Lattice-based cryptography, day 1: simplicity

D. J. Bernstein

University of Illinois at Chicago;
Ruhr University Bochum

2000 Cohen cryptosystem

Public key: vector of integers $K = (K_1, \ldots, K_N) \in \{-X, \ldots, X\}^N$.

Encryption:

1. Input message $m \in \{0, 1\}$.

2. Generate $r_1, \ldots, r_N \in \{0, 1\}$.
i.e. $r = (r_1, \ldots, r_N) \in \{0, 1\}^N$.

(Cohen says pick "half of the integers in the public key at random": I guess this means $N \in 2\mathbf{Z}$ and $\sum r_i = N/2$.)

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Source: `www.sage`

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 10493, 17333, 1397, 8656,
 8213, 6370]
```

gers C, s with s > 0,
"C%s" always produces
between 0 and s − 1.

s standard math definition:
$s = C - \lfloor C/s \rfloor s$.

: Typically
roduces C%s < 0
-level languages, so
output leaks input sign.

: For polynomials C,
n make the same mistake.


```
sage: N=10
sage: X=2^50
sage: Y=2^20
sage: Y
1048576
sage: s=randrange(1,Y+1)
sage: s
359512
sage: u=[randrange(
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```

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[870056918917829,
 822006576592695,
 -294765544345815,
 -669275100080982,
 528958455221029,
 426006001074157,
 -641940176080531,
 501543495923784,
 -583064075392587,
 461093902438341]
```

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```
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=2^50
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=[ui+s*randrange(
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918917829,
576592695,
554434581 5,
5100080982,
455221029,
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0176080531,
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```
ndrange(
-(X+ui)/s),
((X-ui)/s)+1)
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,
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47024
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Chosen-ciphertext attack against this system:
Decrypt $-C$. Flip result.

(Works whenever $C \neq 0$.)

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856043576
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B-bit input message
$m = (m_1, \ldots, m_B) \in \{0,1\}^B$.
For each $i \in \{1, \ldots, B\}$:
Generate $r_{i,1}, \ldots, r_{i,N} \in \{0,1\}$.

Ciphertext C:
$(-1)^{m_1}(r_{1,1}K_1 + \cdots + r_{1,N}K_N)$,
$\ldots$,
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Ciphertext C:
$(-1)^{m_1}(r_{1,1}K_1 + \cdots + r_{1,N}K_N)$,
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$r_{N/2+1}, \ldots, r_N)$.

$r_1 K_1 + \cdots + r_{N/2} K_{N/2}$ in
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sage: N=10
sage: E=2^10
sage: Y=2^50
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If u_i/s is small enough then 2009 DGHV system is homomorphic.

Take two ciphertexts:
$C = m + 2\epsilon + sq$,
$C' = m' + 2\epsilon' + sq'$
with small $\epsilon, \epsilon' \in \mathbf{Z}$.

$C + C' = m + m' + 2(\epsilon + \epsilon') +$ $s(q + q')$. This decrypts to $m + m'$ mod 2 if $\epsilon + \epsilon'$ is small.

$CC' = mm' + 2(\epsilon m' + \epsilon' m + 2\epsilon\epsilon') +$ $s(\cdots)$. This decrypts to mm' if $\epsilon m' + \epsilon' m + 2\epsilon\epsilon'$ is small.


```
sage: N=10
sage: E=2^10
sage: Y=2^50
sage: X=2^80
sage:
```

Homomorphic encryption

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```
sage: N=10
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sage: Y=2^50
sage: X=2^80
sage: s=1+2*randrange(Y/4,Y/2)
sage: s
984887308997925
sage:
```

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984887308997925
sage: u=[randrange(E)
....:    for i in range(N)]
sage: u
[247, 418, 365, 738, 123, 735,
 772, 209, 673, 47]
sage:
```

orphic encryption

s small enough then 2009
ystem is homomorphic.

o ciphertexts:
$+ 2\epsilon + sq$,
$+ 2\epsilon' + sq'$
all $\epsilon, \epsilon' \in \mathbf{Z}$.

$= m + m' + 2(\epsilon + \epsilon') +$
). This decrypts to
mod 2 if $\epsilon + \epsilon'$ is small.

$mm' + 2(\epsilon m' + \epsilon' m + 2\epsilon\epsilon') +$
This decrypts to
$\epsilon m' + \epsilon' m + 2\epsilon\epsilon'$ is small.


```
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sage: E=2^10
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sage: u
[247, 418, 365, 738, 123, 735,
 772, 209, 673, 47]
sage:
```

```
sage:
```

ryption

ough then 2009
omomorphic.

xts:

q'
$\mathbf{Z}$.

$+ 2(\epsilon + \epsilon') +$
ecrypts to
$+ \epsilon'$ is small.

$n' + \epsilon' m + 2\epsilon\epsilon') +$
pts to
$+ 2\epsilon\epsilon'$ is small.


```
sage: N=10
sage: E=2^10
sage: Y=2^50
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....:    for i in range(N)]
sage: u
[247, 418, 365, 738, 123, 735,
 772, 209, 673, 47]
sage:
```

```
sage:
```

2009

hic.

$) +$

nall.

$2\epsilon\epsilon') +$

small.


```
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984887308997925
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sage: u
[247, 418, 365, 738, 123, 735,
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sage:
```

```
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```

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984887308997925
sage: u=[randrange(E)
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sage: u
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 772, 209, 673, 47]
sage:
```



```
sage:
```

```
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sage: u
[247, 418, 365, 738, 123, 735,
 772, 209, 673, 47]
sage:
```



```
sage: K=[2*ui+s*randrange(
....:       ceil(-(X+2*ui)/s),
....:       floor((X-2*ui)/s)+1)
....:     for ui in u]
sage:
```

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sage: N=10
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sage: s=1+2*randrange(Y/4,Y/2)
sage: s
984887308997925
sage: u=[randrange(E)
....:     for i in range(N)]
sage: u
[247, 418, 365, 738, 123, 735,
 772, 209, 673, 47]
sage:
```



```
sage: K=[2*ui+s*randrange(
....:        ceil(-(X+2*ui)/s),
....:        floor((X-2*ui)/s)+1)
....:      for ui in u]
sage: K
[587473338058640662659869,
 -1111539179100720083770339,
 794301459533783434896055,
 688178021083749589017751,
 742362470968200823035396,
 1023345827831539515054795,
 -3571686793985588767300006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381]
```

```
=10
=2^10
=2^50
=2^80
=1+2*randrange(Y/4,Y/2)

08997925
=[randrange(E)
  for i in range(N)]

18, 365, 738, 123, 735,
09, 673, 47]
```



```
sage: K=[2*ui+s*randrange(
....:        ceil(-(X+2*ui)/s),
....:        floor((X-2*ui)/s)+1)
....:     for ui in u]
sage: K
[587473338058640662659869,
 -1111539179100720083770339,
 794301459533783434896055,
 68817802108374958901751,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381]
```

```
sage: m
sage: r
....:
sage:
```

```
range(Y/4,Y/2)

ge(E)
n range(N)]

738, 123, 735,
47]
```



```
sage: K=[2*ui+s*randrange(
....:        ceil(-(X+2*ui)/s),
....:        floor((X-2*ui)/s)+1)
....:     for ui in u]
sage: K
[587473338058640662659869,
 -111153917910072008377033 9,
 794301459533783434896055,
 688178021083749589017 51,
 742362470968200823035396,
 1023345827831539515054795,
 -3571686793985588767300 06,
 1121421619119964601051443,
 -1109674862276222495587129,
 -2356289377850037705233 81]
```

```
sage: m=randrang
sage: r=[randran
....:     for i i
sage:
```

```
,Y/2)

)]

735,
```



```
sage: K=[2*ui+s*randrange(
....:       ceil(-(X+2*ui)/s),
....:       floor((X-2*ui)/s)+1)
....:     for ui in u]
sage: K
[58747333805864066265986 9,
 -11115391791007200837703 39,
 79430145953378343489605 5,
 68817802108374958901751,
 74236247096820082303539 6,
 10233458278315395150547 95,
 -35716867939855887673000 6,
 11214216191199646010514 43,
 -11096748622762224955871 29,
 -23562893778500377052338 1]
```

```
sage: m=randrange(2)
sage: r=[randrange(2)
....:     for i in range(N
sage:
```

```
sage: K=[2*ui+s*randrange(
....:      ceil(-(X+2*ui)/s),
....:      floor((X-2*ui)/s)+1)
....:    for ui in u]
sage: K
[5874733380586406626 59869,
 -11115391791007200837 70339,
 79430145953378343489 6055,
 68817802108374958901 751,
 74236247096820082303 5396,
 10233458278315395150 54795,
 -35716867939855887673 0006,
 11214216191199646010 51443,
 -11096748622762224955 87129,
 -23562893778500377052 3381]
```



```
sage: m=randrange(2)
sage: r=[randrange(2)
....:    for i in range(N)]
sage:
```

```
sage: K=[2*ui+s*randrange(
....:      ceil(-(X+2*ui)/s),
....:      floor((X-2*ui)/s)+1)
....:    for ui in u]
sage: K
[587473338058640662659869,
 -1111539179100720083770339,
 794301459533783434896055,
 68817802108374958901751,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381]
```



```
sage: m=randrange(2)
sage: r=[randrange(2)
....:    for i in range(N)]
sage: C=m+sum(r[i]*K[i]
....:    for i in range(N))
sage: C
2094088748748247210016703
sage:
```

```
sage: K=[2*ui+s*randrange(
....:      ceil(-(X+2*ui)/s),
....:      floor((X-2*ui)/s)+1)
....:    for ui in u]
sage: K
[58747338058640662659869,
 -1111539179100720083770339,
 794301459533783434896055,
 68817802108374958901751,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381]
```



```
sage: m=randrange(2)
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sage: C
2094088748748247210016703
sage: C%s
2703
sage:
```

```
sage: K=[2*ui+s*randrange(
....:        ceil(-(X+2*ui)/s),
....:        floor((X-2*ui)/s)+1)
....:     for ui in u]
sage: K
[58747338058640662659869,
 -1111539179100720083770339,
 79430145953783434896055,
 68817802108374958901751,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381]
```



```
sage: m=randrange(2)
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....:     for i in range(N))
sage: C
2094088748748247210016703
sage: C%s
2703
sage: (C%s)%2
1
sage:
```

```
sage: K=[2*ui+s*randrange(
....:      ceil(-(X+2*ui)/s),
....:      floor((X-2*ui)/s)+1)
....:    for ui in u]
sage: K
[58747338058640662659869,
 -1111539179100720083770339,
 794301459533783434896055,
 68817802108374958901751,
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 1121421619119964601051443,
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```



```
sage: m=randrange(2)
sage: r=[randrange(2)
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sage: C=m+sum(r[i]*K[i]
....:     for i in range(N))
sage: C
2094088748748247210016703
sage: C%s
2703
sage: (C%s)%2
1
sage: m
1
sage:
```

```
=[2*ui+s*randrange(
    ceil(-(X+2*ui)/s),
    floor((X-2*ui)/s)+1)
  for ui in u]

338058640662659869,
39179100720083770339,
459533783434896055,
02108374958901751,
470968200823035396,
5827831539515054795,
8679398558876730006,
1619119964601051443,
74862276222495587129,
893778500370523381]
```



```
sage: m=randrange(2)
sage: r=[randrange(2)
....:     for i in range(N)]
sage: C=m+sum(r[i]*K[i]
....:     for i in range(N))
sage: C
2094088748748247210016703
sage: C%s
2703
sage: (C%s)%2
1
sage: m
1
sage: 
```

```
sage: m
sage: r
....:
sage:
```

```
randrange(
-(X+2*ui)/s),
((X-2*ui)/s)+1)
in u]

662659869,
20083770339,
434896055,
58901751,
823035396,
9515054795,
8876730006,
4601051443,
22495587129,
3770523381]
```



```
sage: m=randrange(2)
sage: r=[randrange(2)
....:    for i in range(N)]
sage: C=m+sum(r[i]*K[i]
....:    for i in range(N))
sage: C
2094088748748247210016703
sage: C%s
2703
sage: (C%s)%2
1
sage: m
1
sage:
```

```
sage: m2=randran
sage: r2=[randra
....:     for i
sage:
```

```
e(
/s),
/s)+1)

),
39,
5,

3,
5,
6,
3,
29,
1]
```



```
sage: m=randrange(2)
sage: r=[randrange(2)
....:    for i in range(N)]
sage: C=m+sum(r[i]*K[i]
....:    for i in range(N))
sage: C
20940887487482472100167O3
sage: C%s
2703
sage: (C%s)%2
1
sage: m
1
sage:
```

```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:    for i in range(
sage:
```

```
sage: m=randrange(2)
sage: r=[randrange(2)
....:    for i in range(N)]
sage: C=m+sum(r[i]*K[i]
....:    for i in range(N))
sage: C
2094088748748247210016703
sage: C%s
2703
sage: (C%s)%2
1
sage: m
1
sage:
```



```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:     for i in range(N)]
sage:
```

```
sage: m=randrange(2)
sage: r=[randrange(2)
....:     for i in range(N)]
sage: C=m+sum(r[i]*K[i]
....:     for i in range(N))
sage: C
2094088748748247210016703
sage: C%s
2703
sage: (C%s)%2
1
sage: m
1
sage:
```



```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:     for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:     for i in range(N))
sage: C2
-51722353737982737270129
sage:
```

```
sage: m=randrange(2)
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sage: C=m+sum(r[i]*K[i]
....:      for i in range(N))
sage: C
20940887487482472100167O3
sage: C%s
2703
sage: (C%s)%2
1
sage: m
1
sage:
```



```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:      for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:      for i in range(N))
sage: C2
-5172235373798273727O129
sage: C2%s
4971
sage:
```

```
sage: m=randrange(2)
sage: r=[randrange(2)
....:     for i in range(N)]
sage: C=m+sum(r[i]*K[i]
....:     for i in range(N))
sage: C
2094088748748247210016703
sage: C%s
2703
sage: (C%s)%2
1
sage: m
1
sage:
```



```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:     for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:     for i in range(N))
sage: C2
-5172253737982737270129
sage: C2%s
4971
sage: (C2%s)%2
1
sage:
```

```
sage: m=randrange(2)
sage: r=[randrange(2)
....:     for i in range(N)]
sage: C=m+sum(r[i]*K[i]
....:     for i in range(N))
sage: C
2094088748748247210016703
sage: C%s
2703
sage: (C%s)%2
1
sage: m
1
sage:
```



```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:     for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:     for i in range(N))
sage: C2
-5172235373798273727O129
sage: C2%s
4971
sage: (C2%s)%2
1
sage: m2
1
sage:
```

```
=randrange(2)
=[randrange(2)
  for i in range(N)]
=m+sum(r[i]*K[i]
  for i in range(N))

74874824721001670З
%s

C%s)%2
```



```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:       for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:       for i in range(N))
sage: C2
-51722353737982737270129
sage: C2%s
4971
sage: (C2%s)%2
1
sage: m2
1
sage:
```

```
sage: (
7674
sage: (
1343661
sage:
```

```
e(2)
ge(2)
n range(N)]
i]*K[i]
n range(N))

210016703
```



```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:     for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:     for i in range(N))
sage: C2
-51722353737982737270129
sage: C2%s
4971
sage: (C2%s)%2
1
sage: m2
1
sage:
```

```
sage: (C+C2)%s
7674
sage: (C*C2)%s
13436613
sage:
```

```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:     for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:     for i in range(N))
sage: C2
-51722353737982737270129
sage: C2%s
4971
sage: (C2%s)%2
1
sage: m2
1
sage:
```



```
sage: (C+C2)%s
7674
sage: (C*C2)%s
13436613
sage:
```

```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:     for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:     for i in range(N))
sage: C2
-51722353737982737270129
sage: C2%s
4971
sage: (C2%s)%2
1
sage: m2
1
sage:
```



```
sage: (C+C2)%s
7674
sage: (C*C2)%s
13436613
sage:
```

```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:     for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:     for i in range(N))
sage: C2
-5172235373798273727O129
sage: C2%s
4971
sage: (C2%s)%2
1
sage: m2
1
sage:
```



```
sage: (C+C2)%s
7674
sage: (C*C2)%s
13436613
sage:
```

Because C mod s and C' mod s are small enough compared to s, have $C + C'$ mod $s = (C \bmod s) + (C' \bmod s)$ and CC' mod $s = (C \bmod s)(C' \bmod s)$.

```
sage: m2=randrange(2)
sage: r2=[randrange(2)
....:     for i in range(N)]
sage: C2=m2+sum(r2[i]*K[i]
....:     for i in range(N))
sage: C2
-5172235373798273727O129
sage: C2%s
4971
sage: (C2%s)%2
1
sage: m2
1
sage:
```



```
sage: (C+C2)%s
7674
sage: (C*C2)%s
13436613
sage:
```

Because $C \bmod s$ and C' mod s are small enough compared to s, have $C + C' \bmod s = (C \bmod s) + (C' \bmod s)$ and $CC' \bmod s = (C \bmod s)(C' \bmod s)$.

Refinements: add more noise to ciphertexts, bootstrap (2009 Gentry) to control noise, etc.

```
2=randrange(2)
2=[randrange(2)
   for i in range(N)]
2=m2+sum(r2[i]*K[i]
   for i in range(N))
2
537379827372701 29
2%s

C2%s)%2

2
```



```
sage: (C+C2)%s
7674
sage: (C*C2)%s
13436613
sage:
```

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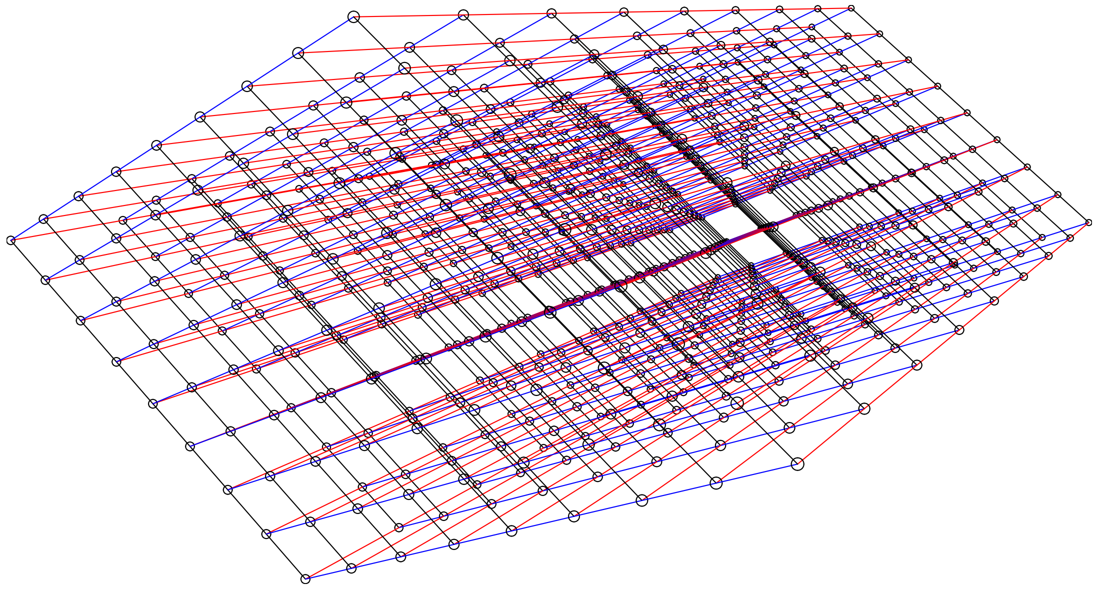

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C+C2)%s
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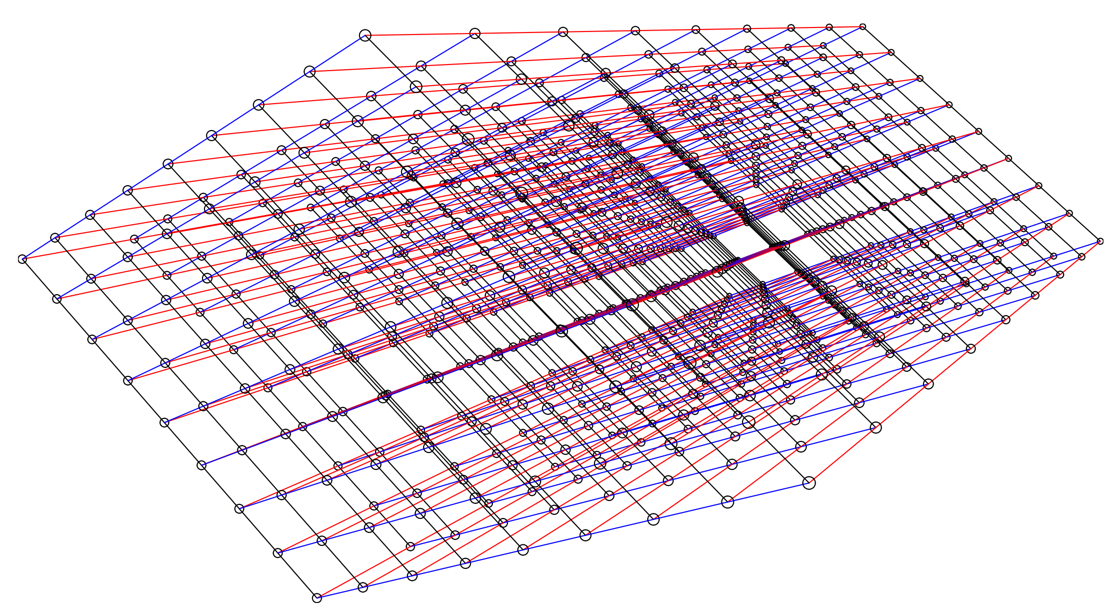

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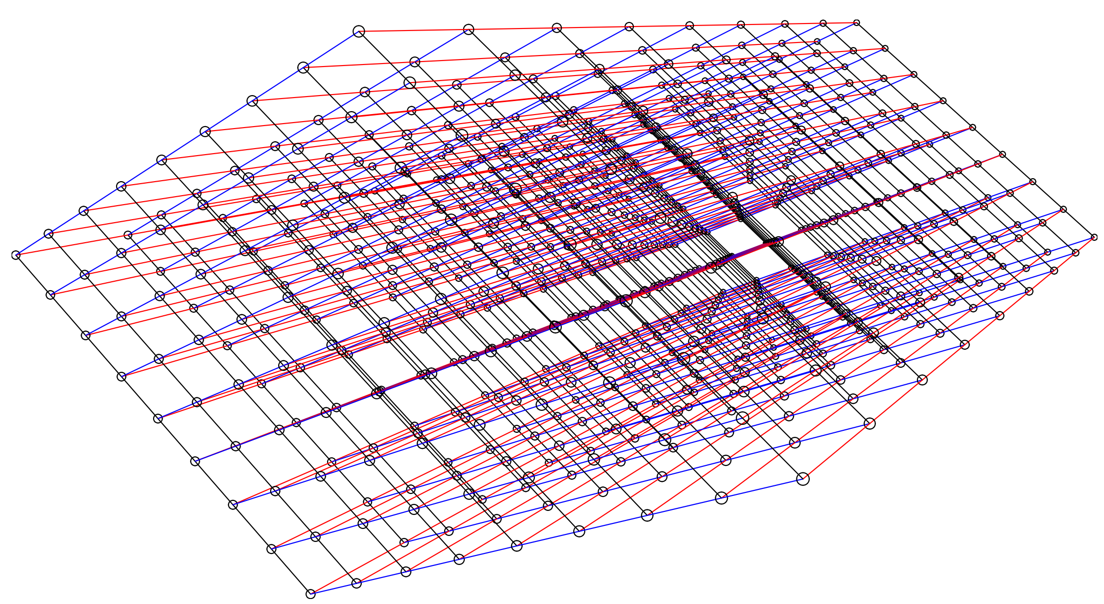

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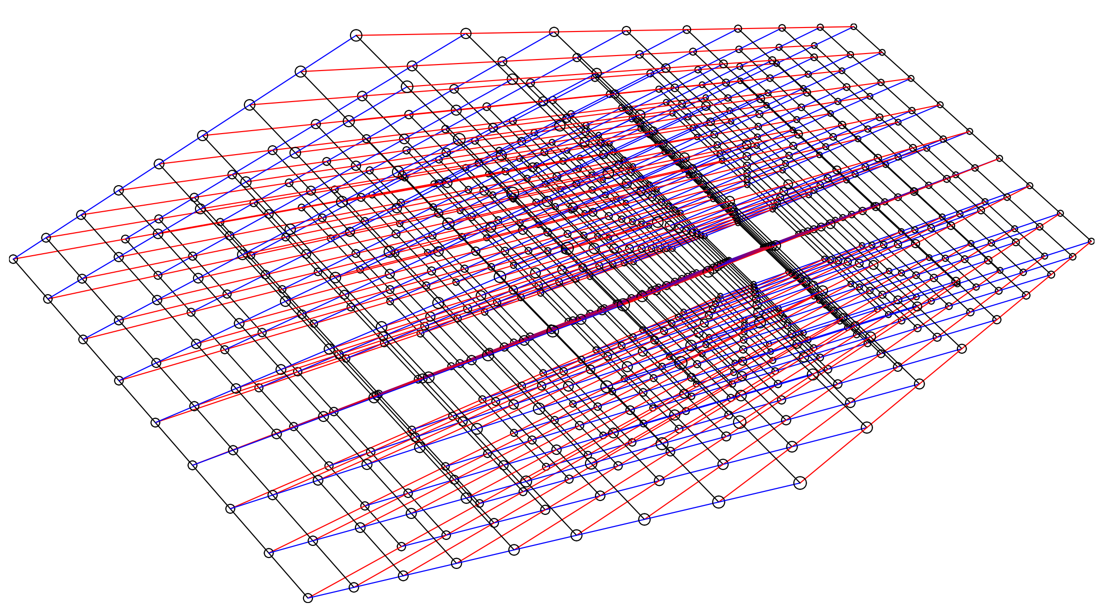

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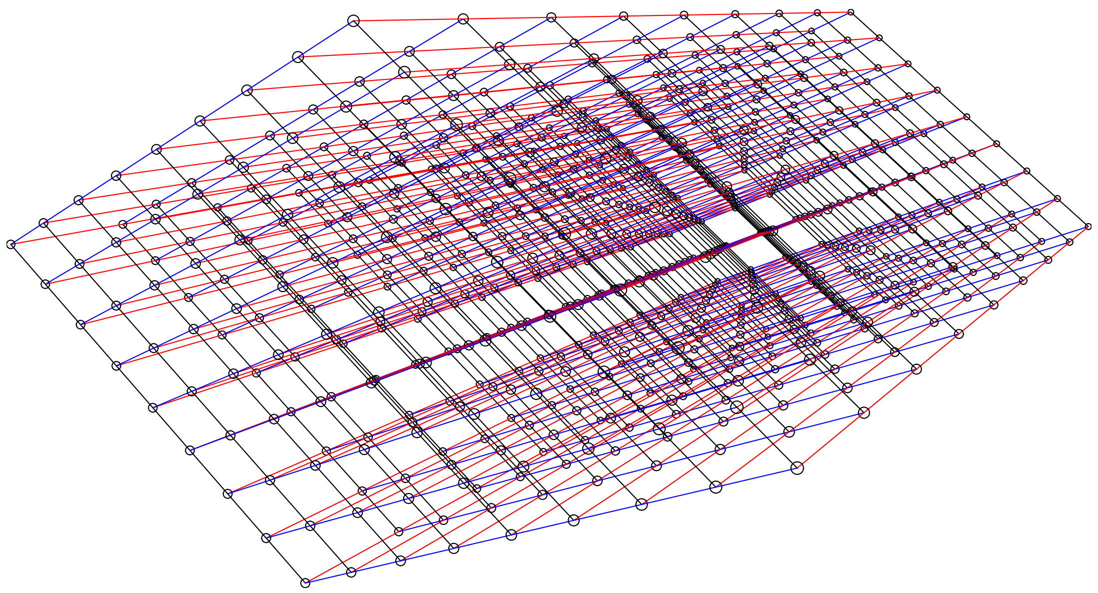

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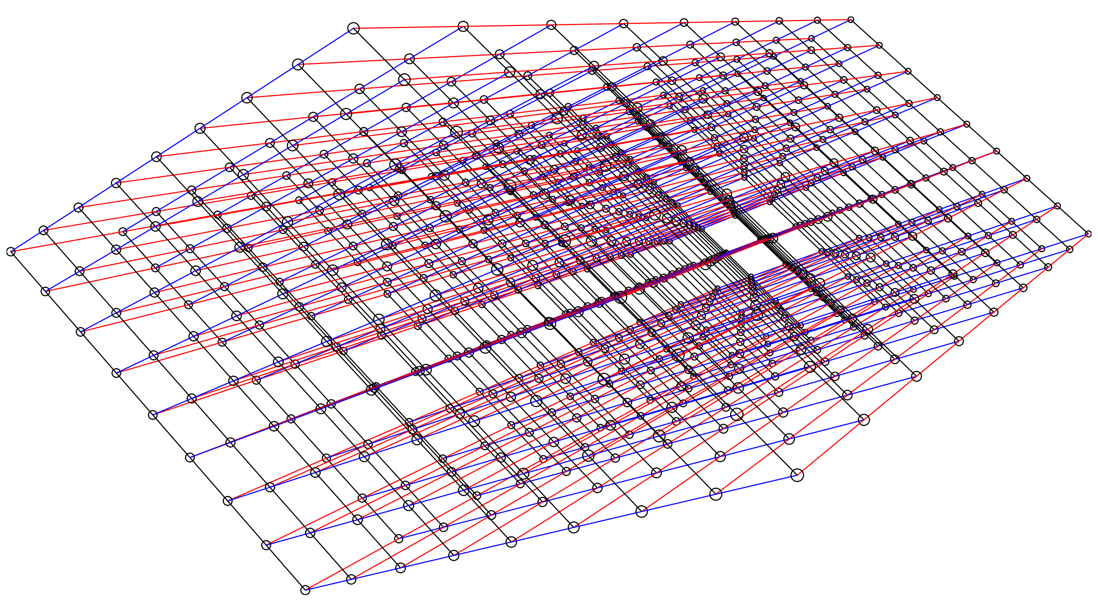

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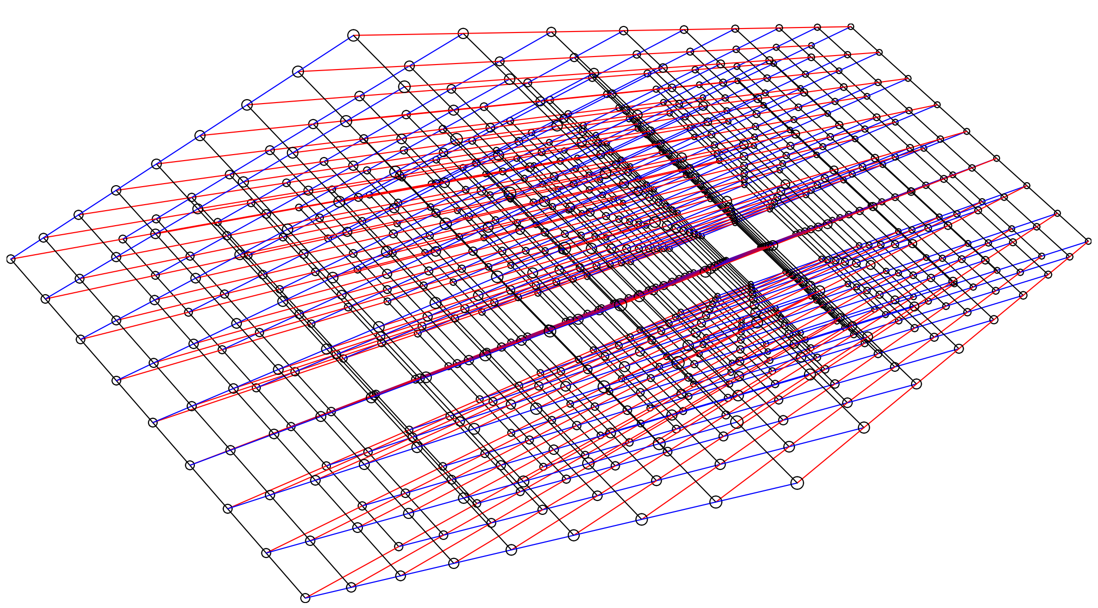

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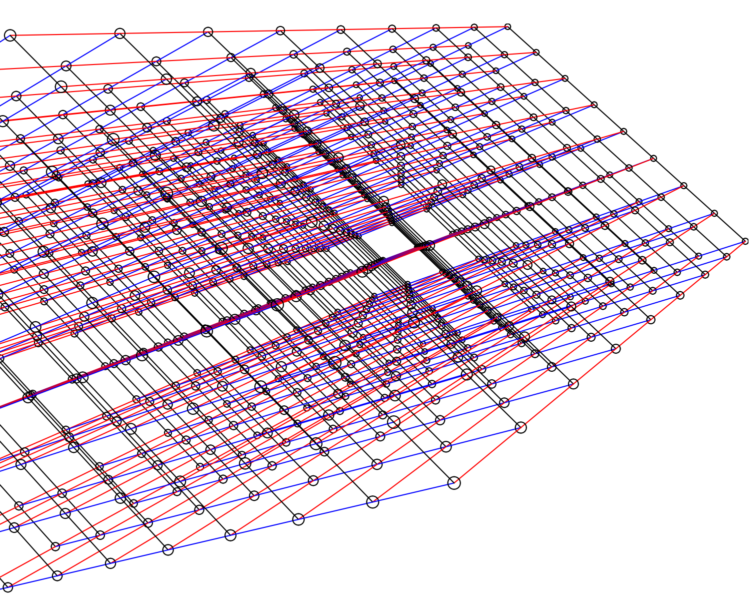

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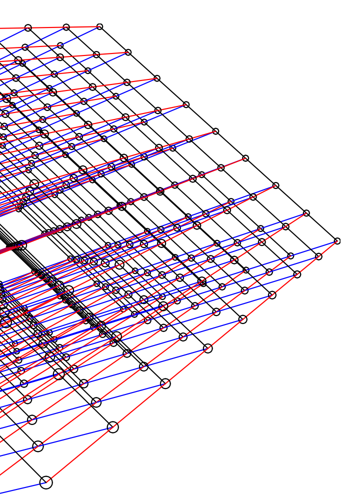

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One way to find $(r_1, \ldots, r_N)$ where $C = r_1 K_1 + \cdots + r_N K_N$:

Choose λ. Define
$V_0 = (-C, 0, 0, \ldots, 0)$,
$V_1 = (K_1, \lambda, 0, \ldots, 0)$,
$V_2 = (K_2, 0, \lambda, \ldots, 0)$,
$\ldots$,
$V_N = (K_N, 0, 0, \ldots, \lambda)$.

Define $L = \mathbf{Z}V_0 + \cdots + \mathbf{Z}V_N$.
L contains the short vector
$V_0 + r_1 V_1 + \cdots + r_N V_N = (0, r_1\lambda, \ldots, r_N\lambda)$.

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```

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596487875
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sage: q0
596487875
sage: round(K[0]/q0)
984887308997925
sage:
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$K_i - q_iK_j = 2q_ju_i - 2q_iu_j$.

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$-K_1, 0, \ldots, 0)$;

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sage: s
984887308997925
sage:
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sage: V
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 -11115
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 688178
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 102334
 -35716
 112142
 -11096
 -23562
sage:
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DGHV keys

$sq_i \approx sq_i$.

$u_i < E$.

$= 2q_j u_i - 2q_i u_j$.

$\ldots, K_N)$;

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$\cdots + q_N V_N =$

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 -11096748622762
 -23562893778500
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$2q_i u_j$.

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sage: V=matrix.identity(N)
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sage: s
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sage:
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 6881780210837495890175l,
 74236247096820082303539б
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 -35716867939855887673000
 11214216191199646010514
 -11096748622762224955871
 -23562893778500377052338
sage:
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984887308997925
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984887308997925
sage:
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sage: V[0]
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 688178021083749589017 51,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381)
sage:
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sage: V[0]
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 68817802108374958901751,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381)
sage: V[1]
(0, -587473338058640662659869,
 0, 0, 0, 0, 0, 0, 0, 0)
sage:
```

```
=matrix.identity(N)
=-K[0]*V
top=copy(K)
top[0]=E
[0]=Vtop
0=V.LLL()[0][0]/E
0
75
ound(K[0]/q0)
08997925

08997925
```



```
sage: V[0]
(1024,
 -1111539179100720083770339,
 794301459533783434896055,
 68817802108374958901751,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381)
sage: V[1]
(0, -587473338058640662659869,
 0, 0, 0, 0, 0, 0, 0, 0)
sage:
```

```
sage: V
(610803
 370302
 -22561
 110012
 135946
sage:
```

```
dentity(N)

K)

[0][0]/E

/q0)
```



```
sage: V[0]
(1024,
 -1111539179100720083770339,
 794301459533783434896055,
 68817802108374958901751,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381)
sage: V[1]
(0, -587473338058640662659869,
 0, 0, 0, 0, 0, 0, 0, 0)
sage:
```

```
sage: V.LLL()[0]
(610803584000, 1
 37030242384, 84
 -225618319442,
 1100126026284,
 1359463649048,
sage:
```

)


```
sage: V[0]
(1024,
 -1111539179100720083770339,
 794301459533783434896055,
 68817802108374958901751,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381)
sage: V[1]
(0, -58747338058640662659869,
 0, 0, 0, 0, 0, 0, 0, 0)
sage:
```



```
sage: V.LLL()[0]
(610803584000, 1056189937
 37030242384, 84589845469
 -225618319442, 363547143
 1100126026284, -31315097
 1359463649048, 174256676
sage:
```

```
sage: V[0]
(1024,
 -1111539179100720083770339,
 794301459533783434896055,
 688178021083749589017S1,
 742362470968200823035396,
 1023345827831539515054795,
 -3571686793985588767300O6,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381)
sage: V[1]
(0, -5874733380586406626598 69,
 0, 0, 0, 0, 0, 0, 0, 0)
sage:
```



```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage:
```

```
sage: V[0]
(1024,
 -1111539179100720083770339,
 7943014595337834348960 55,
 6881780210837495890175 1,
 7423624709682008230353 96,
 1023345827831539515054795,
 -3571686793985588767300 06,
 1121421619119964601051443,
 -1109674862276222495587129,
 -2356289377850037705233 81)
sage: V[1]
(0, -5874733380586406626598 69,
 0, 0, 0, 0, 0, 0, 0, 0)
sage:
```



```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage:
```

```
sage: V[0]
(1024,
 -1111539179100720083770339,
 794301459533783434896055,
 688178021083749589017516,
 742362470968200823035396,
 1023345827831539515054795,
 -3571686793985588767300006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381)
sage: V[1]
(0, -587473338058640662659869,
 0, 0, 0, 0, 0, 0, 0, 0)
sage:
```



```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage:
```

```
sage: V[0]
(1024,
 -1111539179100720083770339,
 7943014595337834348960 55,
 6881780210837495890175 1,
 7423624709682008230353 96,
 1023345827831539515054795,
 -3571686793985588767300 06,
 1121421619119964601051443,
 -1109674862276222495587129,
 -2356289377850037705233 81)
sage: V[1]
(0, -5874733380586406626598 69,
 0, 0, 0, 0, 0, 0, 0, 0)
sage:
```



```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage:
```

```
sage: V[0]
(1024,
 -1111539179100720083770339,
 794301459533783434896055,
 68817802108374958901751,
 742362470968200823035396,
 1023345827831539515054795,
 -357168679398558876730006,
 1121421619119964601051443,
 -1109674862276222495587129,
 -235628937785003770523381)
sage: V[1]
(0, -58747338058640662659869,
 0, 0, 0, 0, 0, 0, 0, 0)
sage:
```



```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage: q[0]*K[9]-q[9]*K[0]
174256676348
sage:
```

```
[0]

39179100720083770339,
459533783434896055,
02108374958901751,
470968200823035396,
5827831539515054795,
8679398558876730006,
1619119964601051443,
74862276222495587129,
8937785003770523381)
[1]
7473338058640662659869,
0, 0, 0, 0, 0, 0)
```



```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage: q[0]*K[9]-q[9]*K[0]
174256676348
sage:
```

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```
20083770339,
434896055,
58901751,
823035396,
9515054795,
8876730006,
4601051443,
22495587129,
3770523381)

8640662659869,
0, 0, 0)
```



```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage: q[0]*K[9]-q[9]*K[0]
174256676348
sage:
```

2009 DGHV analy
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```
339,
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31)
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```



```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage: q[0]*K[9]-q[9]*K[0]
174256676348
sage:
```

2009 DGHV analysis:
can choose key sizes where
these lattice attacks fail.

```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage: q[0]*K[9]-q[9]*K[0]
174256676348
sage:
```


2009 DGHV analysis:
can choose key sizes where
these lattice attacks fail.

```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage: q[0]*K[9]-q[9]*K[0]
174256676348
sage:
```


2009 DGHV analysis: can choose key sizes where these lattice attacks fail.

2011 Coron–Mandal–Naccache–Tibouchi: reduce key sizes by modifying DGHV. “This shows that fully homomorphic encryption can be implemented with a simple scheme.”

```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage: q[0]*K[9]-q[9]*K[0]
174256676348
sage:
```


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e.g. all attacks take $\geq 2^{72}$ cycles with public keys only

```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage: q[0]*K[9]-q[9]*K[0]
174256676348
sage:
```


2009 DGHV analysis: can choose key sizes where these lattice attacks fail.

2011 Coron–Mandal–Naccache–Tibouchi: reduce key sizes by modifying DGHV. “This shows that fully homomorphic encryption can be implemented with a simple scheme.”

e.g. all attacks take $\geq 2^{72}$ cycles with public keys only 802MB.

```
sage: V.LLL()[0]
(610803584000, 1056189937254,
 37030242384, 845898454698,
 -225618319442, 363547143644,
 1100126026284, -313150978512,
 1359463649048, 174256676348)
sage: q=[Ki//s for Ki in K]
sage: q[0]*E
610803584000
sage: q[0]*K[1]-q[1]*K[0]
1056189937254
sage: q[0]*K[9]-q[9]*K[0]
174256676348
sage:
```


2009 DGHV analysis:
can choose key sizes where
these lattice attacks fail.

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e.g. all attacks take $\geq 2^{72}$ cycles with public keys only 802MB.

2012 Chen–Nguyen: faster attack.
Need bigger DGHV/CMNT keys.

```
.LLL()[0]
584000, 1056189937254,
42384, 845898454698,
8319442, 363547143644,
6026284, -313150978512,
3649048, 174256676348)
=[Ki//s for Ki in K]
[0]*E
84000
[0]*K[1]-q[1]*K[0]
937254
[0]*K[9]-q[9]*K[0]
76348
```


2009 DGHV analysis: can choose key sizes where these lattice attacks fail.

2011 Coron–Mandal–Naccache–Tibouchi: reduce key sizes by modifying DGHV. "This shows that fully homomorphic encryption can be implemented with a simple scheme."

e.g. all attacks take $\geq 2^{72}$ cycles with public keys only 802MB.

2012 Chen–Nguyen: faster attack. Need bigger DGHV/CMNT keys.

Big atta

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```
056189937254,
5898454698,
363547143644,
-31315097851 2,
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or Ki in K]
```

```
q[1]*K[0]
```

```
q[9]*K[0]
```


2009 DGHV analysis:
can choose key sizes where
these lattice attacks fail.

2011 Coron–Mandal–Naccache–Tibouchi: reduce key sizes by modifying DGHV. "This shows that fully homomorphic encryption can be implemented with a simple scheme."

e.g. all attacks take $\geq 2^{72}$ cycles with public keys only 802MB.

2012 Chen–Nguyen: faster attack. Need bigger DGHV/CMNT keys.

Big attack surface

1991 Chaum–van
Pfitzmann: choose
define $C(x, y) = 4$
for suitable ranges

Simple, beautiful,
Very easy security
finding C collision
computing a discr

/254,
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2009 DGHV analysis:
can choose key sizes where
these lattice attacks fail.

2011 Coron–Mandal–Naccache–Tibouchi: reduce key sizes by modifying DGHV. "This shows that fully homomorphic encryption can be implemented with a simple scheme."

e.g. all attacks take $\geq 2^{72}$ cycles with public keys only 802MB.

2012 Chen–Nguyen: faster attack. Need bigger DGHV/CMNT keys.

Big attack surfaces are dang

1991 Chaum–van Heijst–
Pfitzmann: choose p sensibl
define $C(x, y) = 4^x 9^y \bmod$
for suitable ranges of x and

Simple, beautiful, structured
Very easy security reduction
finding C collision implies
computing a discrete logarit

2009 DGHV analysis:
can choose key sizes where
these lattice attacks fail.

2011 Coron–Mandal–Naccache–Tibouchi: reduce key sizes by modifying DGHV. "This shows that fully homomorphic encryption can be implemented with a simple scheme."

e.g. all attacks take $\geq 2^{72}$ cycles with public keys only 802MB.

2012 Chen–Nguyen: faster attack.
Need bigger DGHV/CMNT keys.

Big attack surfaces are dangerous

1991 Chaum–van Heijst–Pfitzmann: choose p sensibly; define $C(x, y) = 4^x 9^y \bmod p$ for suitable ranges of x and y.

Simple, beautiful, structured.
Very easy security reduction:
finding C collision implies
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2009 DGHV analysis:
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Simple, beautiful, structured. Very easy security reduction: finding C collision implies computing a discrete logarithm.

Typical exaggerations:
C is "provably secure"; C is "cryptographically collision-free"; "security follows from rigorous mathematical proofs".

GHV analysis:
ose key sizes where
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i: reduce key sizes
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attacks take $\geq 2^{72}$ cycles
blic keys only 802MB.

en–Nguyen: faster attack.
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Big attack surfaces are dangerous

1991 Chaum–van Heijst–Pfitzmann: choose p sensibly; define $C(x, y) = 4^x 9^y \bmod p$ for suitable ranges of x and y.

Simple, beautiful, structured. Very easy security reduction: finding C collision implies computing a discrete logarithm.

Typical exaggerations:
C is “provably secure”; C is “cryptographically collision-free”; “security follows from rigorous mathematical proofs”.

Security losses in C
1922 Kraitchik (in
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Big attack surfaces are dangerous

1991 Chaum–van Heijst–Pfitzmann: choose p sensibly; define $C(x, y) = 4^x 9^y \bmod p$ for suitable ranges of x and y.

Simple, beautiful, structured. Very easy security reduction: finding C collision implies computing a discrete logarithm.

Typical exaggerations: C is "provably secure"; C is "cryptographically collision-free"; "security follows from rigorous mathematical proofs".

Security losses in C include
1922 Kraitchik (index calcul
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1993 Schirokauer (faster NF
1994 Shor (quantum poly ti
many subsequent attack spe
from people who care about
pre-quantum security.

C is very bad cryptography.
No matter what user's cost
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function designs such as BL

Big attack surfaces are dangerous

1991 Chaum–van Heijst–Pfitzmann: choose p sensibly; define $C(x, y) = 4^x 9^y \bmod p$ for suitable ranges of x and y.

Simple, beautiful, structured. Very easy security reduction: finding C collision implies computing a discrete logarithm.

Typical exaggerations: C is "provably secure"; C is "cryptographically collision-free"; "security follows from rigorous mathematical proofs".

Security losses in C include
1922 Kraitchik (index calculus);
1986 Coppersmith–Odlyzko–Schroeppel (NFS predecessor);
1993 Gordon (general DL NFS);
1993 Schirokauer (faster NFS);
1994 Shor (quantum poly time);
many subsequent attack speedups from people who care about pre-quantum security.

C is very bad cryptography. No matter what user's cost limit is, obtain better security with "unstructured" compression-function designs such as BLAKE.

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C collision implies
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Security losses in C include
1922 Kraitchik (index calculus);
1986 Coppersmith–Odlyzko–
Schroeppel (NFS predecessor);
1993 Gordon (general DL NFS);
1993 Schirokauer (faster NFS);
1994 Shor (quantum poly time);
many subsequent attack speedups
from people who care about
pre-quantum security.

C is very bad cryptography.
No matter what user's cost limit
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"unstructured" compression-
function designs such as BLAKE.

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Security losses in C include
1922 Kraitchik (index calculus);
1986 Coppersmith–Odlyzko–
Schroeppel (NFS predecessor);
1993 Gordon (general DL NFS);
1993 Schirokauer (faster NFS);
1994 Shor (quantum poly time);
many subsequent attack speedups
from people who care about
pre-quantum security.

C is very bad cryptography.
No matter what user’s cost limit
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“unstructured” compression-
function designs such as BLAKE.

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Security losses in C include
1922 Kraitchik (index calculus);
1986 Coppersmith–Odlyzko–Schroeppel (NFS predecessor);
1993 Gordon (general DL NFS);
1993 Schirokauer (faster NFS);
1994 Shor (quantum poly time);
many subsequent attack speedups from people who care about pre-quantum security.

C is very bad cryptography.
No matter what user's cost limit is, obtain better security with "unstructured" compression-function designs such as BLAKE.

For public-key encryption:
Some mathematical structur
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Security losses in C include
1922 Kraitchik (index calculus);
1986 Coppersmith–Odlyzko–Schroeppel (NFS predecessor);
1993 Gordon (general DL NFS);
1993 Schirokauer (faster NFS);
1994 Shor (quantum poly time);
many subsequent attack speedups from people who care about pre-quantum security.

C is very bad cryptography. No matter what user's cost limit is, obtain better security with "unstructured" compression-function designs such as BLAKE.

For public-key encryption: Some mathematical structure seems to be unavoidable, but pursuing simple structures often leads to security disasters.

Security losses in C include
1922 Kraitchik (index calculus);
1986 Coppersmith–Odlyzko–Schroeppel (NFS predecessor);
1993 Gordon (general DL NFS);
1993 Schirokauer (faster NFS);
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C is very bad cryptography.
No matter what user's cost limit is, obtain better security with "unstructured" compression-function designs such as BLAKE.

For public-key encryption:
Some mathematical structure seems to be unavoidable, but pursuing simple structures often leads to security disasters.

Pre-quantum example: DH is simpler than ECDH, but DH has suffered many more security losses than ECDH. State-of-the-art DH attacks are very complicated.

Security losses in C include
1922 Kraitchik (index calculus);
1986 Coppersmith–Odlyzko–Schroeppel (NFS predecessor);
1993 Gordon (general DL NFS);
1993 Schirokauer (faster NFS);
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For public-key encryption: Some mathematical structure seems to be unavoidable, but pursuing simple structures often leads to security disasters.

Pre-quantum example: DH is simpler than ECDH, but DH has suffered many more security losses than ECDH. State-of-the-art DH attacks are very complicated.

2013 Barbulescu–Gaudry–Joux–Thomé: pre-quantum quasi-poly break of small-characteristic DH.

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2018 Anvers–Vercauteren–Verbauwhede: "an attacker can significantly reduce the security of (Ring/Module)-LWE/LWR based schemes that have a relatively high failure rate".

Frodo, Kyber, LAC, NewHope, Round5, SABER, ThreeBears have nonzero failure rates.

For LAC-128, "the failure rate is 2^{48} times bigger than estimated". Failure rate is also what broke first version of Round5.

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