

**GUARANTEED MESSAGE AUTHENTICATION  
FASTER THAN MD5  
(ABSTRACT)**

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Let  $r_0, r_1, r_2, r_3, x_0, x_1, x_2, x_3, m_0, m_1, m_2, \dots, m_{n-1}$  be integers in  $[-2^{31}, 2^{31} - 1]$ . Define  $r = 2^{96}r_3 + 2^{64}r_2 + 2^{32}r_1 + r_0$ ,  $x = 2^{96}x_3 + 2^{64}x_2 + 2^{32}x_1 + x_0$ , and

$$s = (r^{n+1} + r^n m_0 + r^{n-1} m_1 + \dots + r m_{n-1} + x) \bmod (2^{127} - 1).$$

I can compute  $s$  in about  $330 + 19n$  Pentium cycles, or  $470 + 26n$  UltraSPARC cycles, after a precomputation depending only on  $r$ .

**Applications.** Here's one way to mathematically guarantee the authenticity of a single message  $m = (m_0, m_1, \dots, m_{n-1})$ . The sender and receiver share a secret uniform random pair  $(r, x)$ . The sender computes  $s$  as above and sends  $(m, s)$ . The receiver verifies  $(m, s)$  by recomputing  $s$ . An attacker, given  $(m, s)$ , has a negligible probability of successfully forging a different message.

This system is faster than yesterday's MD5-based systems:  $s = \text{MD5}(r, m, x)$ , for example, or  $s = \text{MD5}(r, \text{MD5}(x, m))$ . It provides essentially the same level of security against today's attacks; unlike MD5, it is also guaranteed secure against tomorrow's attacks. It can easily be extended to handle multiple messages.

The same method can be used to reduce a multiprecision integer modulo a big secret prime. One application is to check ring equations involving large integers—for example, the equation  $s^2 = tn + fh$  in my variant of the Rabin-Williams public-key signature system—with a negligible chance of error.

**Method.** The following comments apply to the Pentium.

The fastest way to add and multiply small integers is with the help of the floating-point unit. For example, it takes just one cycle to compute an exact product of two 32-bit integers in floating-point registers. There is an old trick to split the result into low bits and high bits with a few floating-point additions and subtractions; one need not retrieve integers from the floating-point unit.

In multiprecision arithmetic one should generally use a radix below  $2^{32}$  so that more useful work can be done between carries. I precompute small integers  $c_{i,j}$  such that  $r^i$  is congruent to  $c_{i,0} + 2^{26}c_{i,1} + 2^{52}c_{i,2} + 2^{78}c_{i,3} + 2^{104}c_{i,4}$  modulo  $2^{127} - 1$ . If  $n$  is not too large then the dot products  $\sum_i c_{n-i,0}m_i$ ,  $\sum_i c_{n-i,1}m_i$ , etc. fit safely into floating-point registers. Handling large  $n$  is not much more difficult.

The gcc -O6 optimizer does a poor job of instruction scheduling and register allocation. I use gcc -O1 and schedule instructions manually. The speeds reported above are still not optimal.

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**History.** There are many previous message authentication systems that reduce various rings modulo big secret random prime ideals. This is the first high-security system to break the MD5 speed barrier. The direct ancestor of this system is Shoup's analogous system in characteristic 2.

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