Cryptanalysis of NISTPQC submissions

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PQCRYPTO
ICT-645622

18 August 2018

Workshops on Attacks in Cryptography

NSA announcements

August 11, 2015

> *IAD recognizes that there will be a move, in the not distant future, to a quantum resistant algorithm suite.*

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August 19, 2015

> *IAD will initiate a transition to quantum resistant algorithms in the not too distant future.*

Post-quantum cryptography

- 2015 Finally even NSA admits that the world needs post-quantum crypto.
- 2016 Every agency posts something (NCSC UK, NCSC NL, NSA (broken certificate!)).
- 2016 NIST announces call for submissions to post-quantum project, solicits submissions on signatures, encryption, and key exchange.

Post-quantum cryptography

- 10 years of motivating people to work on post-quantum crypto.
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NIST Post-Quantum "Competition"

December 2016, after public feedback: NIST calls for submissions of post-quantum cryptosystems to standardize.

30 November 2017: NIST receives 82 submissions.

Overview from Dustin Moody's (NIST) talk at Asiacrypt:

	Signatures	KEM/Encryption	Overall
Lattice-based	4	24	28
Code-based	5	19	24
Multi-variate	7	6	13
Hash-based	4		4
Other	3	10	13
Total	**23**	**59**	**82**

“Complete and proper” submissions

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Attack timeline: month 0

2017.12.18 Bernstein–Groot Bruinderink–Panny–Lange:
attack script breaking CCA for HILA5

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☢: submitter has claimed patent on submission.
Warning: Other people could also claim patents.

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Attack timeline: subsequent events

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Color coding: **total break**; **partial break**

HILA5

- HILA5 is a RLWE-based KEM submitted to NISTPQC.

  *This design also provides **IND-CCA secure** KEM-DEM public key encryption if used in conjunction with an appropriate AEAD such as NIST approved AES256-GCM.*

  — HILA5 NIST submission document (v1.0)

- Decapsulation much faster than encapsulation (and faster than any other scheme).
- No mention of a CCA transform (e.g. Fujisaki–Okamoto).

Noisy Diffie–Hellman

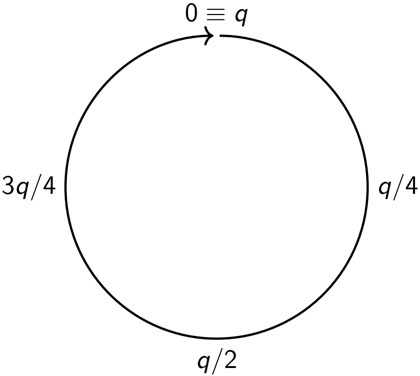
- Have a ring $R = \mathbf{Z}[x]/(q, \varphi)$ where $q \in \mathbf{Z}$ and $\overset{\text{degree } n}{\varphi} \in \mathbf{Z}[x]$.
- Let χ be a narrow distribution around $0 \in R$.
- Fix some "random" element $g \in R$.

$a, e \leftarrow \chi^n$	$b, e' \leftarrow \chi^n$
$A = ga + e$	$B = gb + e'$
$S = Ba = gab + e'a$	$S' = Ab = gab + eb$

$$\implies S - S' = e'a - eb \underset{\chi \text{ small}}{\underset{\uparrow}{\approx}} 0$$

Reconciliation

Alice and Bob obtain close secret vectors $S, S' \in (\mathbf{Z}/q)^n$.
How to map coefficients to bits?



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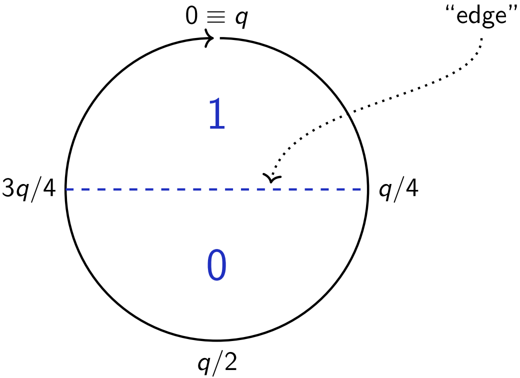

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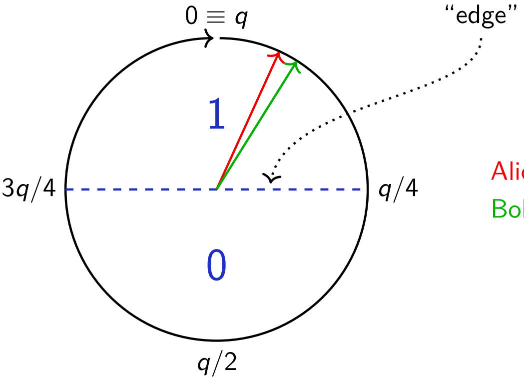

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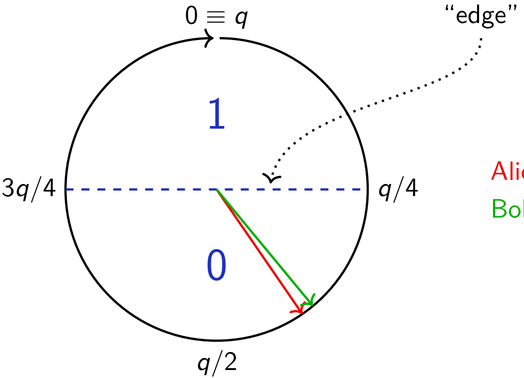

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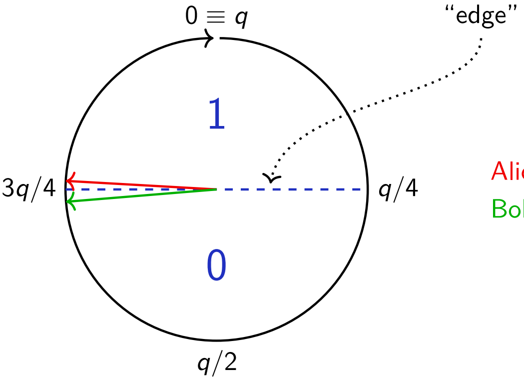

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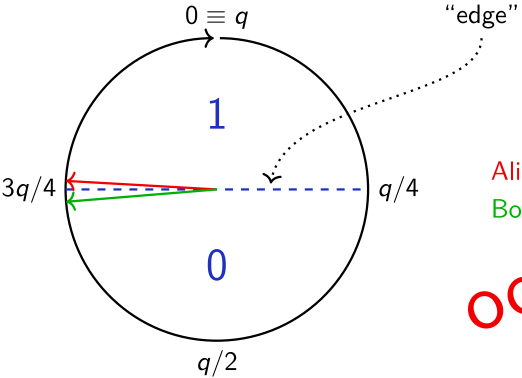

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oops!

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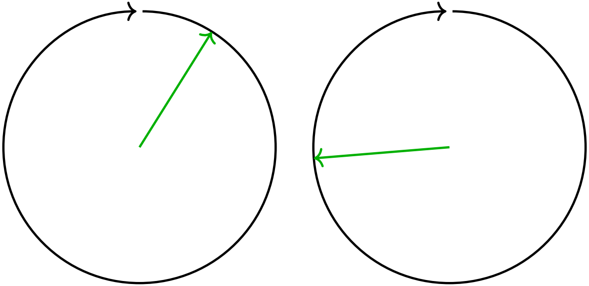

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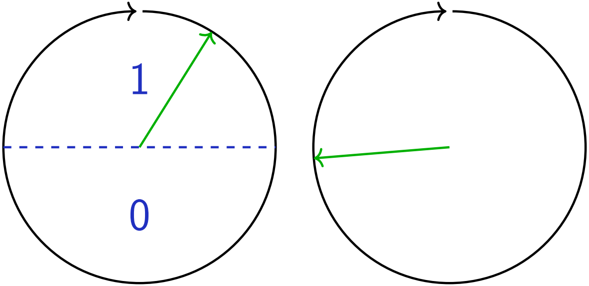

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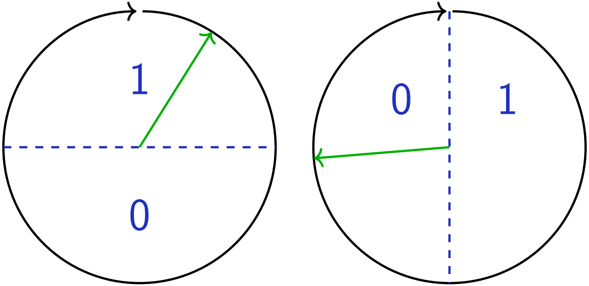

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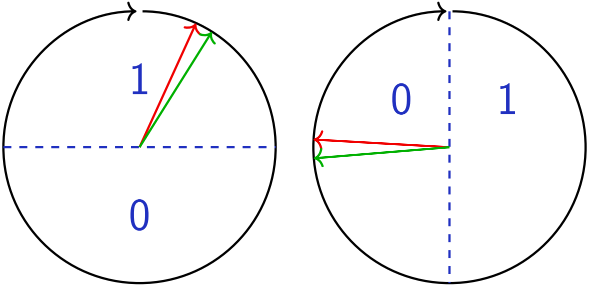

Fluhrer's attack https://ia.cr/2016/085

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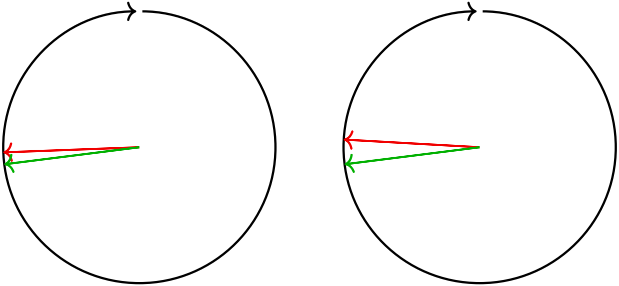

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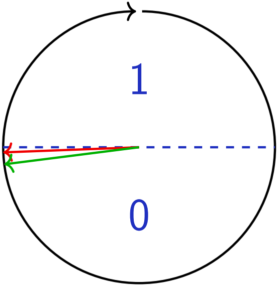

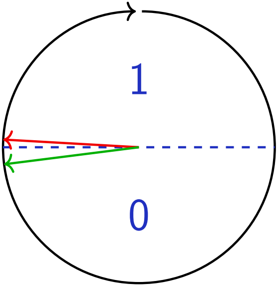

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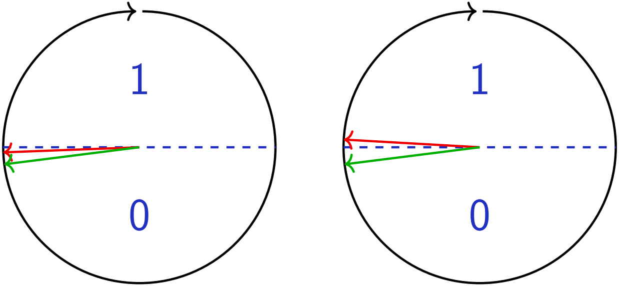

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Evil Bob can distinguish these cases!

(He knows all the other key bits.)

Chosen-ciphertext information leaks

Evil Bob has two guesses k_0, k_1 for what Alice's key k will be given his manipulated public key B.

Alice

Evil Bob

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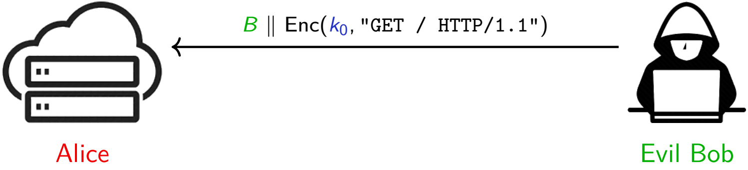

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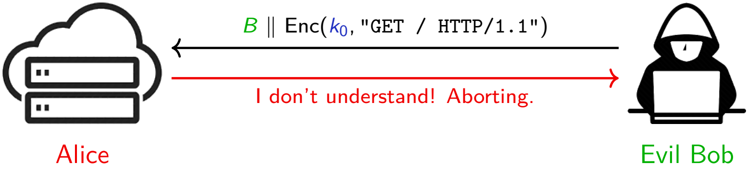

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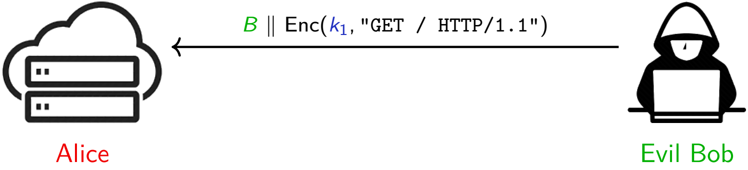

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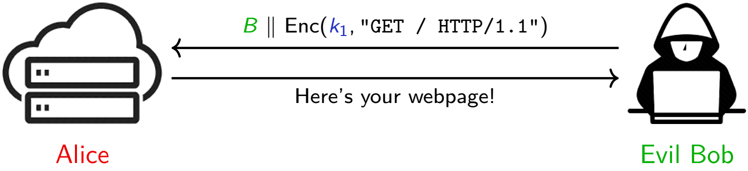

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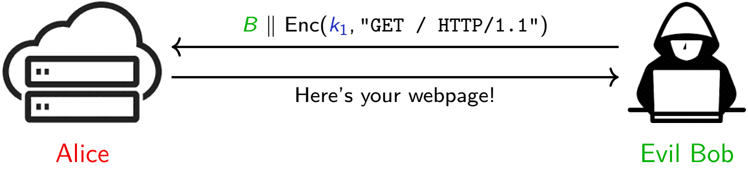

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$B \parallel \mathrm{Enc}(k_0, \texttt{"GET / HTTP/1.1"})$

Decryption failure! Aborting.

Alice

Evil Bob

$\Longrightarrow$ Bob learns that $k = k_1$.

This still works if Enc is an authenticated symmetric cipher!

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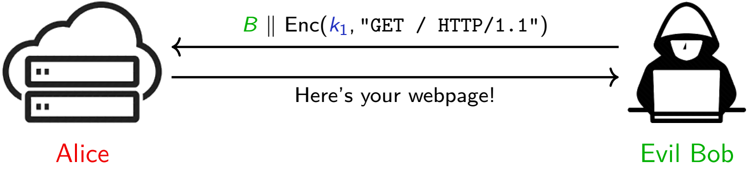

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edge

Suppose Evil Bob knows b_δ such that $g\,ab_\delta[0] = M + \delta$.
$\Longrightarrow$ Querying Alice with $b = b_\delta$ leaks whether $-e'a[0] > \delta$.

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Structure of R
$\rightsquigarrow$ Can choose e' such that $e'a[0] = a[i]$ to recover all of a.

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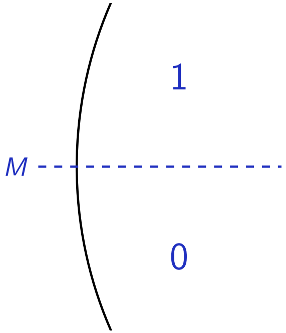
Querying Alice with $b = b_\delta$ and $e' = 1$ leaks whether $-a[0] > \delta$.



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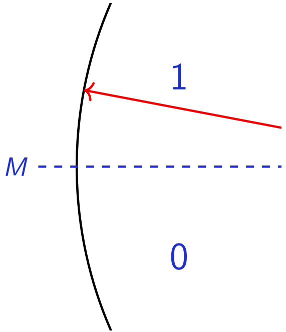

Evil Bob's δ: 0

Alice: 1

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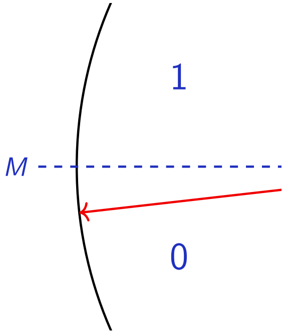

Evil Bob's δ: -8

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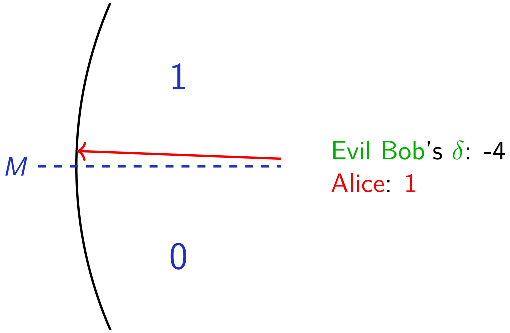

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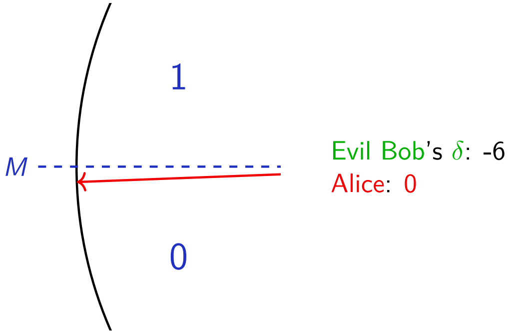

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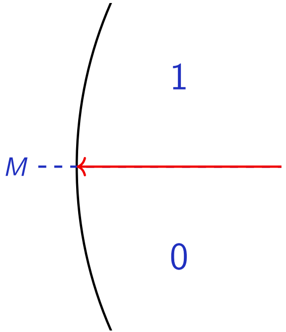

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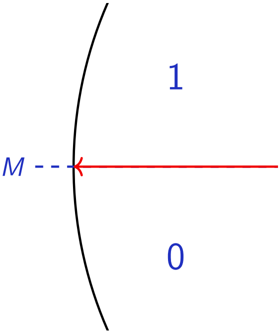

Evil Bob's δ: -5
Alice: 1

$\Longrightarrow$ Evil Bob learns that $a[0] = 5$.

Our work

Adaption of Fluhrer's attack to HILA5 and analysis

HILA5 https://ia.cr/2017/424 https://github.com/mjosaarinen/hila5

- Standard noisy Diffie–Hellman with new reconciliation.

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- Ring: $\mathbf{Z}[x]/(q, x^{1024}+1)$ where $q = 12289$.[1]
- Noise distribution χ: Ψ_{16}.[1]  on $\{-16, \ldots, 16\}$

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- Ring: $\mathbf{Z}[x]/(q, x^{1024}+1)$ where $q = 12289$.[1]
- Noise distribution χ: Ψ_{16}.[1]  on $\{-16, ..., 16\}$
- New reconciliation mechanism:
  - Only use "safe bits" that are far from an edge.
  - Additionally apply an error-correcting code.

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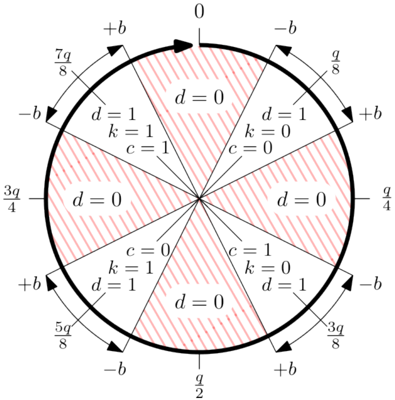
HILA5's reconciliation



(picture: HILA5 documentation)

For each coefficient:

$d = 0$: Discard coefficient.

$d = 1$: Send reconciliation information c; use for key bit k.

Edges:

$c = 0$: $\lceil 3q/8 \rfloor \ldots \lceil 7q/8 \rfloor \rightsquigarrow k = 0$.
$\lceil 7q/8 \rfloor \ldots \lceil 3q/8 \rfloor \rightsquigarrow k = 1$.

$c = 1$: $\lceil q/8 \rfloor \ldots \lceil 5q/8 \rfloor \rightsquigarrow k = 0$.
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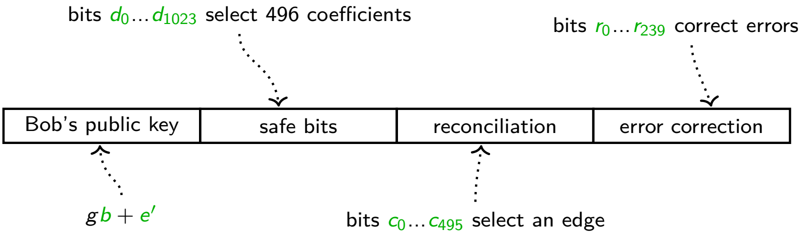
HILA5's packet format



HILA5's packet format

bits $d_0 \ldots d_{1023}$ select 496 coefficients

bits $r_0 \ldots r_{239}$ correct errors

Bob's public key	safe bits	reconciliation	error correction

$g\,b + e'$

bits $c_0 \ldots c_{495}$ select an edge

We're going to manipulate each of these parts.

Unsafe bits

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$\Longrightarrow$ Force $d_0 = 1$ to make Alice use it.

Living on the edge

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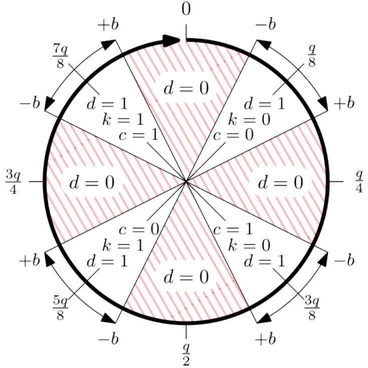
We want to attack the edge at $M = \lceil q/8 \rfloor$.



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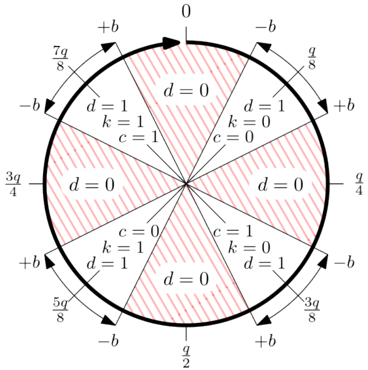

Making errors

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- HILA5 uses a custom linear error-correcting code XE5.
- Encrypted (XOR) using part of Bob's shared secret S'.
- Ten variable-length codewords $R_0 \ldots R_9$.
- Alice corrects $S[0]$ using the first bit of each R_i.
- Capable of correcting (at least) 5-bit errors.

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We want to keep errors in $S[0]$. $\implies$ Flip the first bit of $R_0 \dots R_4$!

All coefficients for the price of one

$gb + e'$	safe bits	reconciliation	error correction

Our binary search recovers $e'a[0]$ from $gab_\delta + e'a$ by varying δ.
How to get $a[1]$, $a[2]$, ..?

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By construction of $R = \mathbf{Z}[x]/(q, x^{1024} + 1)$,
Evil Bob can rotate $a[i]$ into $e'a[0]$ by setting $e' = -x^{1024-i}$.

Running the search for all i yields all coefficients of a.

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If b_0 was wrong, the recovered coefficients are all 0 or -1.
$\Longrightarrow$ easily detectable.

Implementation

- Our code[1] attacks the HILA5 reference implementation.
- 100% success rate in our experiments.
- Less than 6000 queries (virtually always).

  (Note: Evil Bob could recover fewer coefficients and compute the rest by solving a lattice problem of reduced dimension.)

[1] https://helaas.org/hila5-20171218.tar.gz

TIJ/E KWALITEITSGARANTIE
SMEUÏG TOT OP DE BODEM
NOF
BRON VAN BOUWSTOFFEN*
NOF
HILA5
PINDAKAAS
STUKJES PINDA

HK17

"HK17 consists broadly in a Key Exchange Protocol (KEP) based on non-commutative algebra of hypercomplex numbers limited to quaternions and octonions. In particular, this proposal is based on non-commutative and non-associative algebra using octonions."

Security analysis: ". . . In our protocol, we could not find any ways to proceed with any abelianization of our octonions non-associative Moufang loop [29] or reducing of the GSDP problem of polynomial powers of octonions to a finitely generated nilpotent image of the given free group in the cryptosystem and a further nonlinear decomposition attack. We simply conclude that Roman'kov attacks do not affect our proposal."

What are octonions?

$\mathbf{R}$: set of real numbers.

$\mathbf{C}$: set of complex numbers; dim-2 $\mathbf{R}$-vector space.

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Exercise: Every $q \in \mathbf{O}$ has $q^2 = tq - n$ and $q^* = t - q$ for some $t, n \in \mathbf{R}$.

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Use integers modulo prime p instead of real numbers.
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Shared secret: $a^m (b^k r b^\ell) a^n = b^k (a^m r a^n) b^\ell$.

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Does $a^m r a^n$ mean $(a^m r)a^n$, or $a^m(ra^n)$?
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- **Flexible identity**: $x(yx) = (xy)x$.
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- **Moufang identities**: $z(x(zy)) = ((zx)z)y$; $x(z(yz)) = ((xz)y)z$; $(zx)(yz) = (z(xy))z = z((xy)z)$.

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So $a(aa) = (aa)a$; $a(a(aa)) = (aa)(aa) = ((aa)a)a$; etc.

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- **Flexible identity**: $x(yx) = (xy)x$.
- **Alternative identity**: $x(xy) = (xx)y$ and $y(xx) = (yx)x$.
- **Moufang identities**: $z(x(zy)) = ((zx)z)y$; $x(z(yz)) = ((xz)y)z$; $(zx)(yz) = (z(xy))z = z((xy)z)$.

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Why does HK17 work?

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$q^m(q^k r q^\ell)q^n = q^k(q^m r q^n)q^\ell$.
$a^m(b^k r b^\ell)a^n = b^k(a^m r a^n)b^\ell$ because a, b are polynomials in q.

A fast attack, and a faster attack

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Even faster: Attacker solves $a^m r a^n = (q+x)r(yq+z)$.
Eight equations in three variables x, y, z; linearize.
This was the second attack script: practically instantaneous.

RaCoSS – Random Code-based Signature Schemes

- System parameters: $n = 2400$, $k = 2060$.
  Random matrix $H \in \mathbf{F}_2^{(n-k)\times n}$.
- Secret key: sparse $S \in \mathbf{F}_2^{n\times n}$.
- Public key: $T = H \cdot S$. (looks pretty random).
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RaCoSS

Implementation bug:

```
unsigned char  c[RACOSS_N];
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for( i=0 ; i<(RACOSS_N/8) ; i++ )
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...compares only the first 300 coefficients!
Thus, a signature with $c[0\ldots299] = 0$ is accepted for

$$\binom{2100}{3} / \binom{2400}{3} \approx 67\%$$

of all messages.

The weight-restricted hash function (`wrhf`)

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$$\binom{2400}{3} = 2301120800 \sim 2^{31.09}$$
possible outputs.
- Slow: 600 to 800 hashes per second and core.
- Expected time for a preimage on $\approx$ 100 cores: 10 hours.
- crashed while brute-forcing: memory leaks
- another message signed by the first KAT:

NISTPQC is so much fun! 10900qmmP

Wait, there is more!

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$$v + Tc = \left(\quad \right) = \left(\qquad\qquad H \qquad\qquad \right) \left(\begin{array}{c} \\ \\ z \\ \\ \\ \end{array} \right)$$

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  Pick $n - k$ columns of H that form an invertible matrix H_1.
- Compute $z = (z_1 || 00 \ldots 0)$ by linear algebra.
- Expected weight of z is $\approx (n - k)/2 = 170 \ll 1564$.
- Properly generated signatures have weight$(z) \approx 261$.

RaCoSS – Summary

- Bug in code: bit vs. byte confusion meant only every 8th bit verified.
- Preimages for RaCoSS' special hash function: only

$$\binom{2400}{3} = 2301120800 \sim 2^{31.09}$$

  possible outputs.
- The code dimensions give a lot of freedom to the attacker – our forged signature is better than a real one!

Code-based encryption

BIG QUAKE
Classic McEliece
LAKE
LOCKER
DAGS
LEDAkem
LEDApkc
Lepton
McNie

Edon-K✝
BIKE☢
HQC☢
NTS-KEM☢
Ouroboros-R☢
QC-MDPC KEM☢
RQC☢
RLCE-KEM☢

✝: submitter has withdrawn submission.

Lattice-based encryption

CRYSTALS-KYBER
EMBLEM and R.EMBLEM
FrodoKEM
KINDI
LAC
LIMA
LOTUS
NewHope
NTRUEncrypt
NTRU-HRSS-KEM
NTRU Prime
Odd Manhattan
SABER
Titanium
HILA5
Ding Key Exchange☢
Lizard☢
KCL OKCN/AKCN/CNKE☢
Round2☢
Compact LWE☢

Other encryption

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Guess Again: hard to classify
HK17✝: hard to classify
RVB✝: hard to classify

Signatures

Gravity-SPHINCS: hash-based
Picnic: hash-based
SPHINCS+: hash-based

DualModeMS: multivariate
GeMSS: multivariate
HiMQ-3: multivariate
LUOV: multivariate
Giophantus: multivariate
Gui☢: multivariate
MQDSS☢: multivariate
Rainbow☢: multivariate

pqRSA: factoring-based

CRYSTALS-DILITHIUM: lattice-based
qTESLA: lattice-based
DRS: lattice-based
FALCON☢: lattice-based
pqNTRUSign☢: lattice-based

pqsigRM: code-based
RaCoSS: code-based
RankSign✝: code-based

WalnutDSA☢: braid-group

Further resources

- https://2017.pqcrypto.org/school: PQCRYPTO summer school with 21 lectures on video + slides + exercises.
- https://2017.pqcrypto.org/exec: Executive school (12 lectures), less math, more overview. So far slides, soon videos.
- https://pqcrypto.org: Our survey site.
  - Many pointers: e.g., to PQCrypto conferences.
  - Bibliography for 4 major PQC systems.
- https://pqcrypto.eu.org: PQCRYPTO EU project.
  - Expert recommendations.
  - Free software libraries.
  - More video presentations, slides, papers.
- https://twitter.com/pqc_eu: PQCRYPTO Twitter feed.
- https://twitter.com/PQCryptoConf: PQCrypto conference Twitter feed.
- https://csrc.nist.gov/projects/post-quantum-cryptography/round-1-submissions NIST PQC competition.