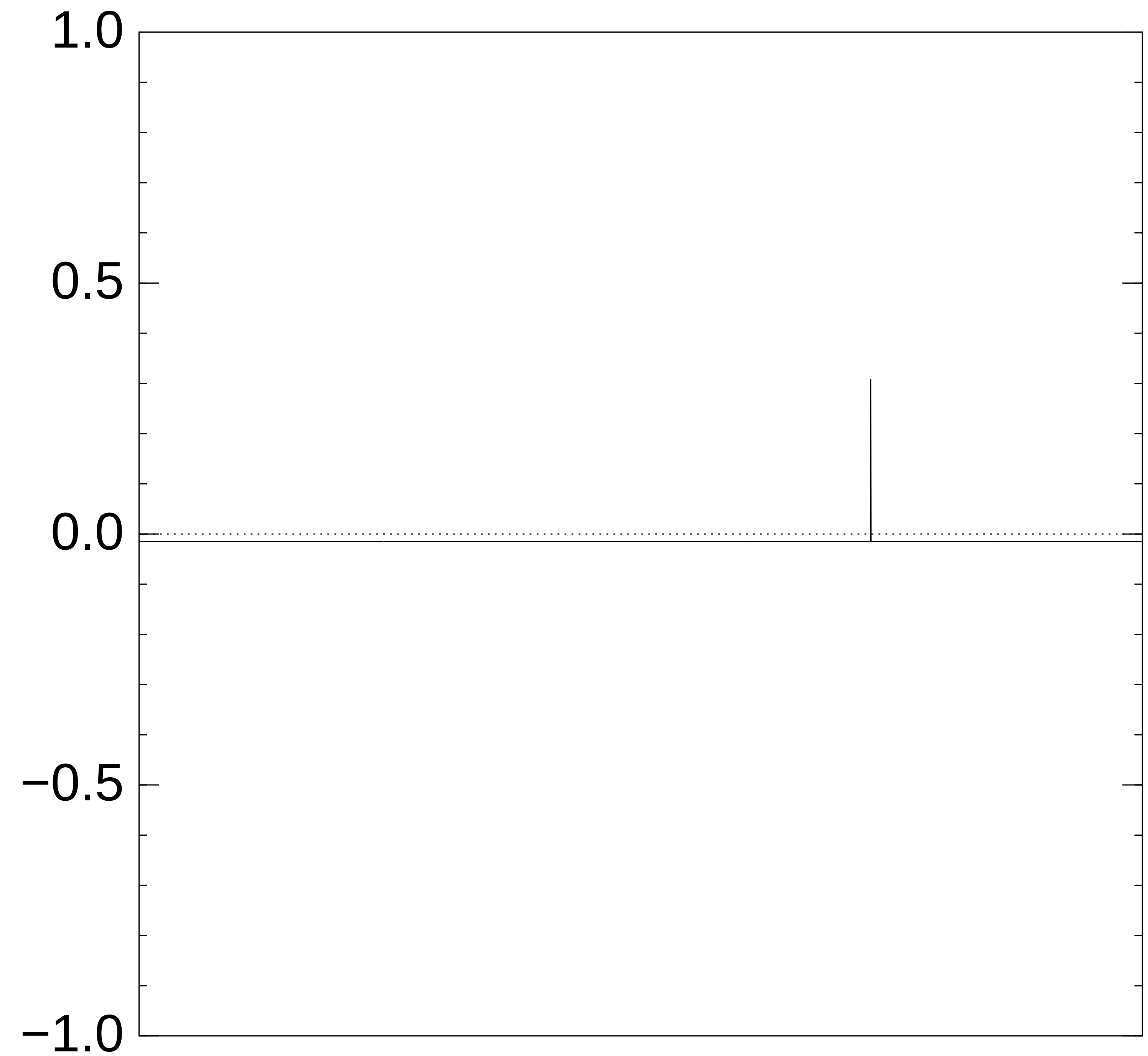
Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
Negate a around its average.
This is also fast.

Repeat Step 1 + Step 2 about $0.58 \cdot 2^{0.5n}$ times.

Measure the n qubits.
With high probability this finds s.

Normalized graph of $q \mapsto a_q$ for an example with $n = 12$ after $90 \times$ (Step 1 + Step 2):



Start from uniform superposition over all n-bit strings q.

Step 1: Set $a \leftarrow b$ where
$b_q = -a_q$ if $f(q) = 0$,
$b_q = a_q$ otherwise.
This is fast.

Step 2: "Grover diffusion".
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This is also fast.

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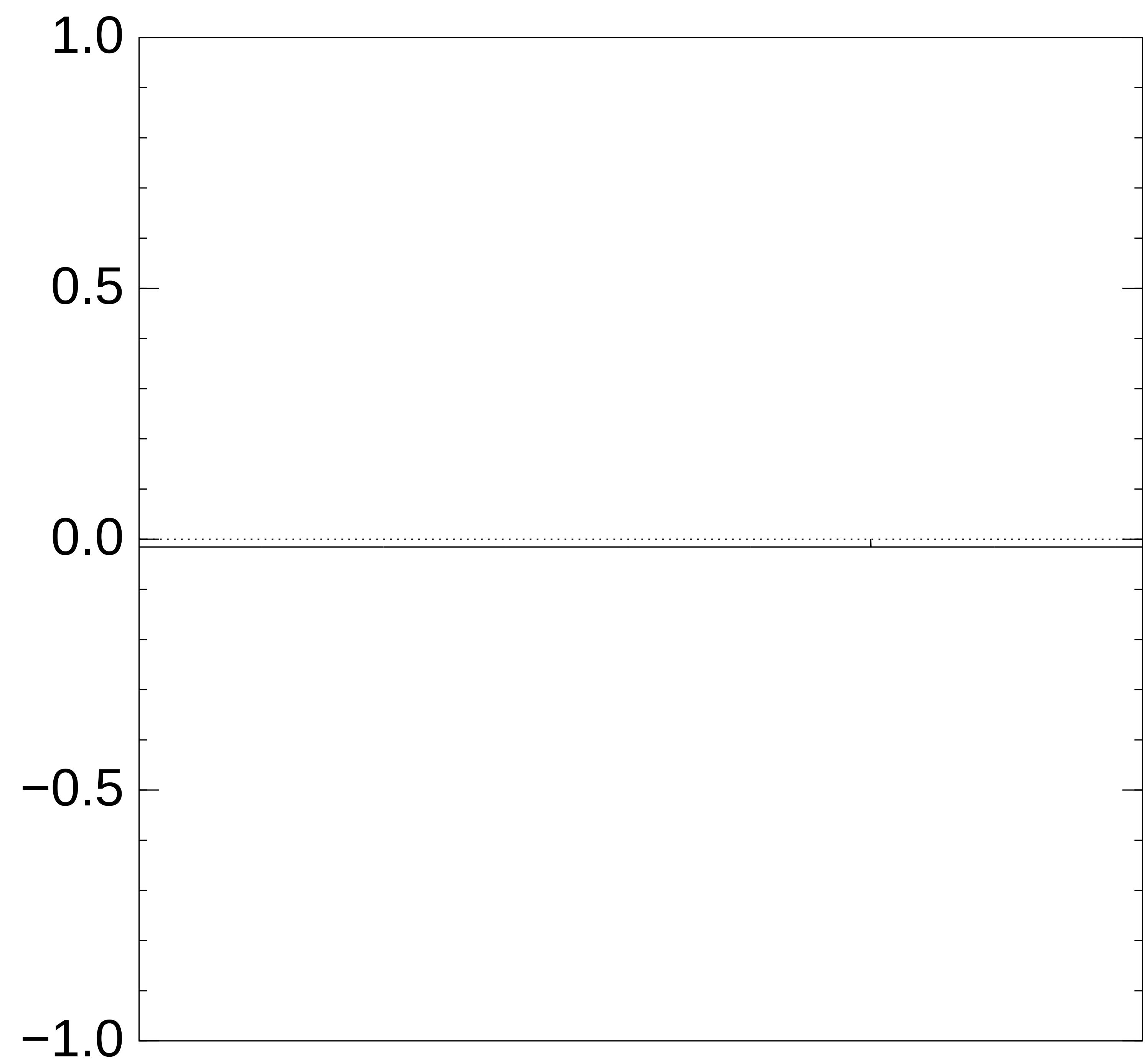

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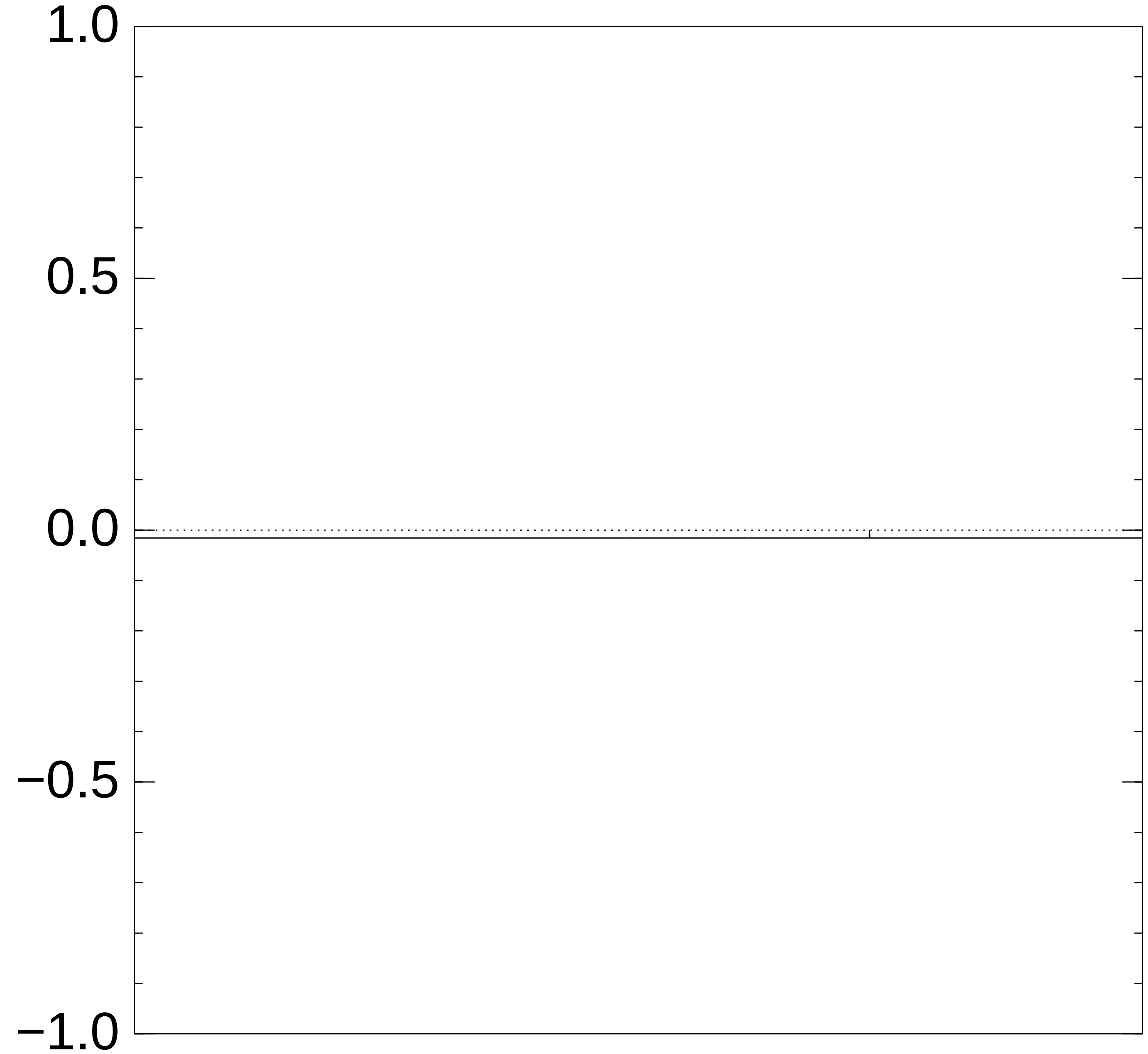

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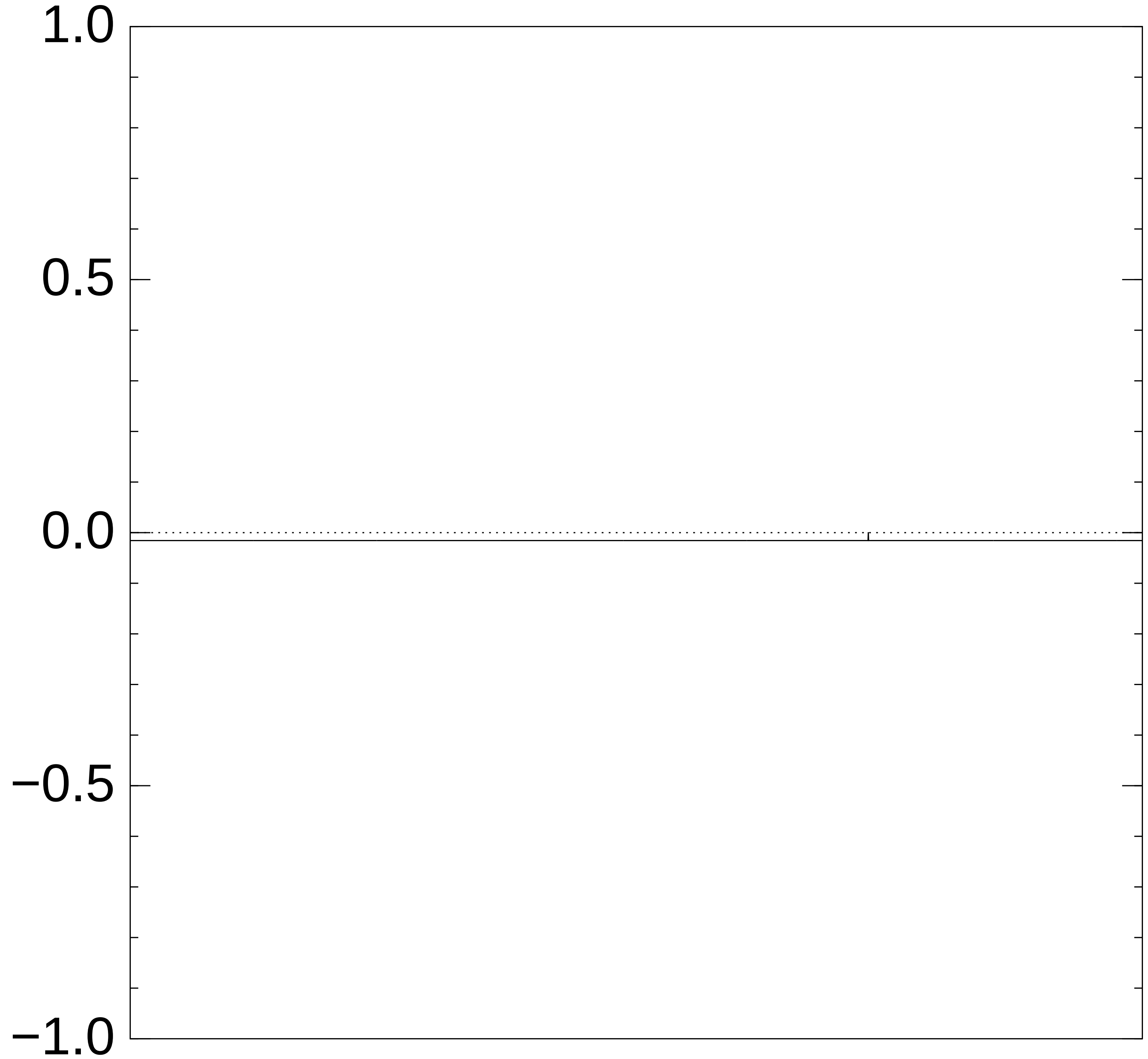

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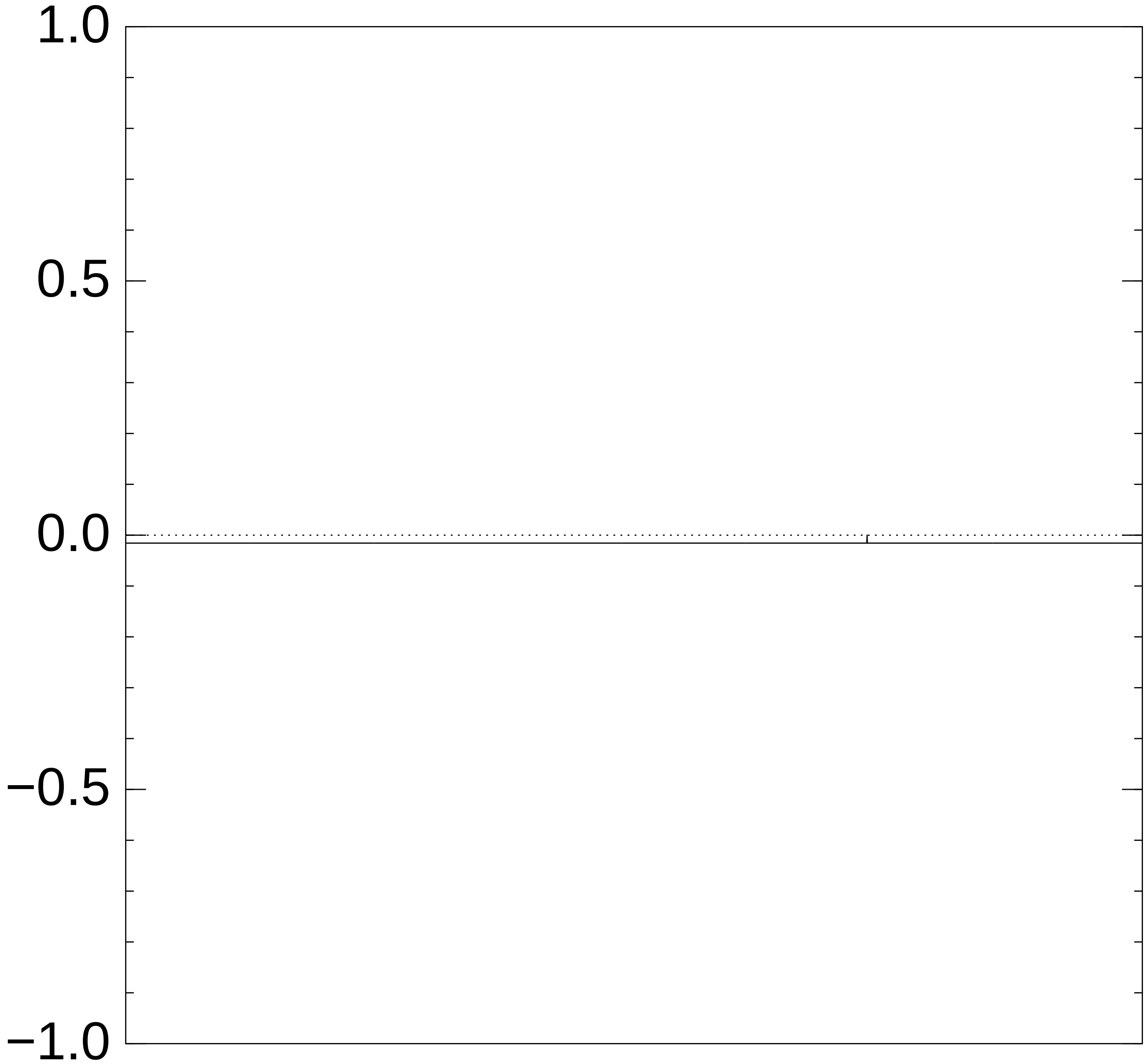

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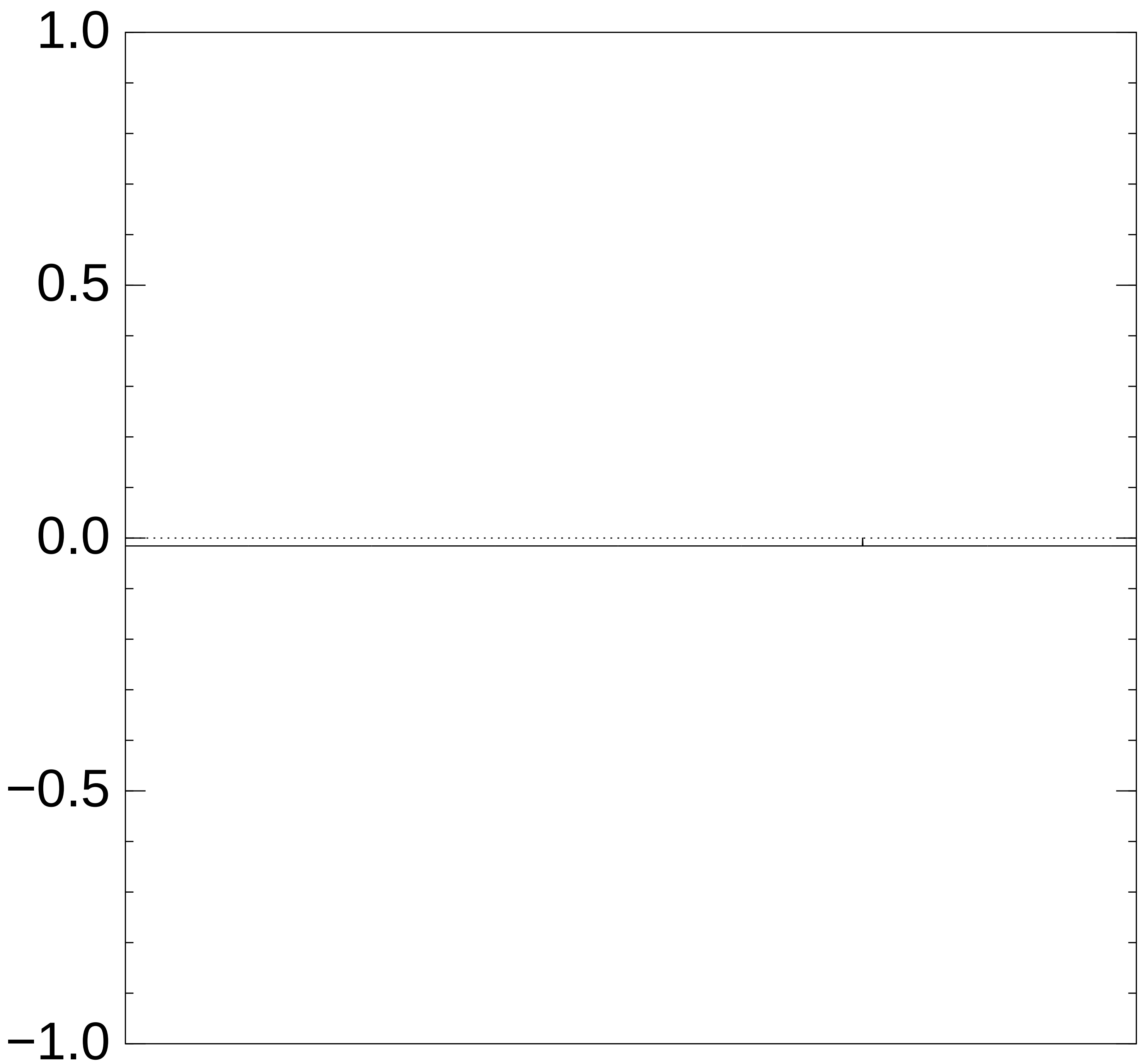

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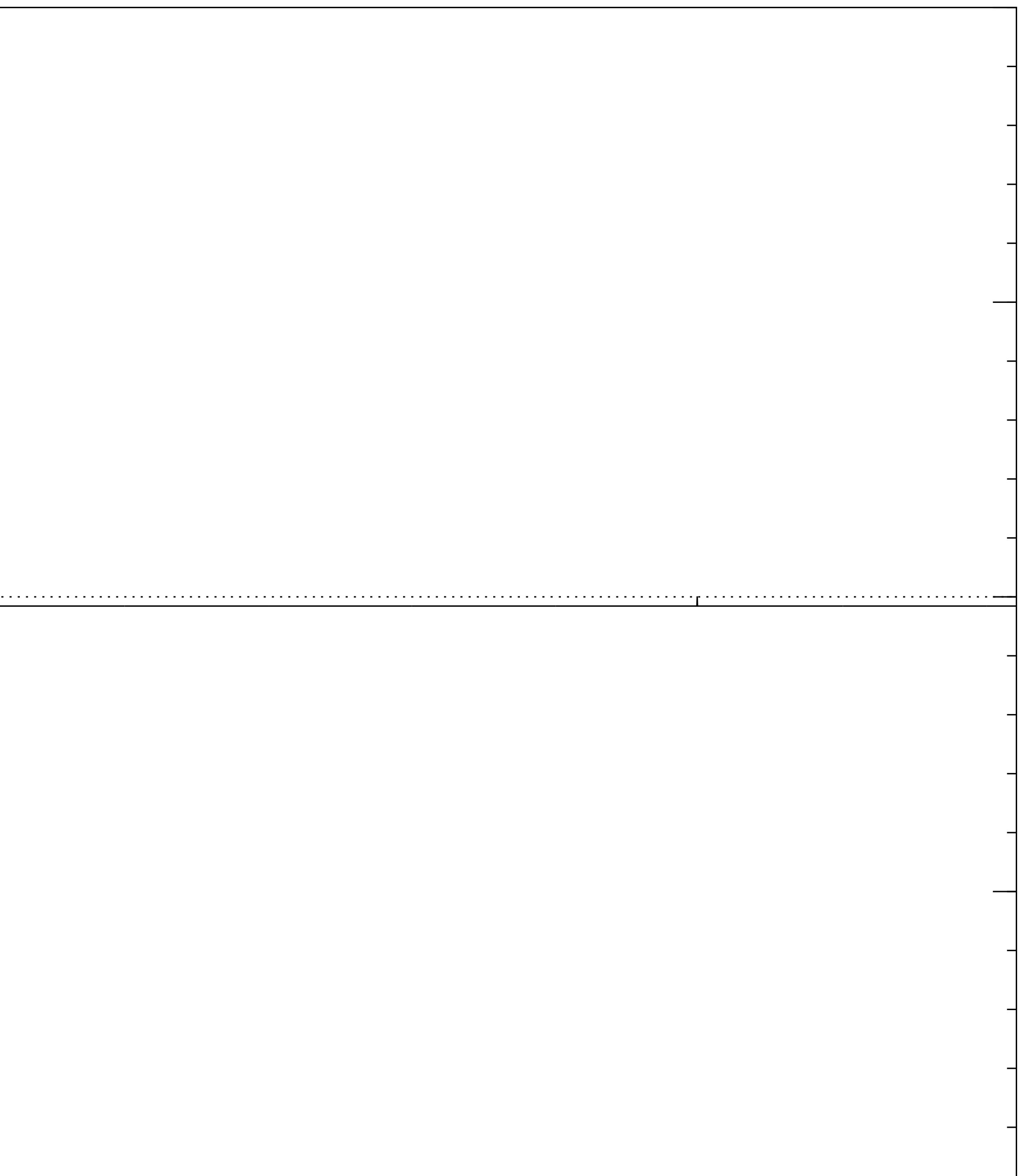

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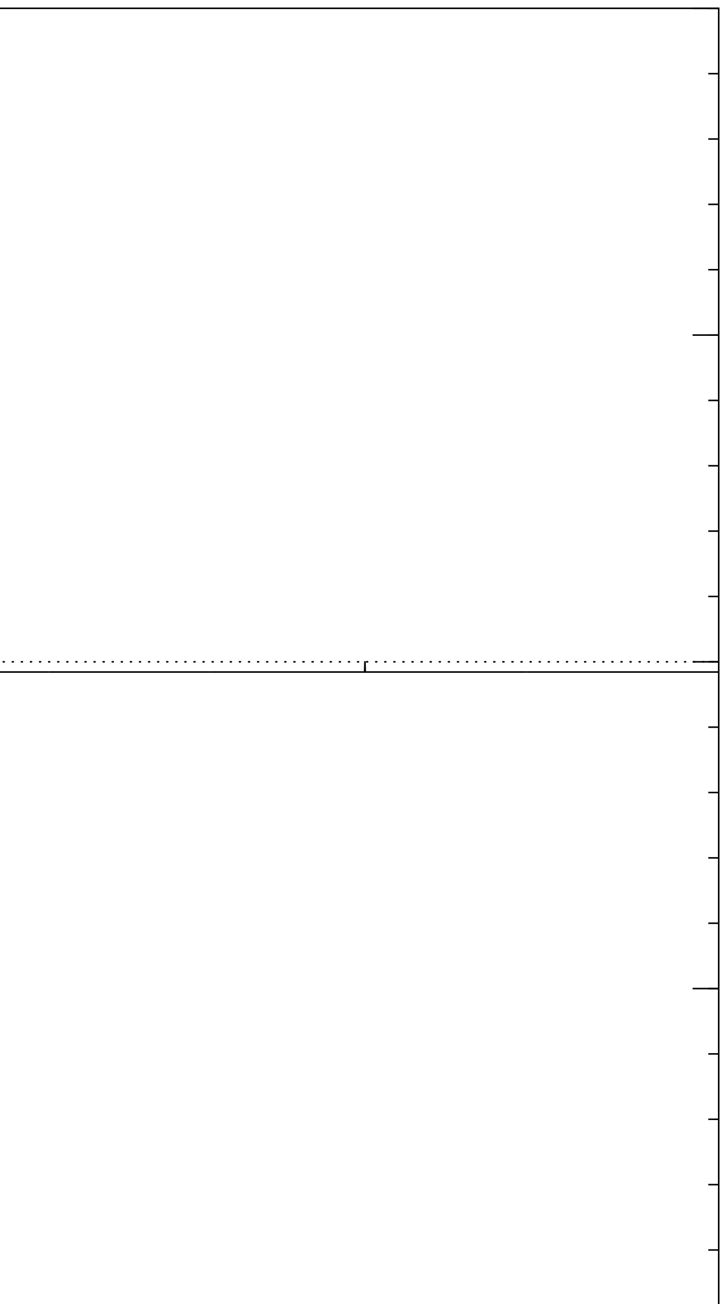

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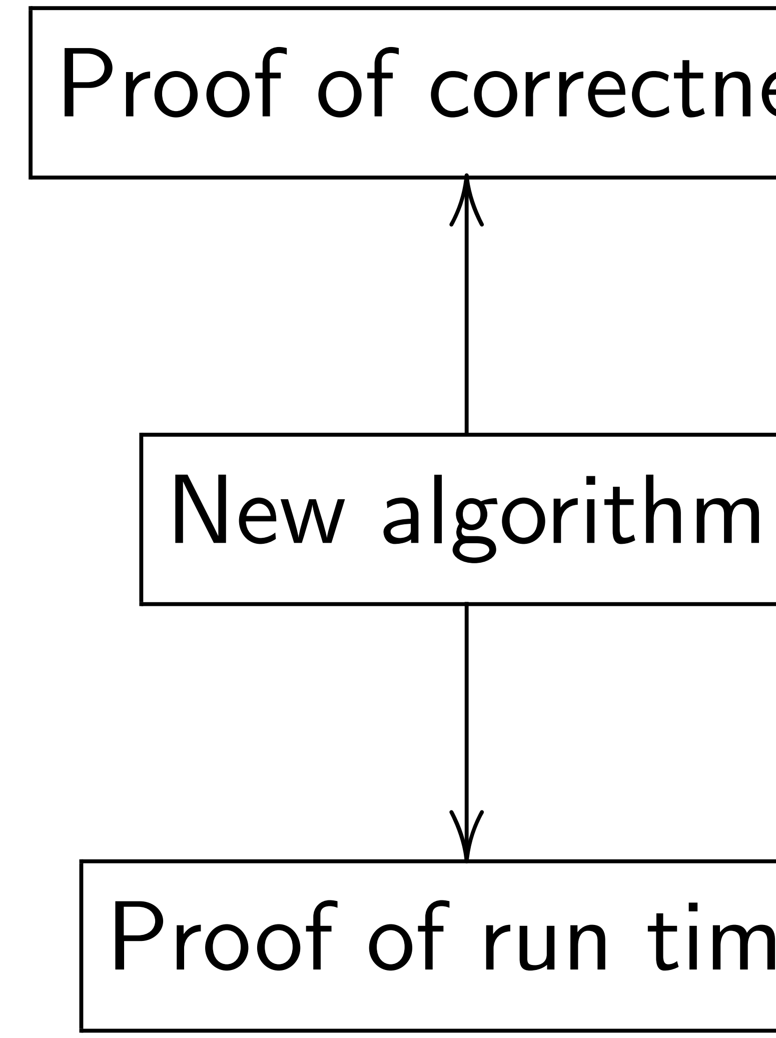

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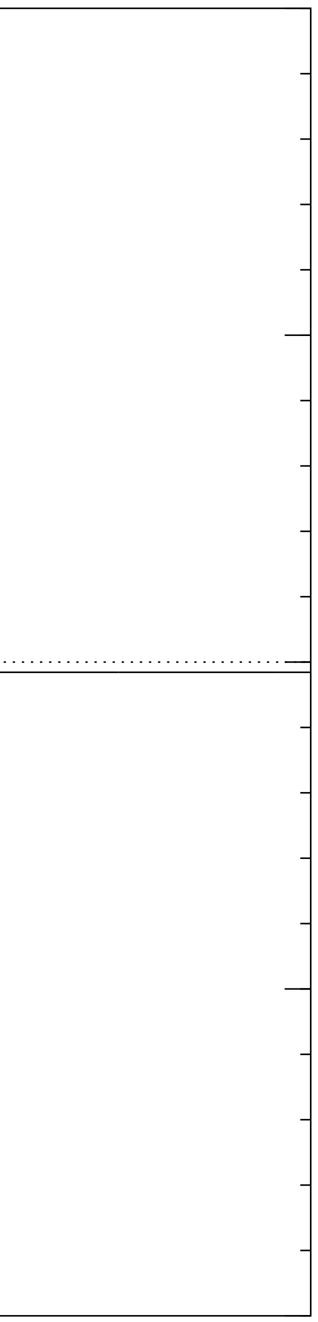

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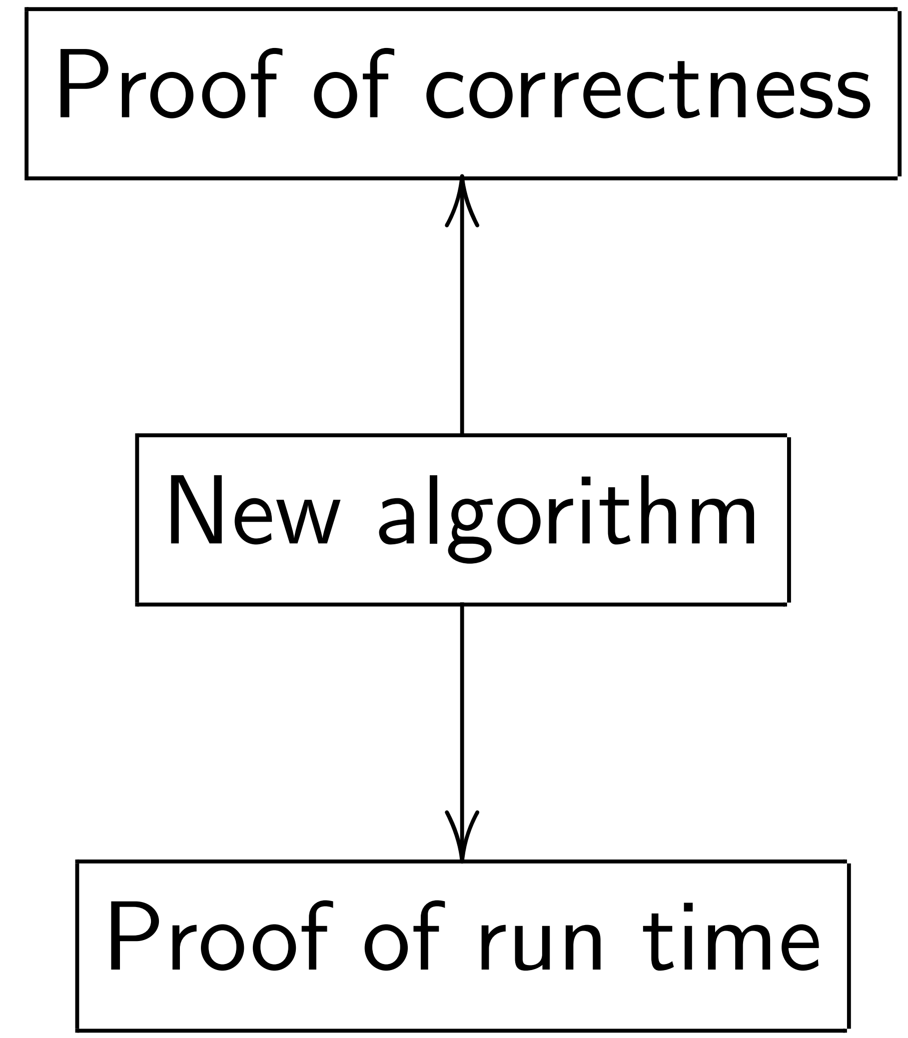

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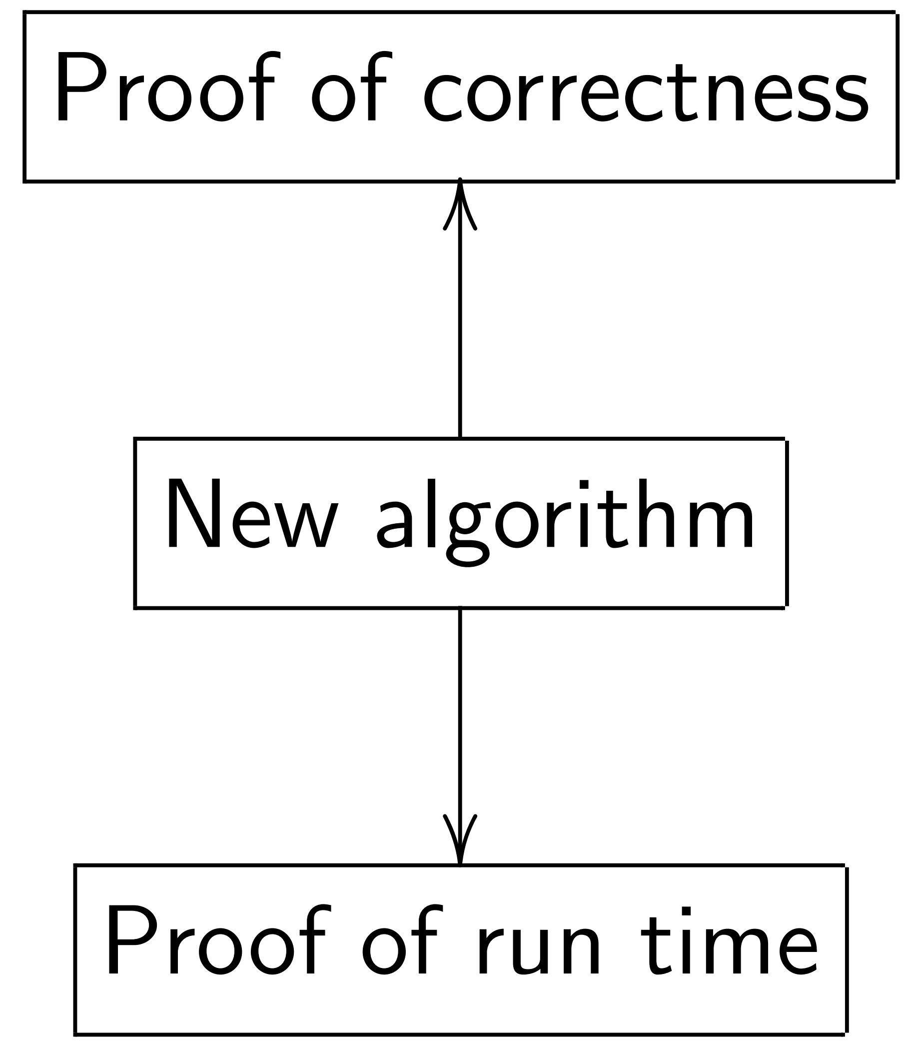

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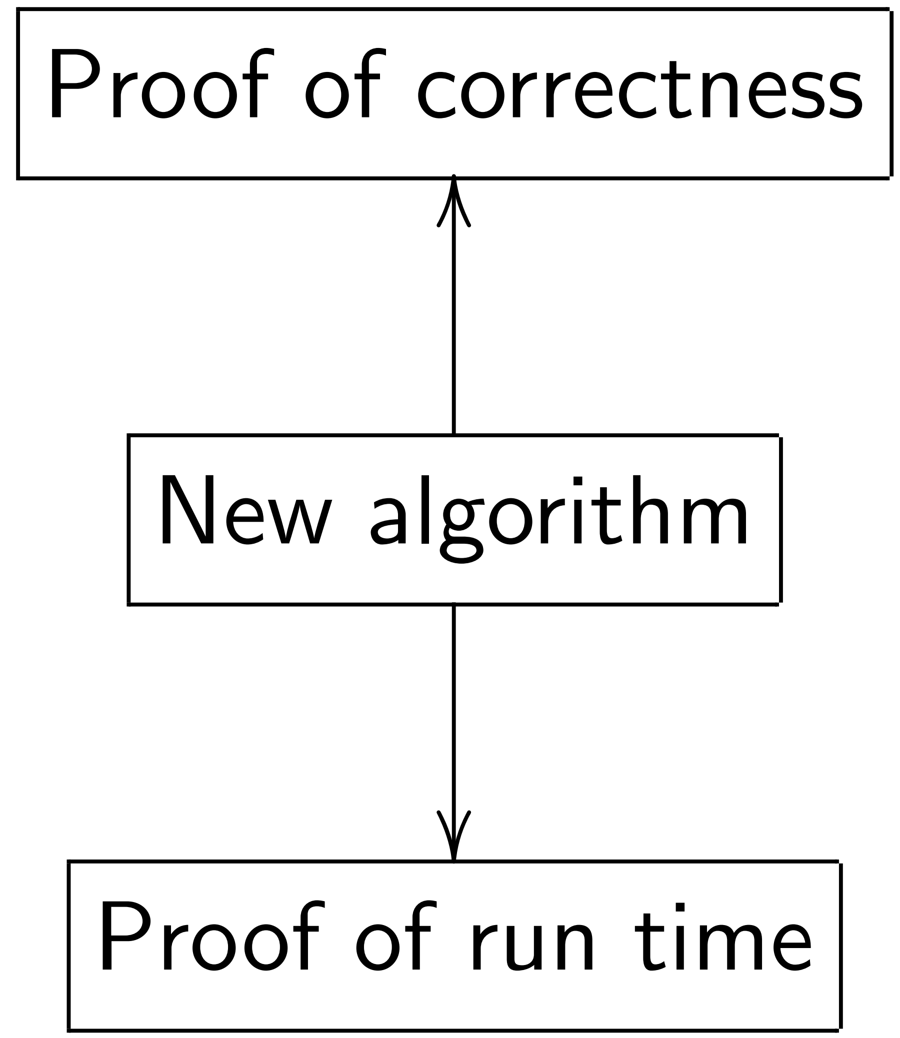

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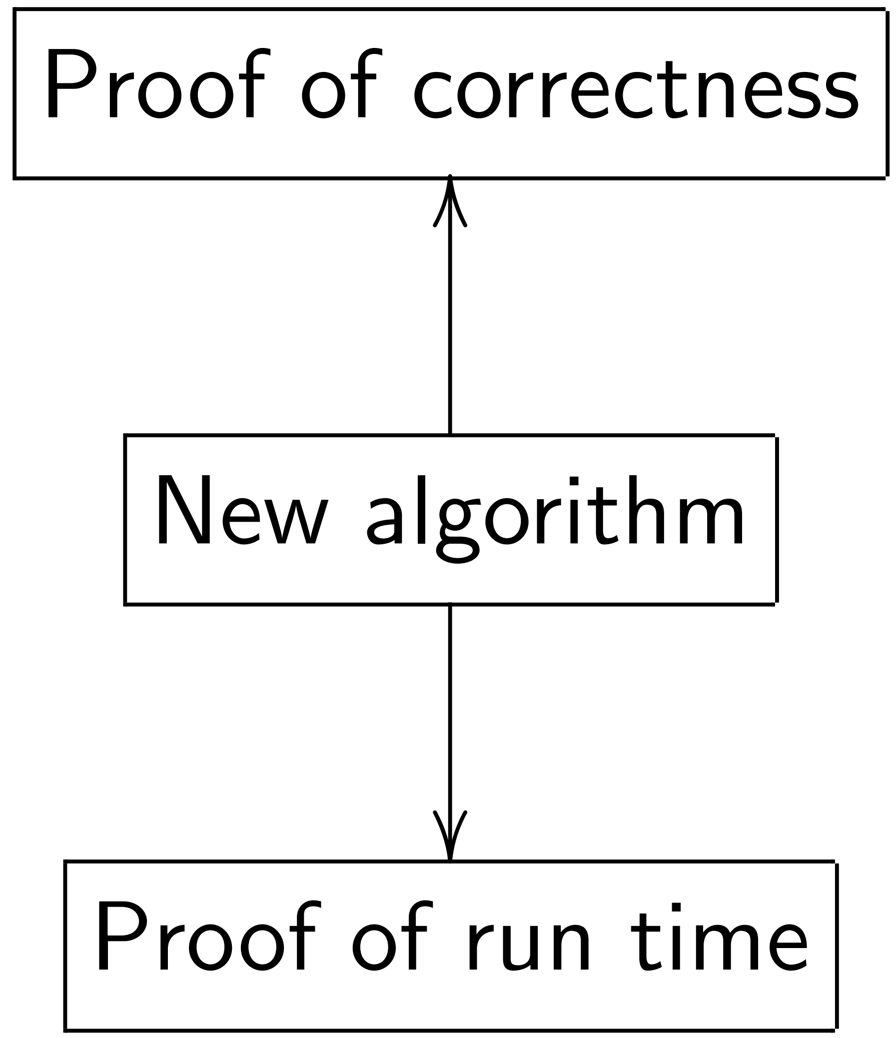

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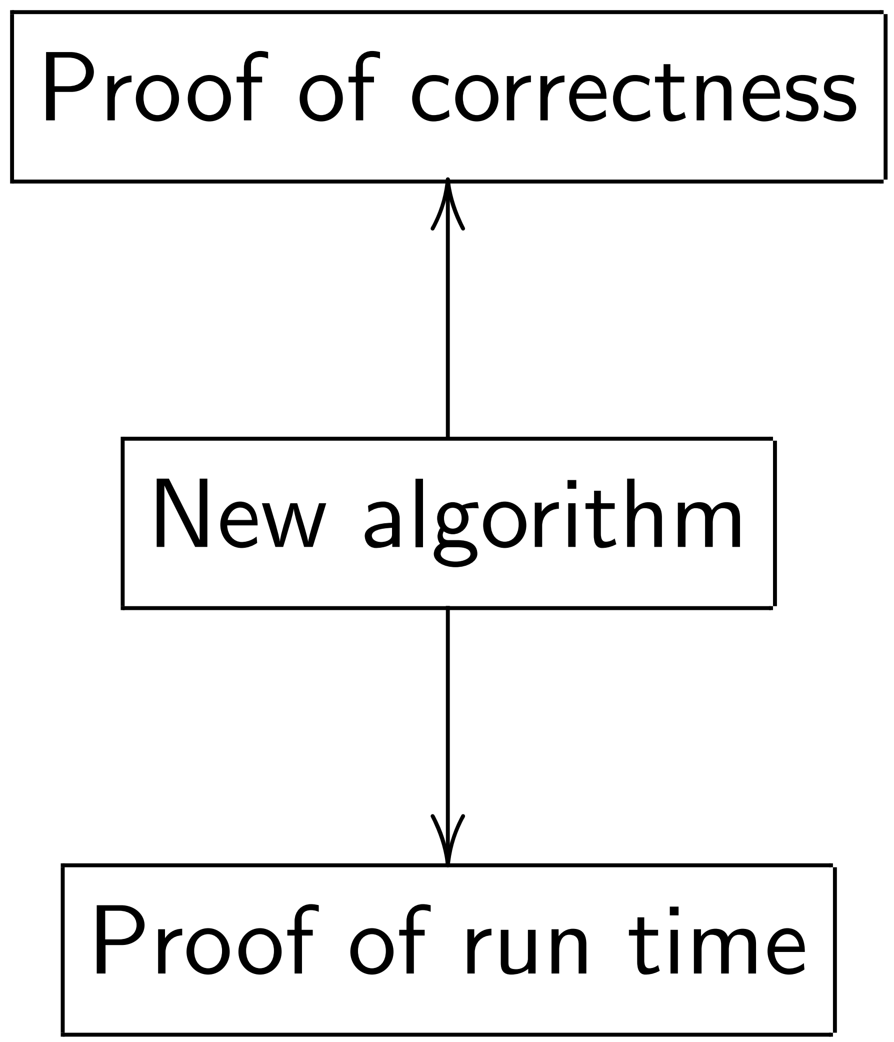

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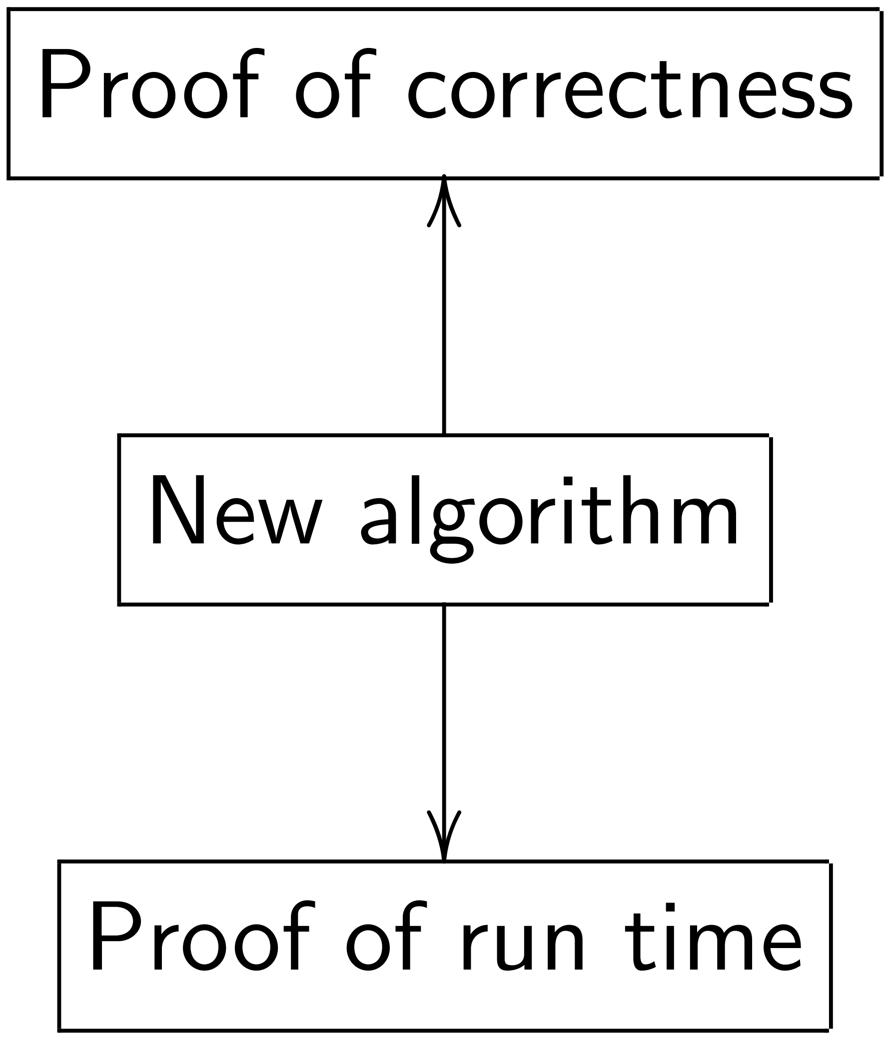

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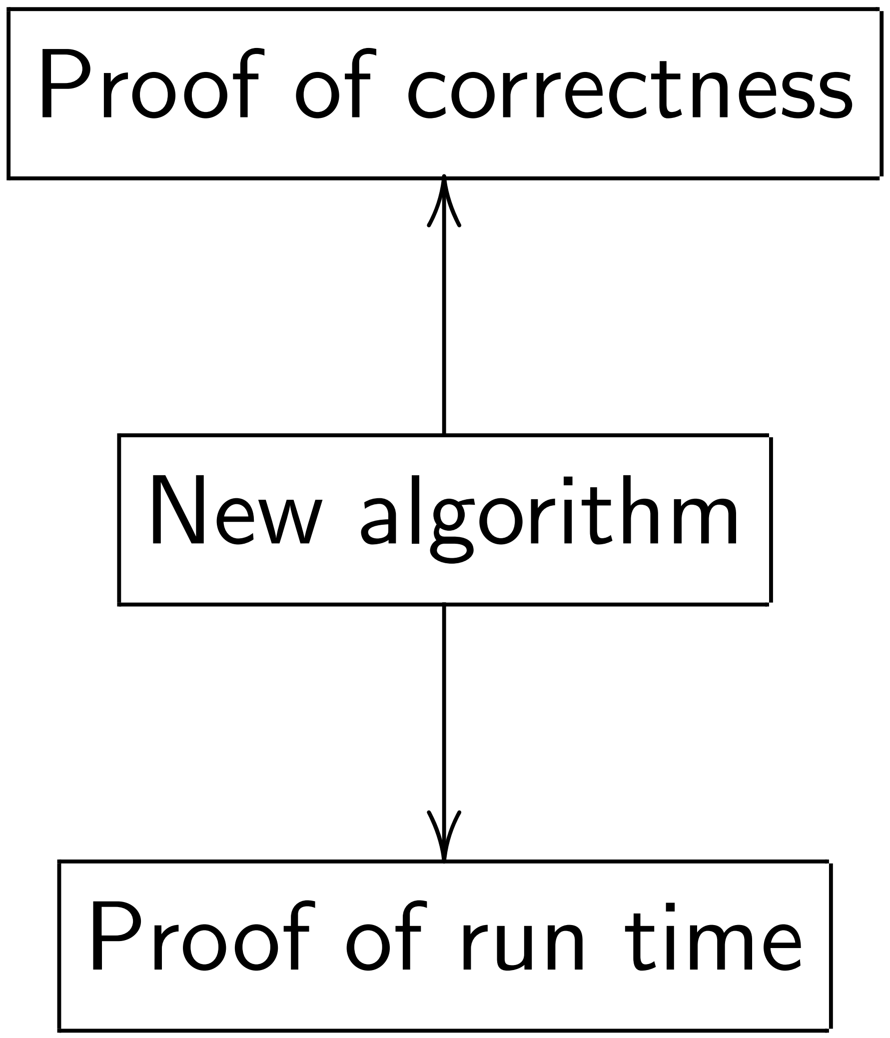

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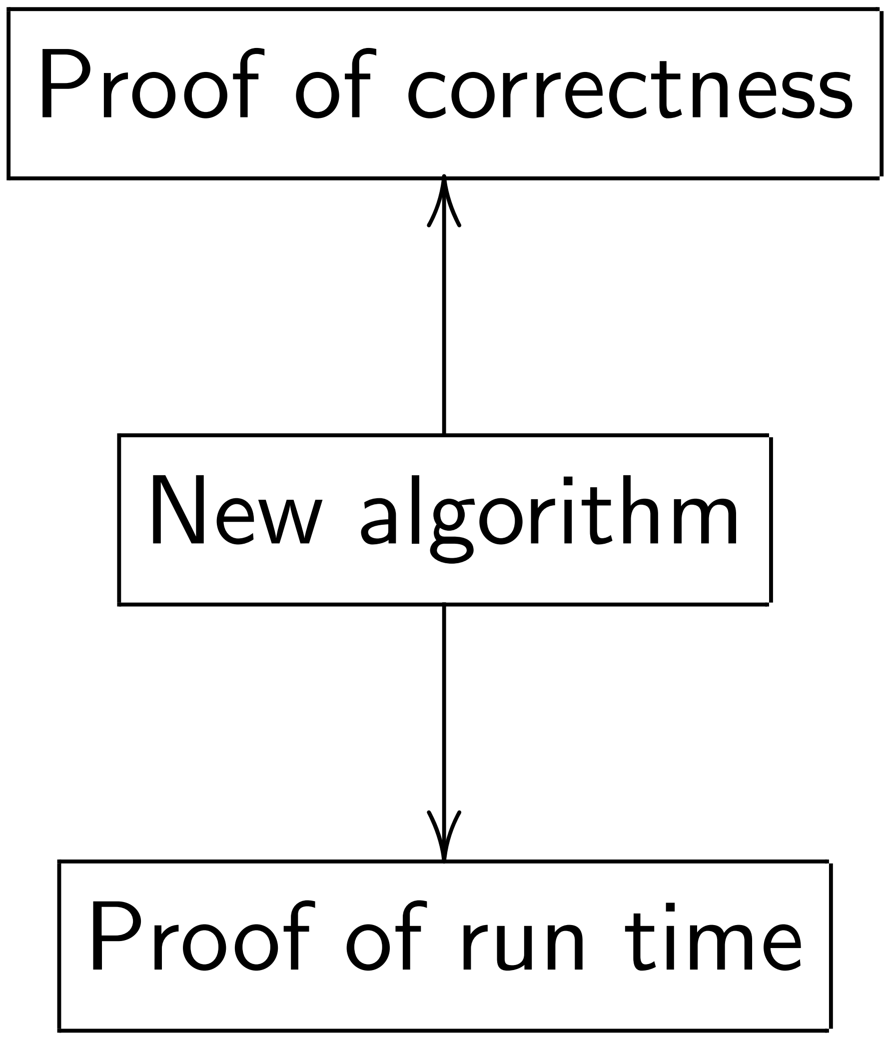

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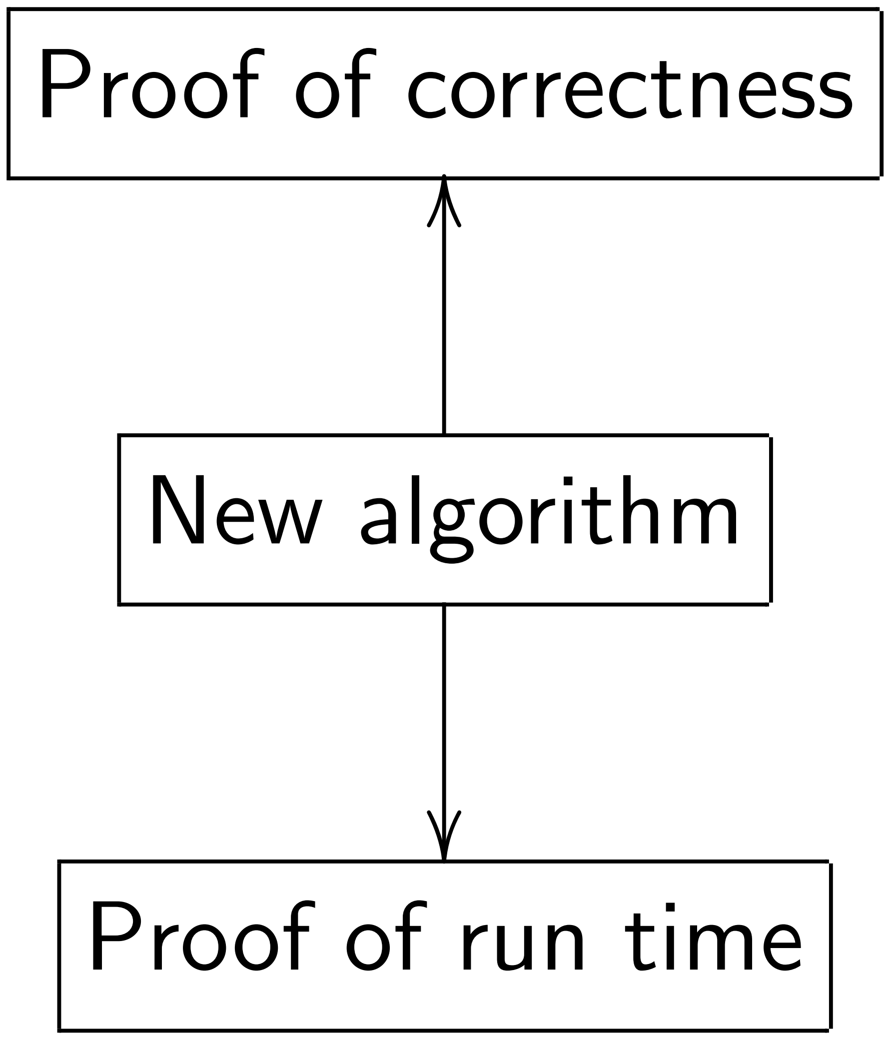

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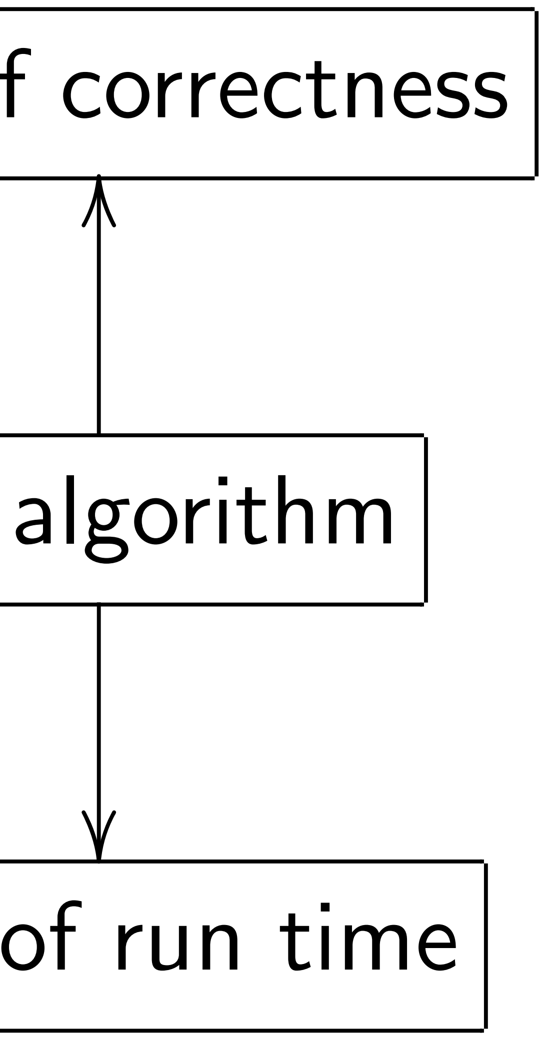

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$\approx 2^{64}$ qubit operations when N is around 1 gigabyte.

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Fix: Switch to AES-256.

AES-256 has 14 rounds. Maybe 12 rounds are enough for 2^{128} post-quantum security? Maybe 10 rounds are enough?

Shor's algorithm (similar to Simon's algorithm) factors RSA modulus N by finding period of $x \mapsto 2^x \bmod N$.

Number of qubit operations $\approx$ number of bit operations to compute $2^x \bmod N$.

$\approx 2^{64}$ qubit operations when N is around 1 gigabyte.

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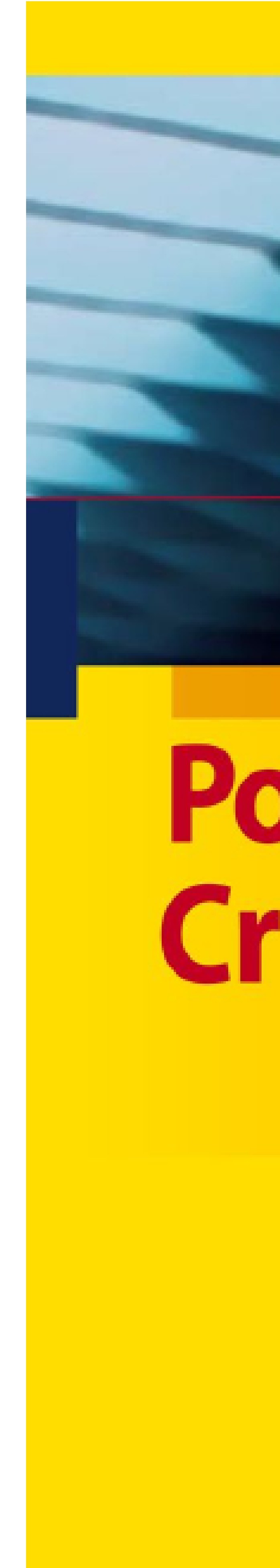

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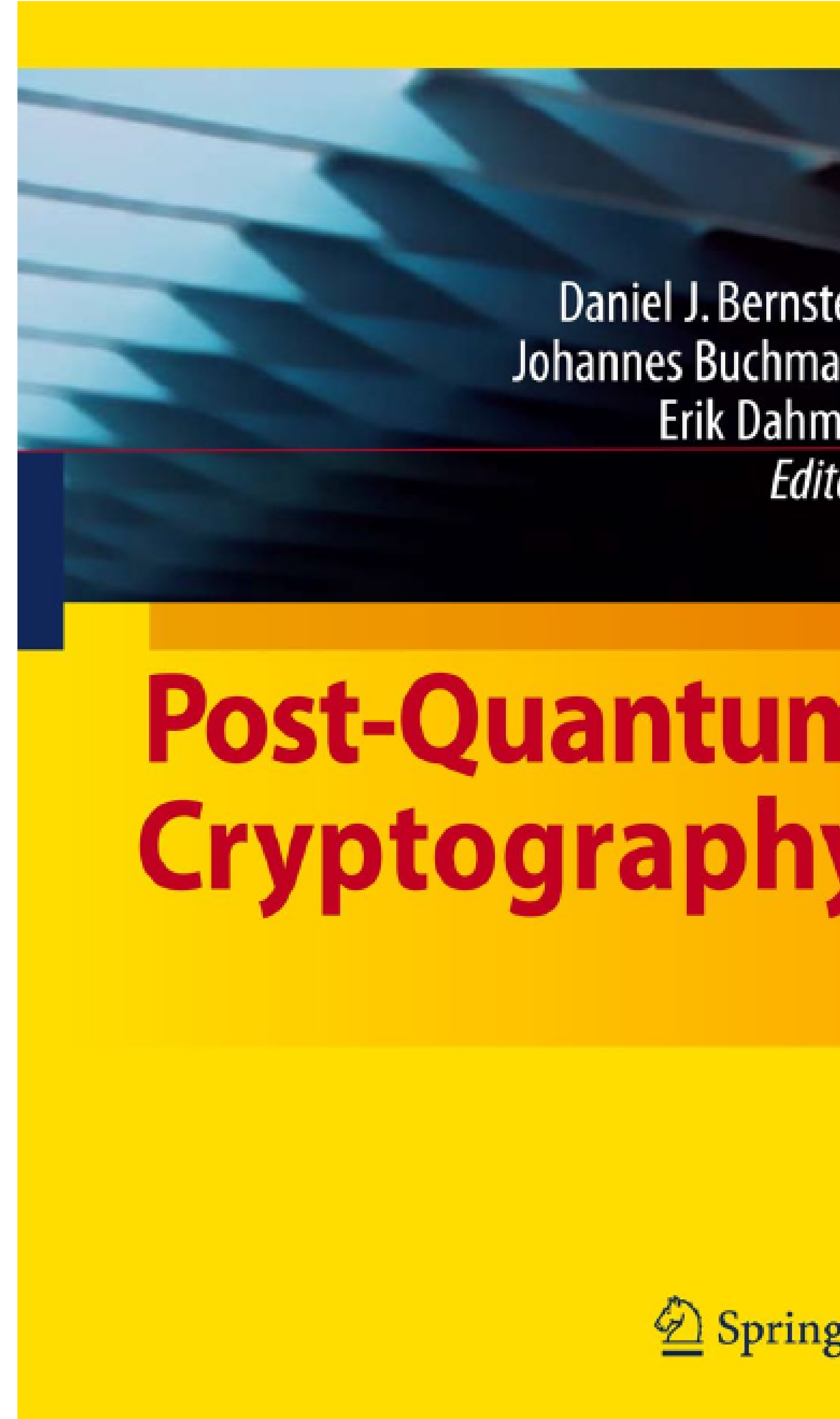

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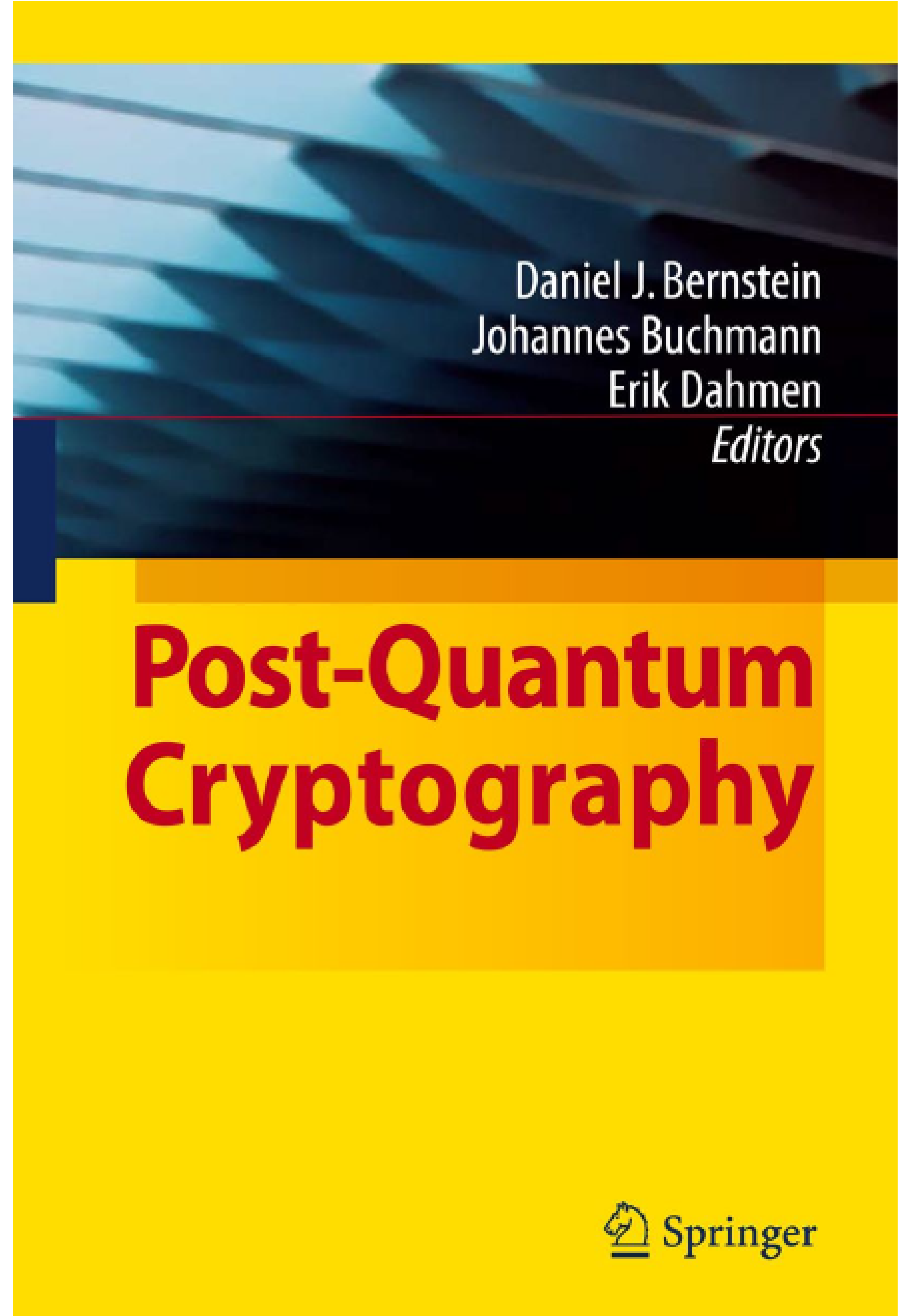

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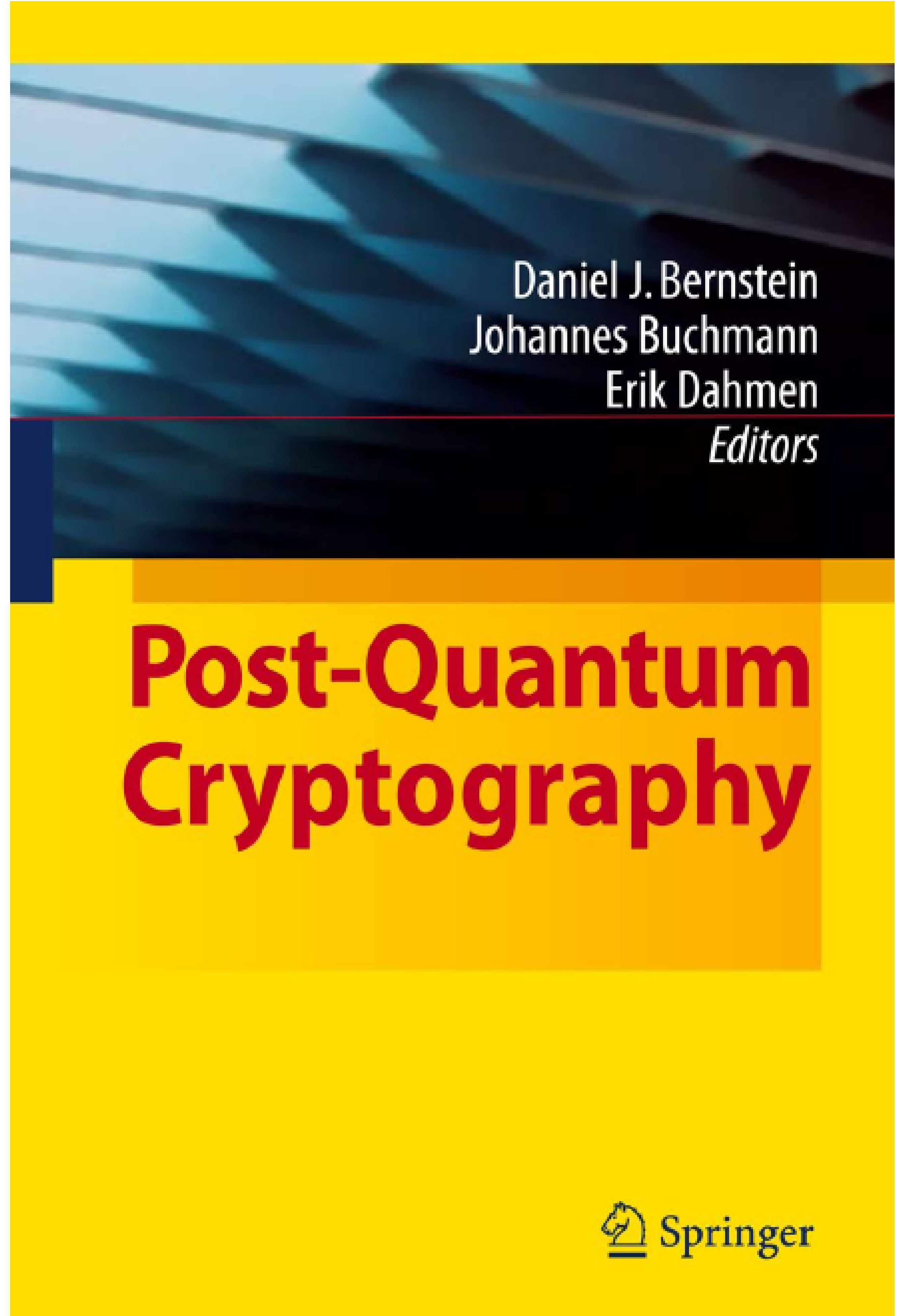

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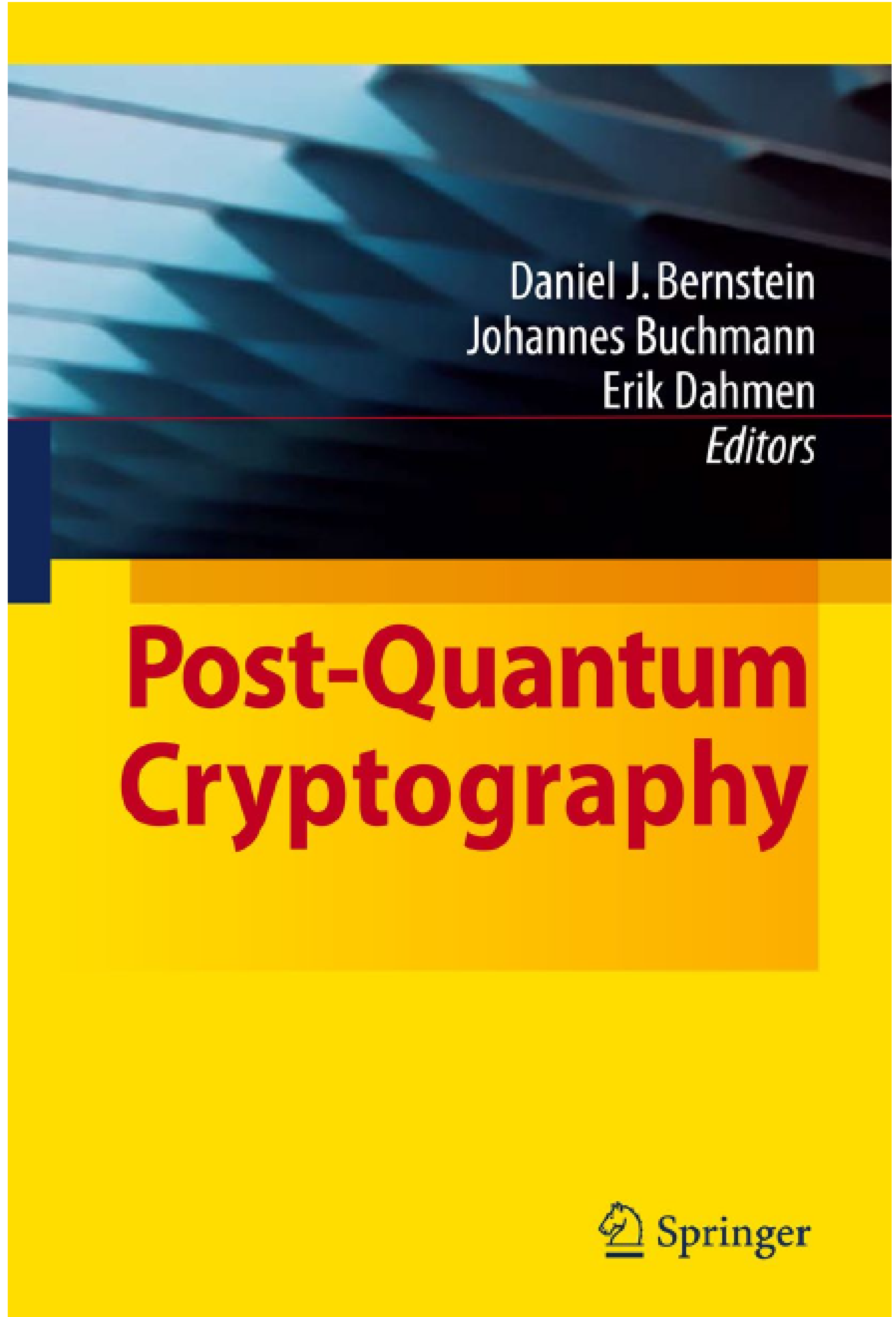

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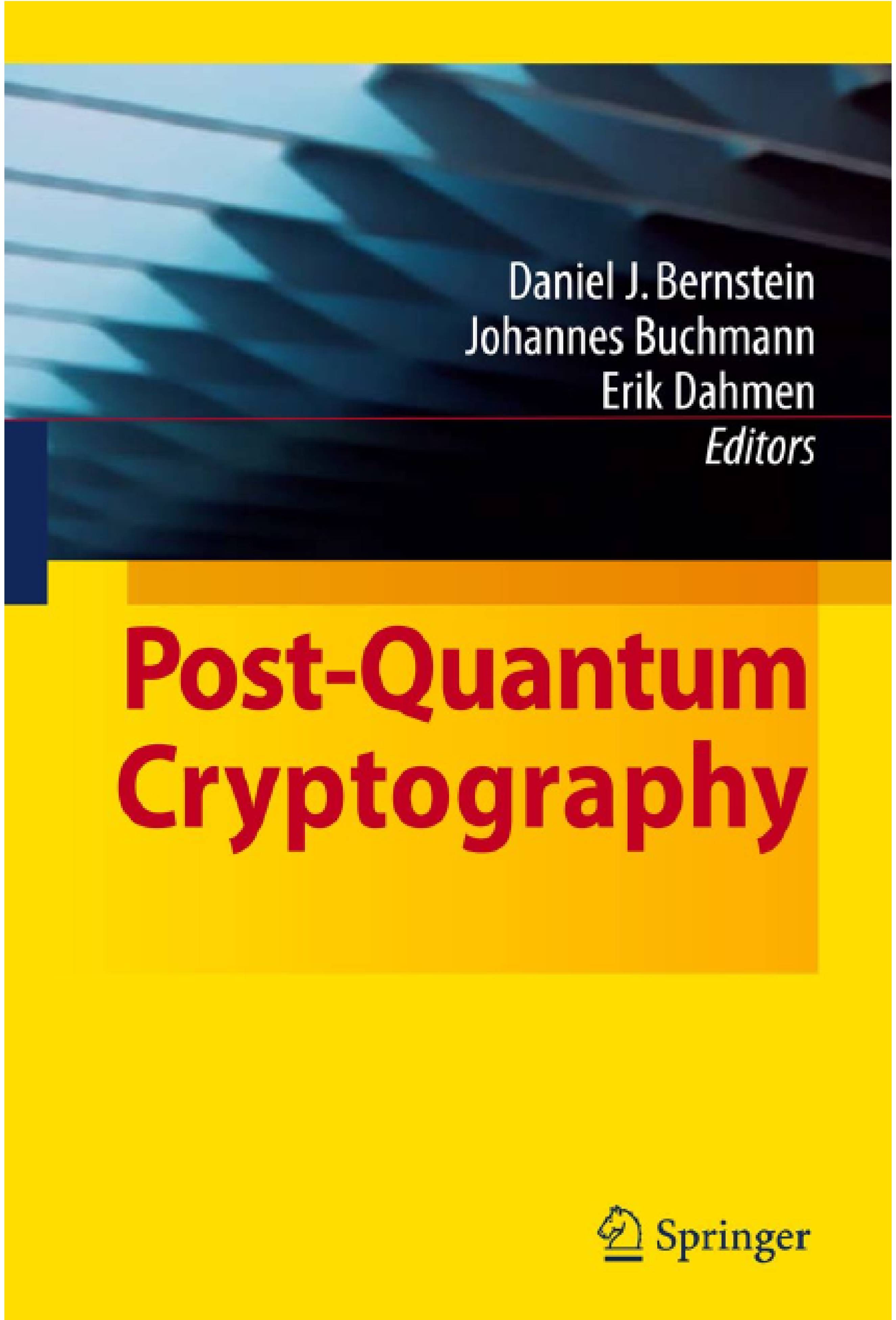

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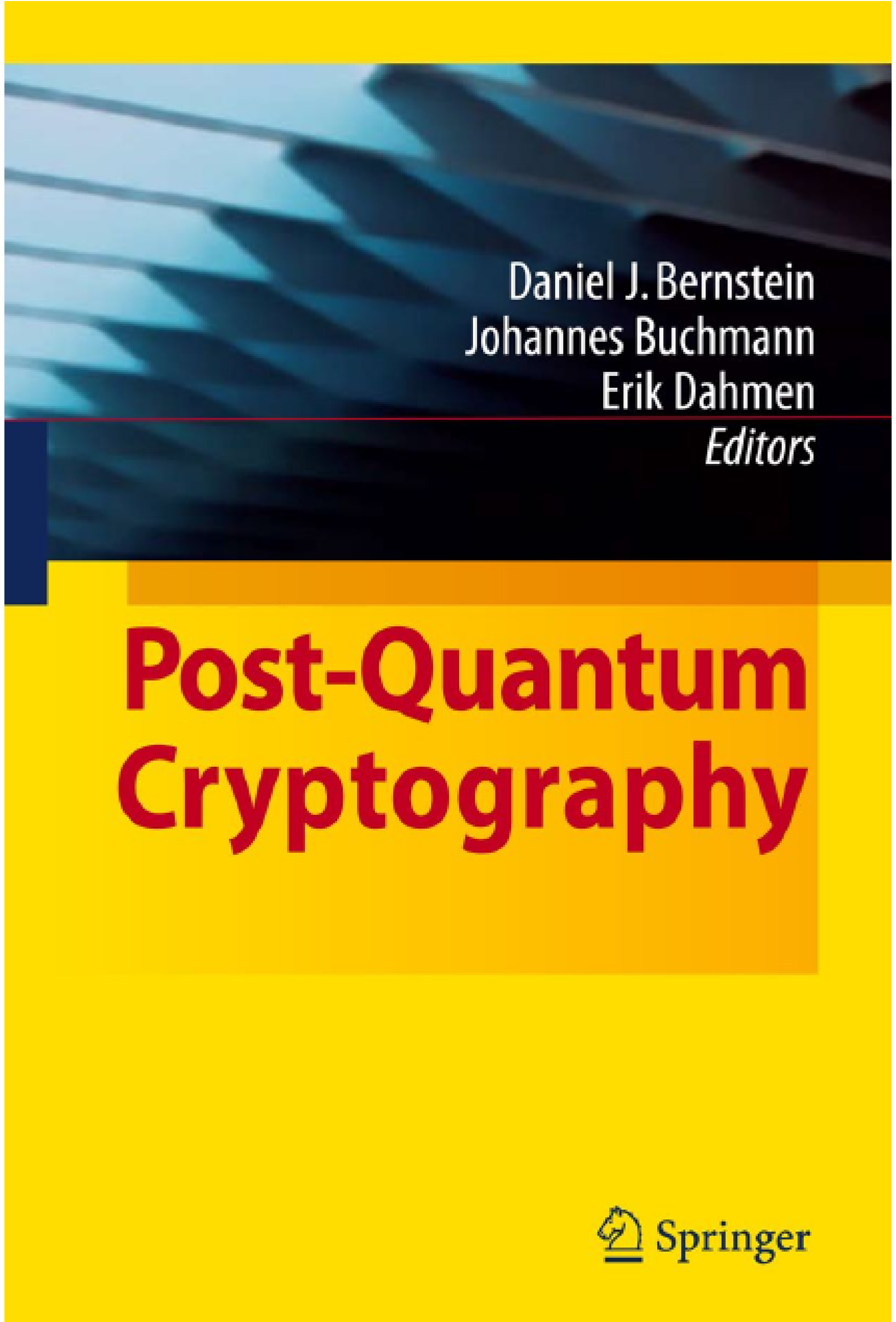

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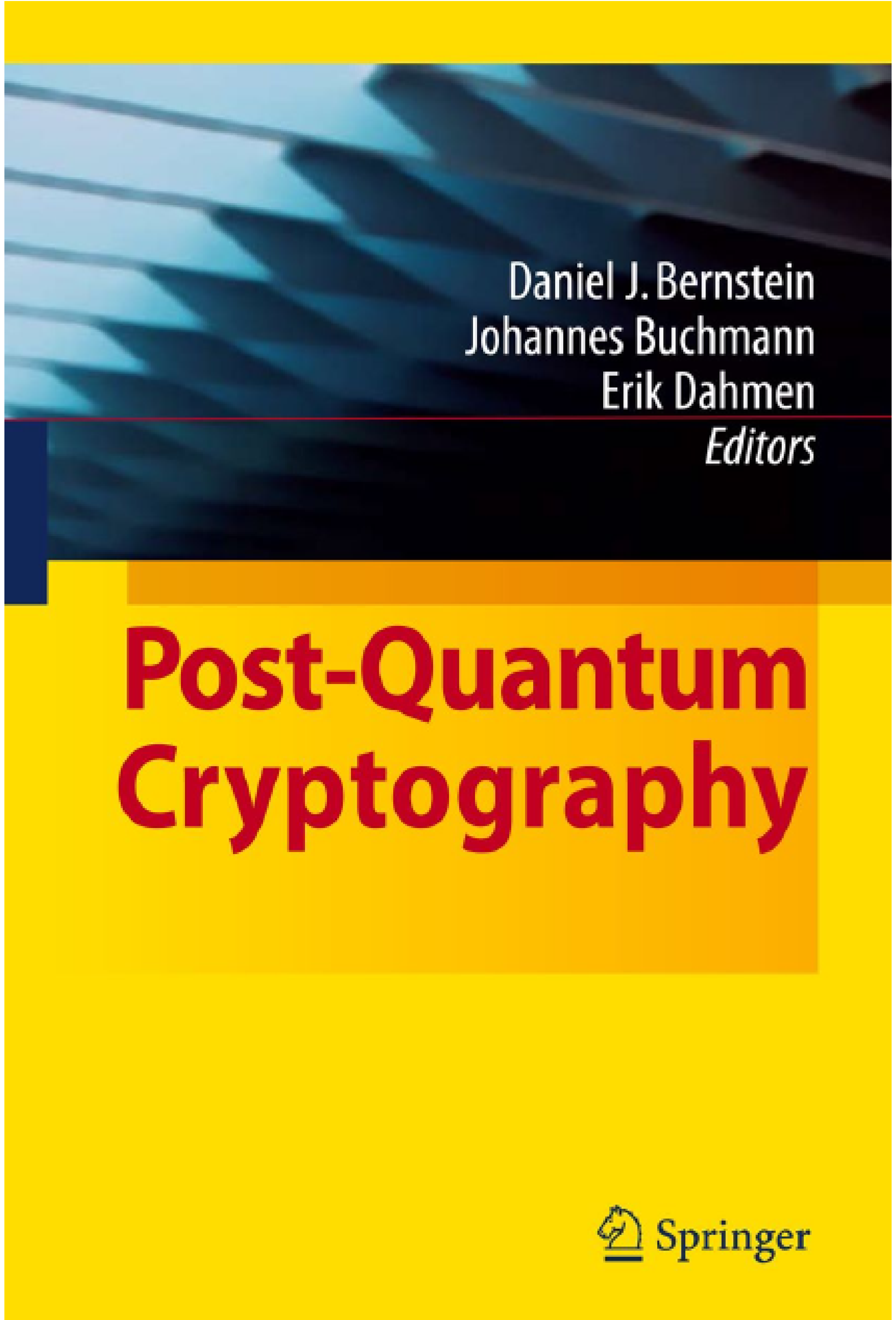

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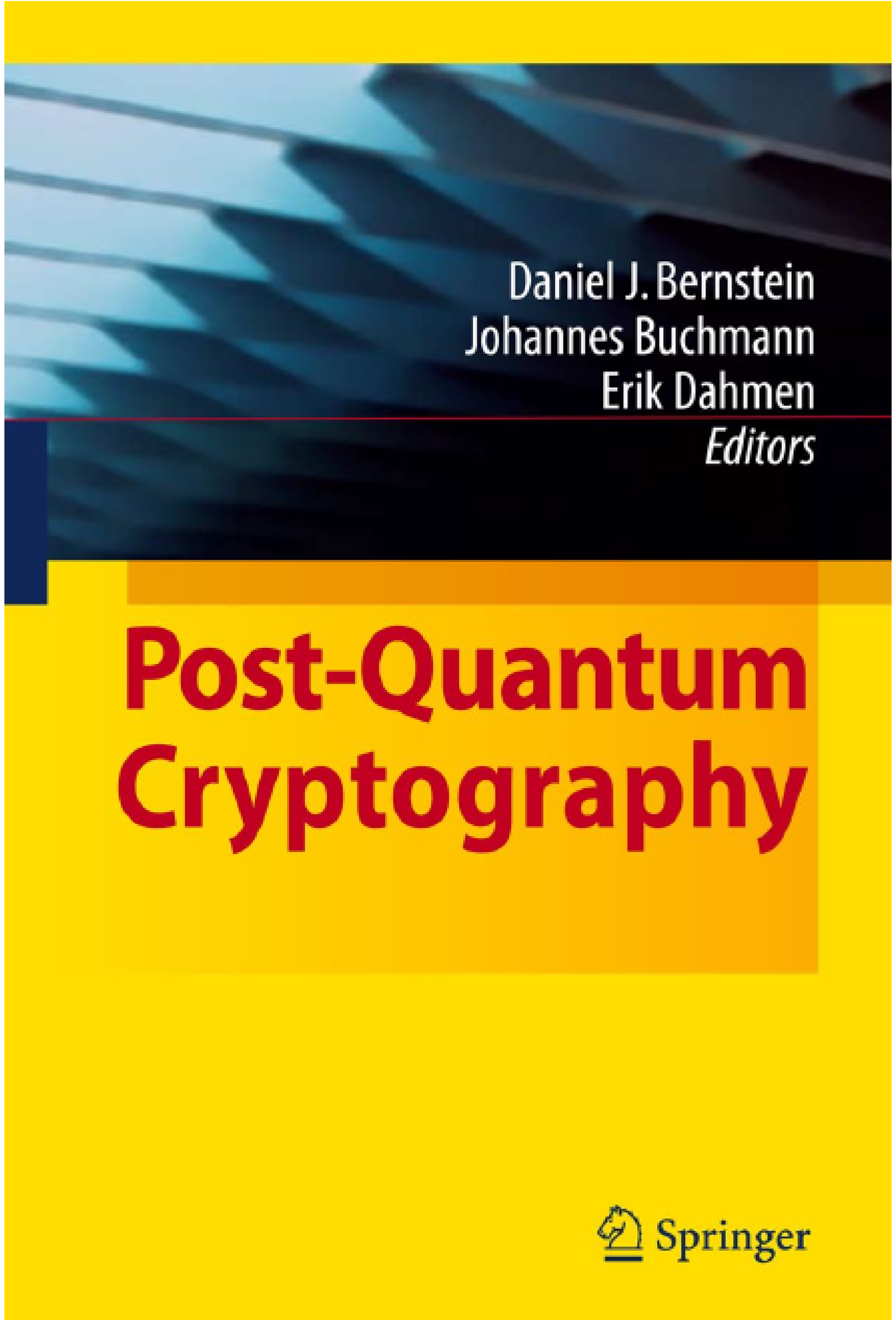

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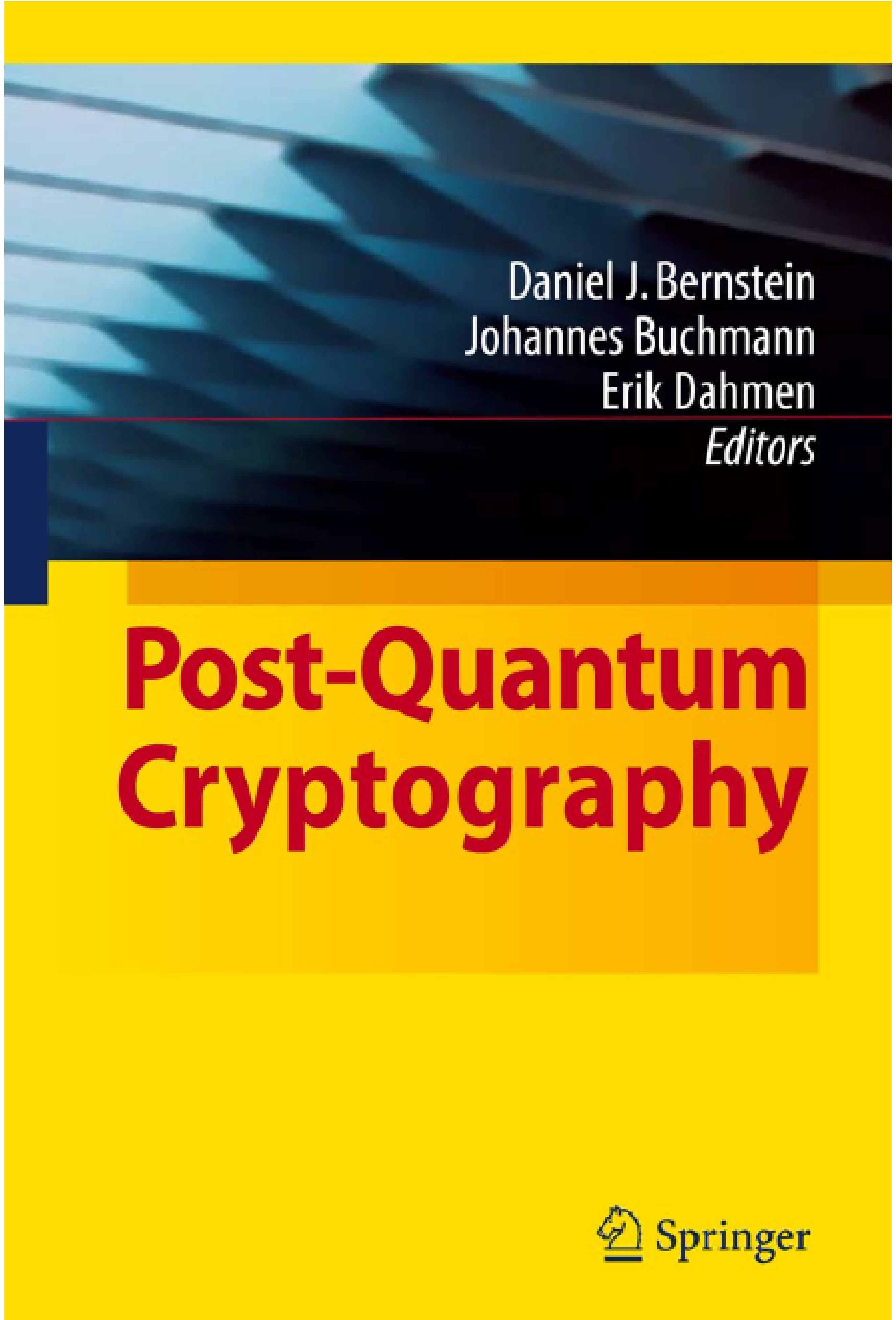

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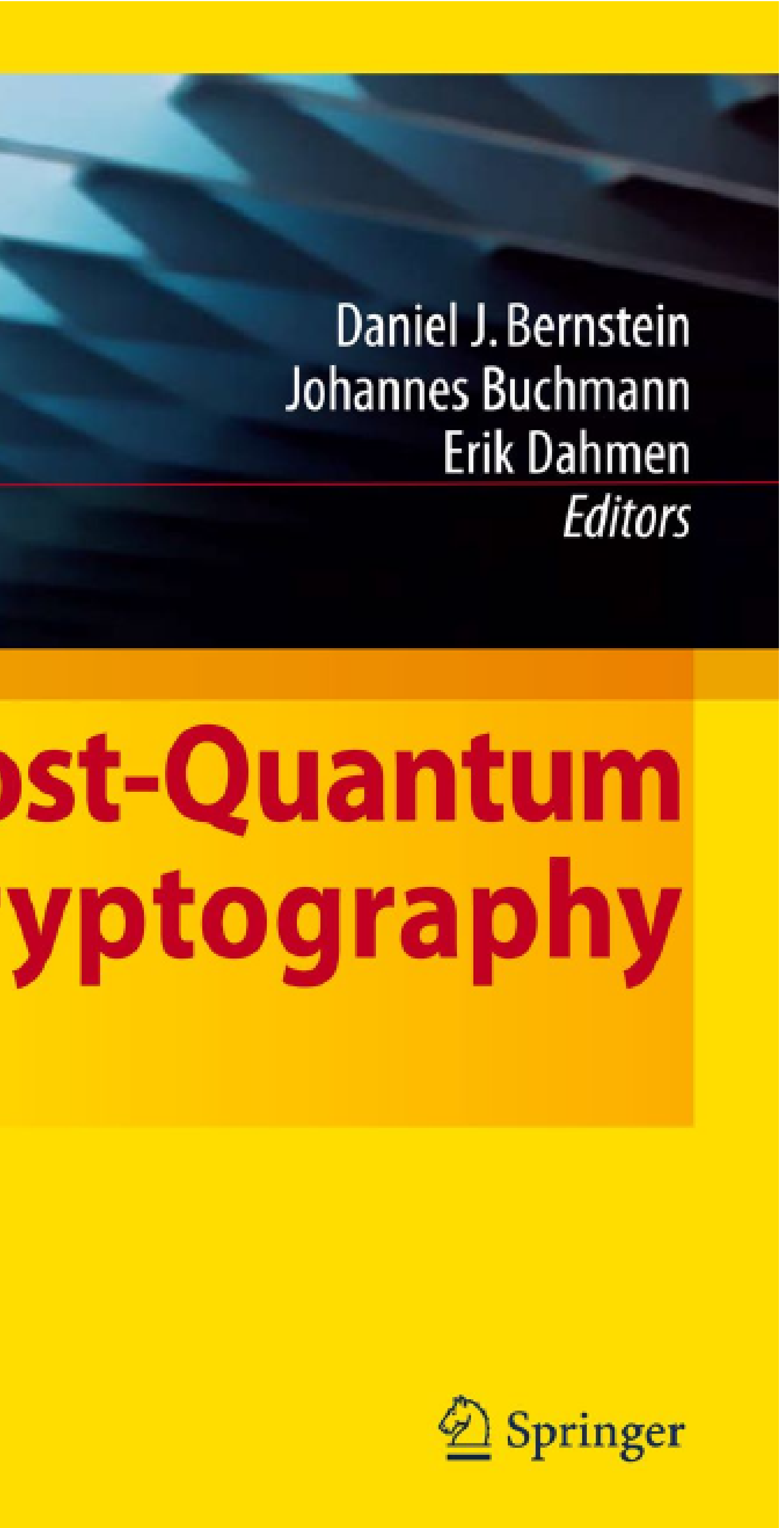

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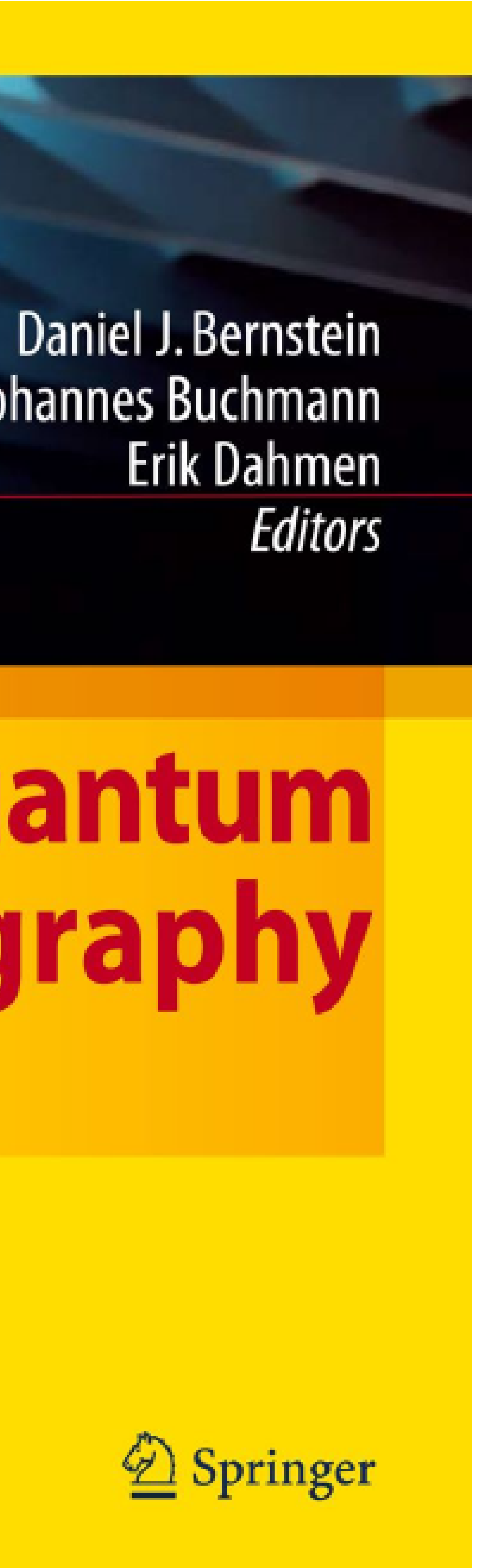

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Attacker wants to decode v: to find element of $\operatorname{Ker} K$ at distance only 50 from v. Presumably unique, revealing e.

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For typical K: Good chance that $\mathbf{F}_2^S \hookrightarrow \mathbf{F}_2^{1024} \xrightarrow{K} \mathbf{F}_2^{500}$ is invertible.

Hope $e \in \mathbf{F}_2^S$; chance $\approx 2^{-53}$. Apply inverse map to Ke, revealing e if $e \in \mathbf{F}_2^S$.

If $e \notin \mathbf{F}_2^S$, try again. $\approx 2^{80}$ bit operations in total.

Bad estimate by McEliece: $\approx 2^{64}$.

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Post-quantum: $2^{(0.5+o(1))w}$. e.g. $\approx 2^{26}$ Grover iterations to search 2^{53} choices of S.