ECCHacks:
a gentle introduction
to elliptic-curve cryptography

Daniel J. Bernstein
University of Illinois at Chicago &
Technische Universiteit Eindhoven

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Technische Universiteit Eindhoven

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Cryptography

Public-key signatures:
e.g., RSA, DSA, ECDSA.
Some uses: signed OS updates,
SSL certificates, e-passports.

Public-key encryption:
e.g., RSA, DH, ECDH.
Some uses: SSL key exchange,
locked iPhone mail download.

Secret-key encryption:
e.g., AES, Salsa20.
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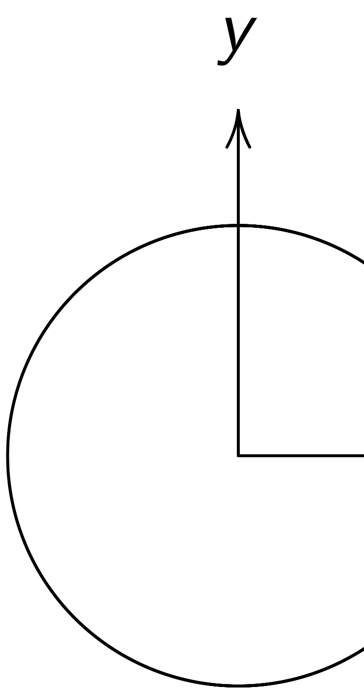

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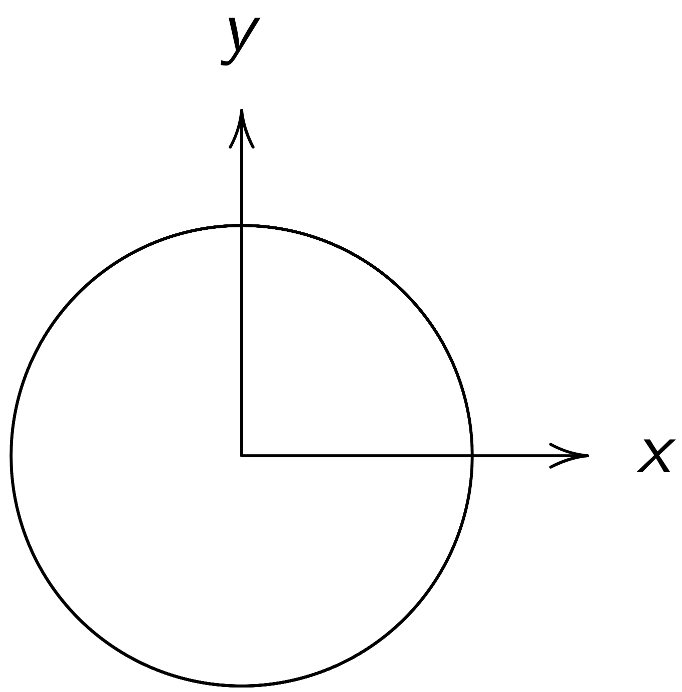

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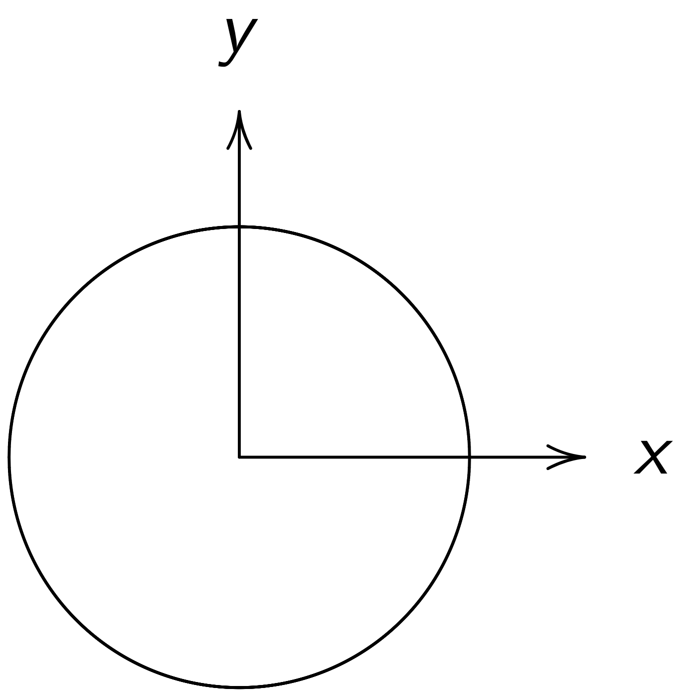

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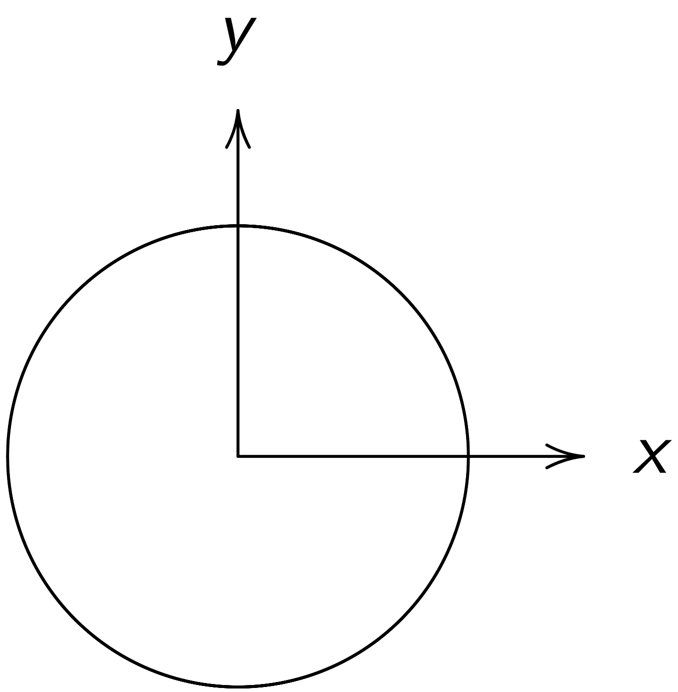

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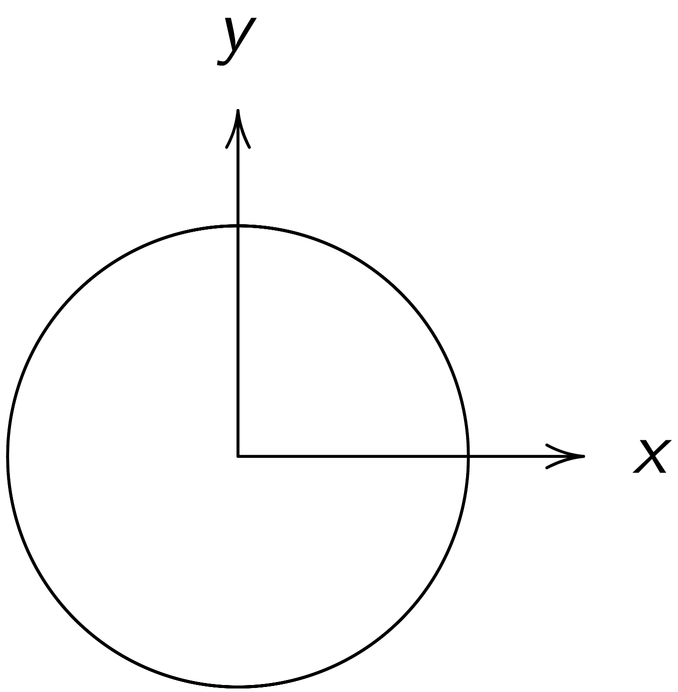

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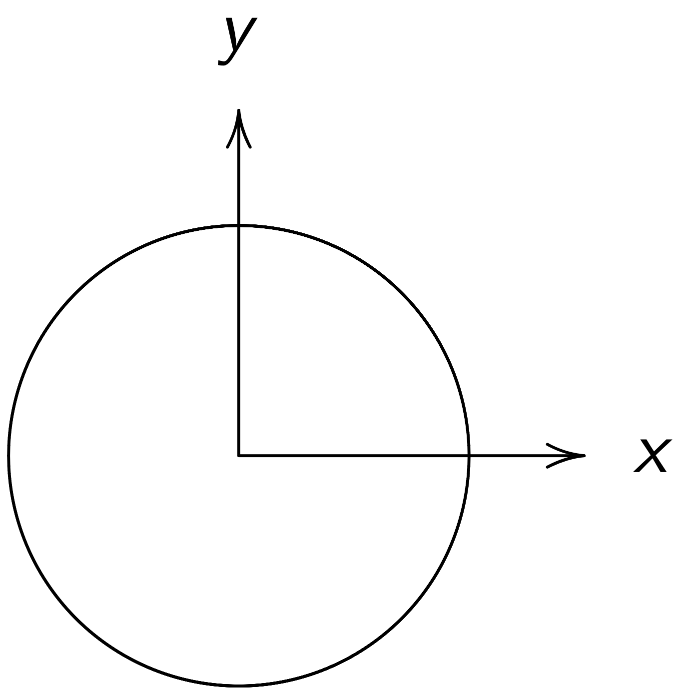

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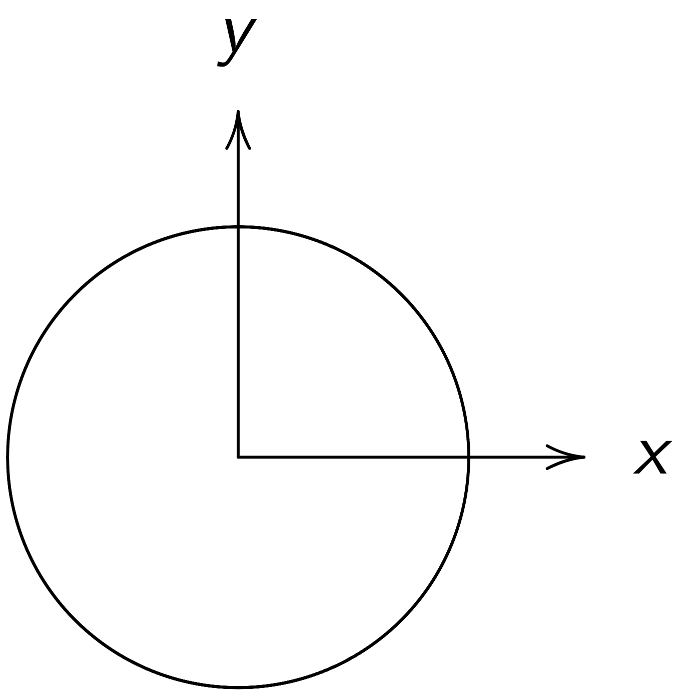

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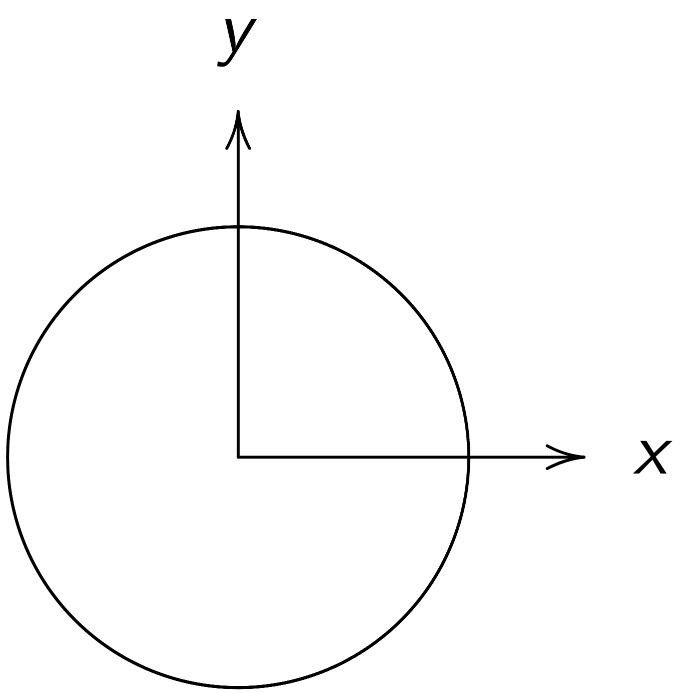

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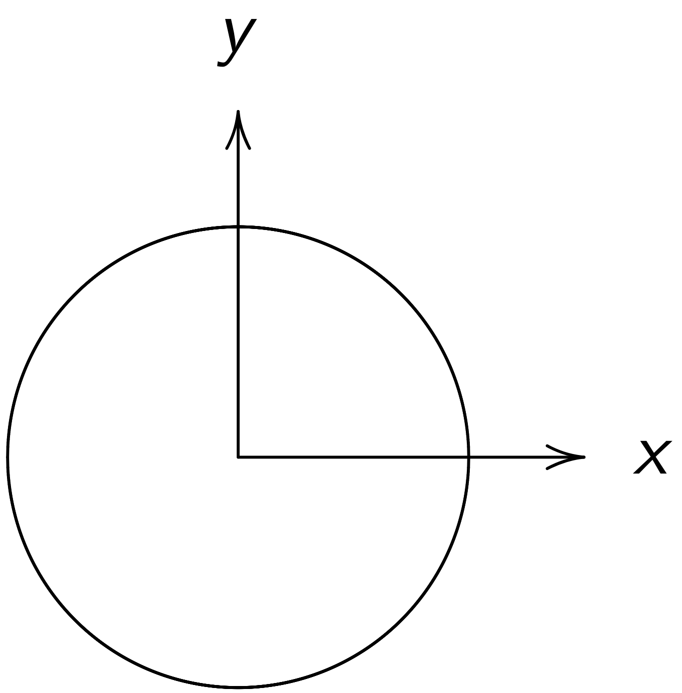

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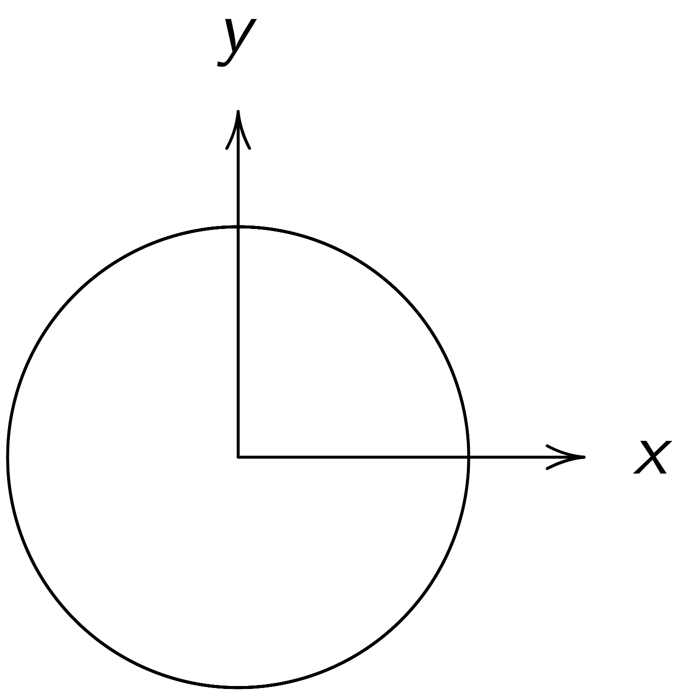

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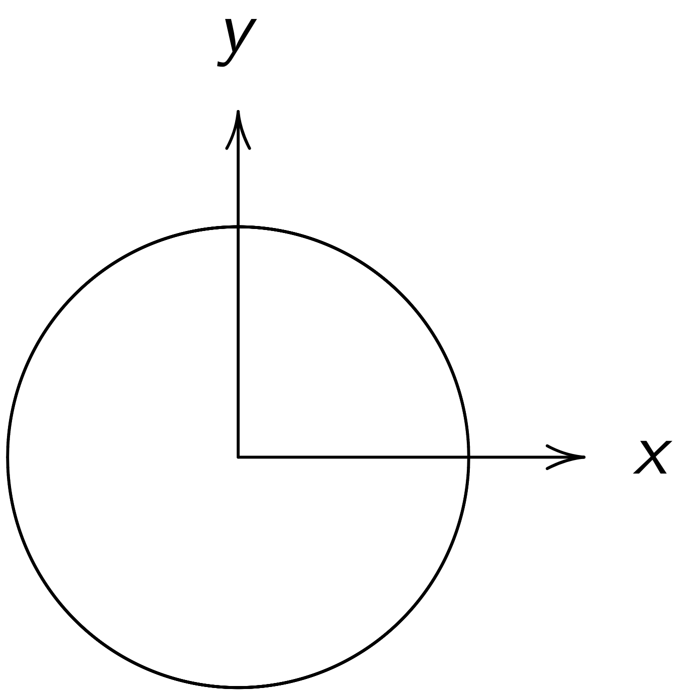

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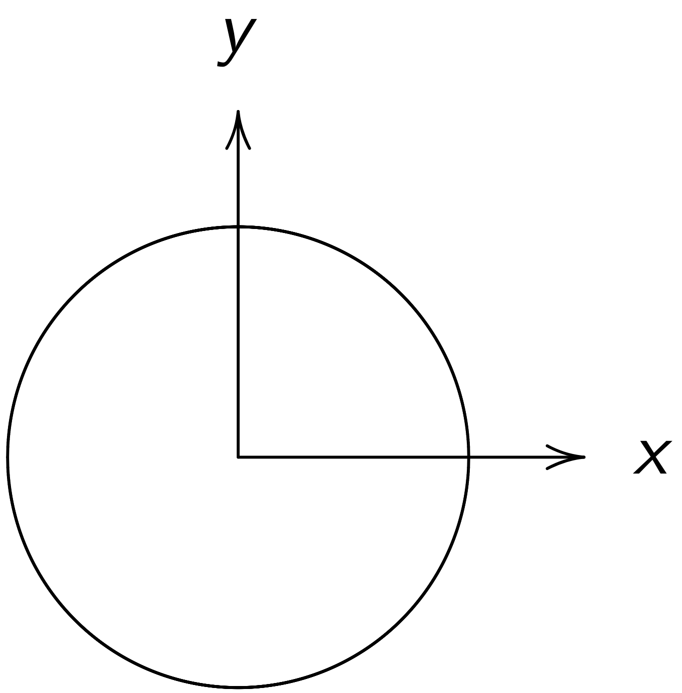

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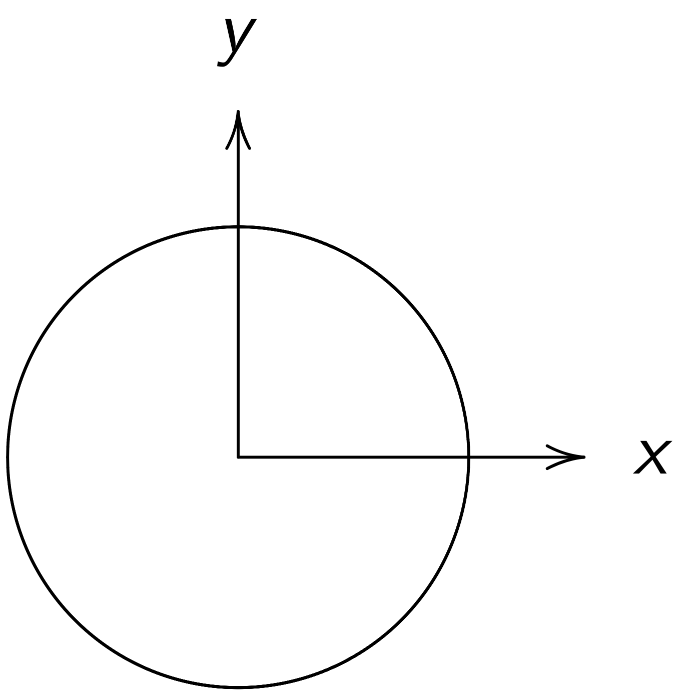

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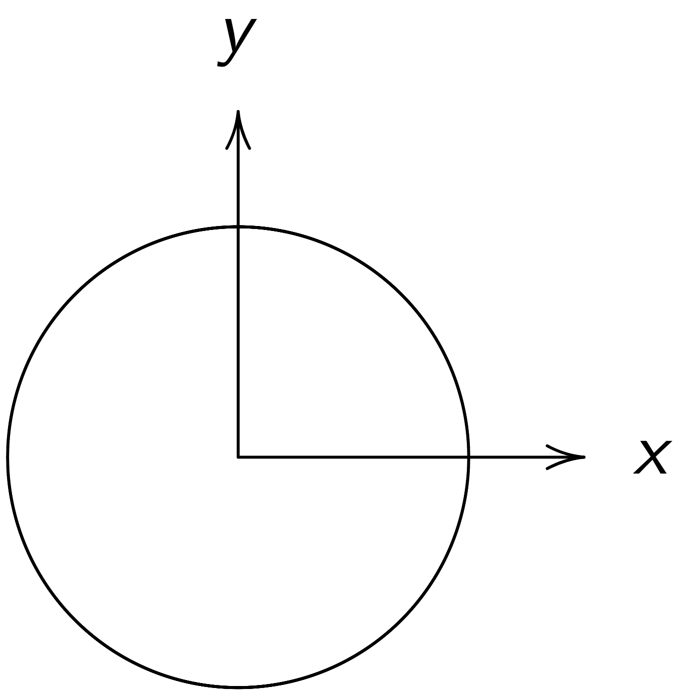

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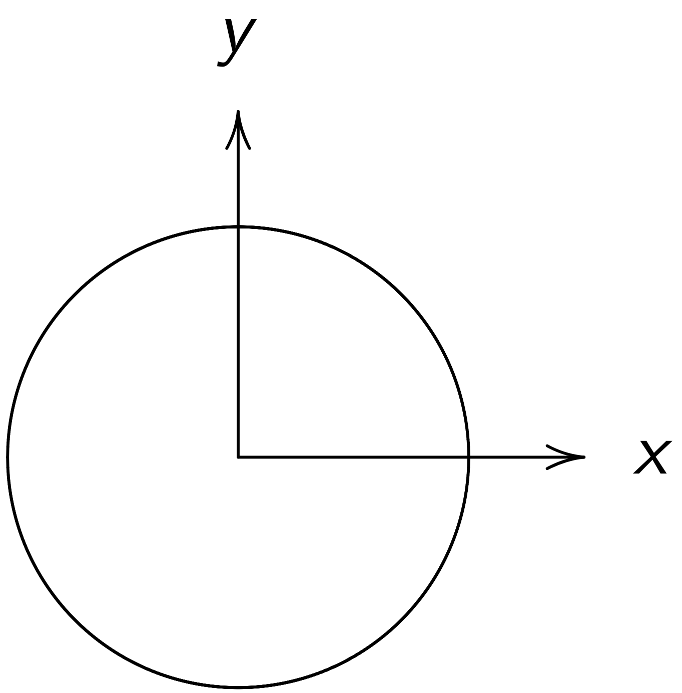

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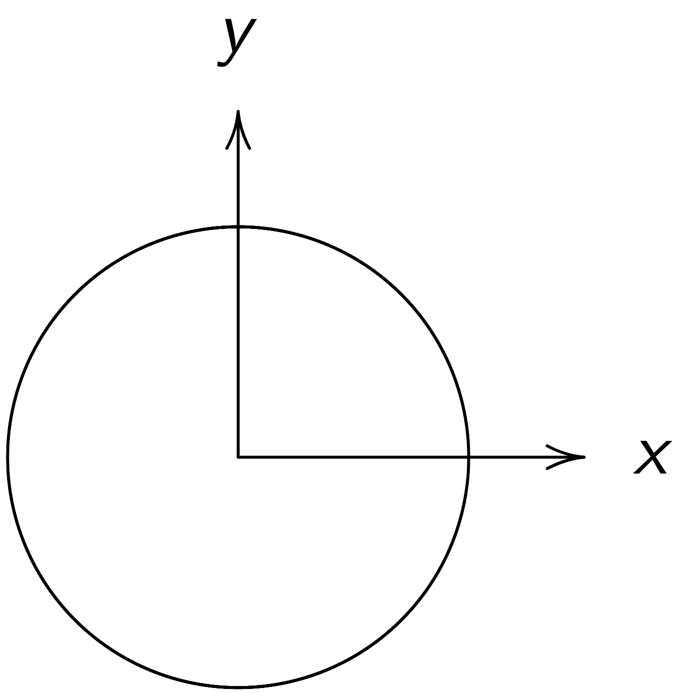

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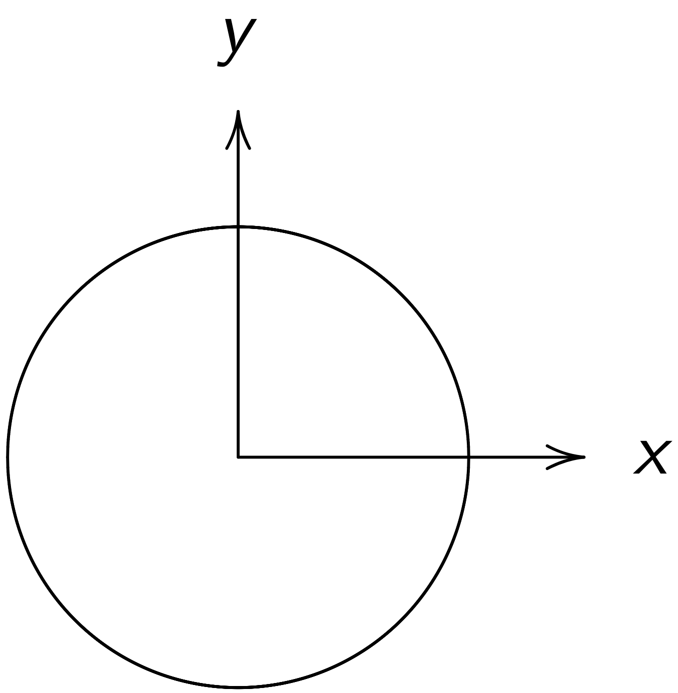

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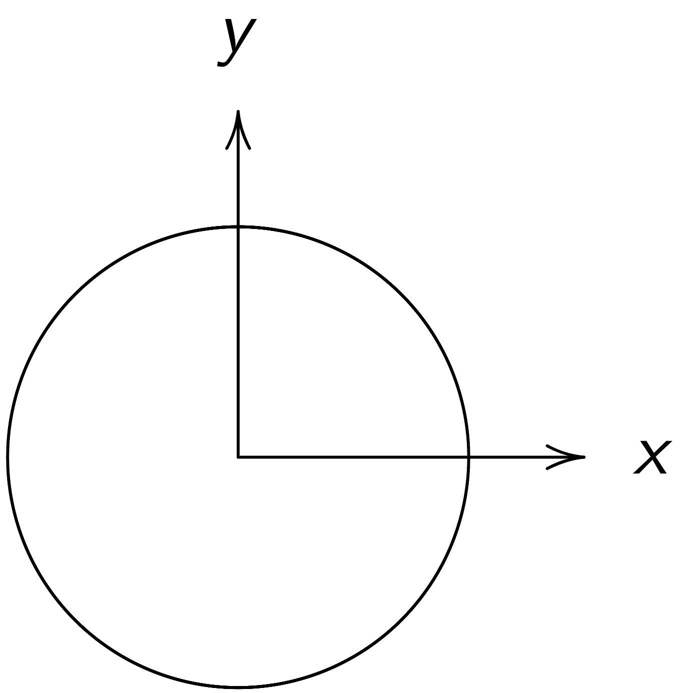

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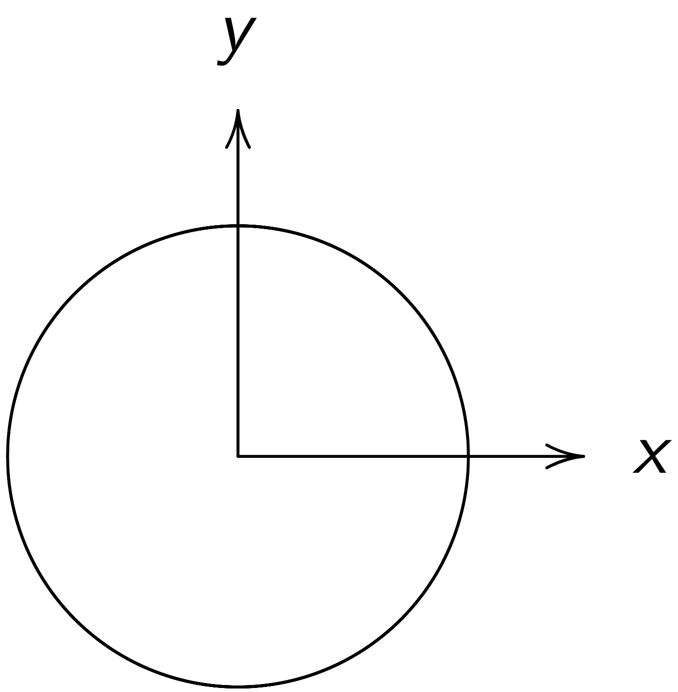

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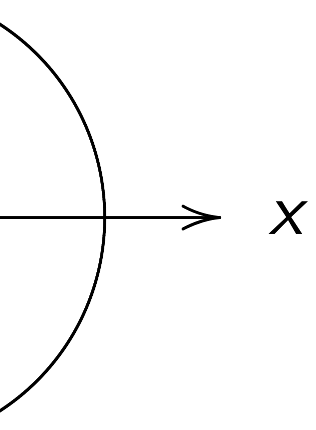

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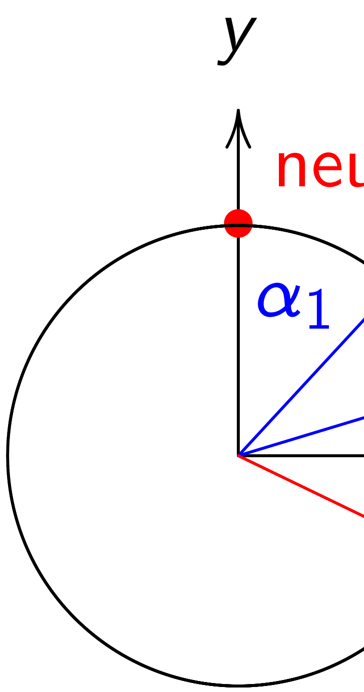

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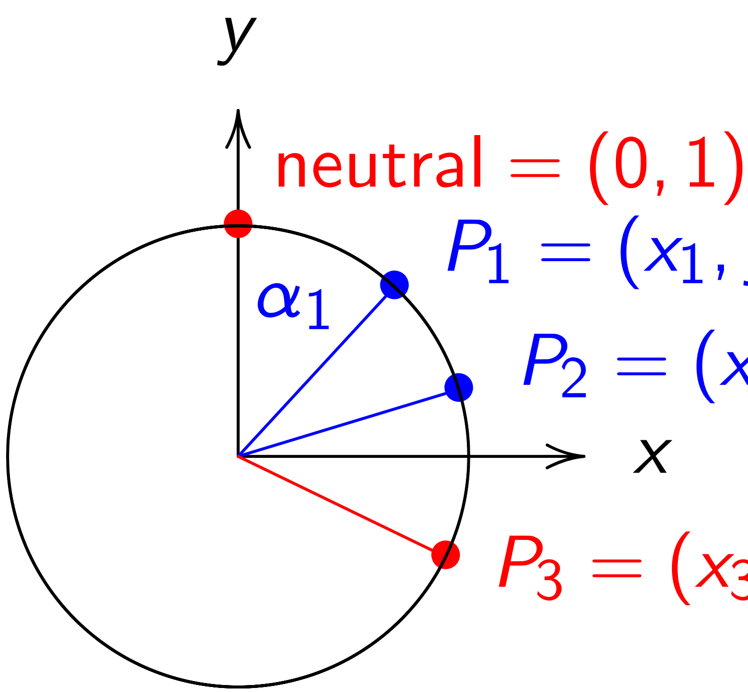
Addition on the clock:



$x^2 + y^2 = 1$, parametrized by

$x = \sin\alpha$, $y = \cos\alpha$.

Examples of points on this curve:

$(0, 1) =$ "12:00".

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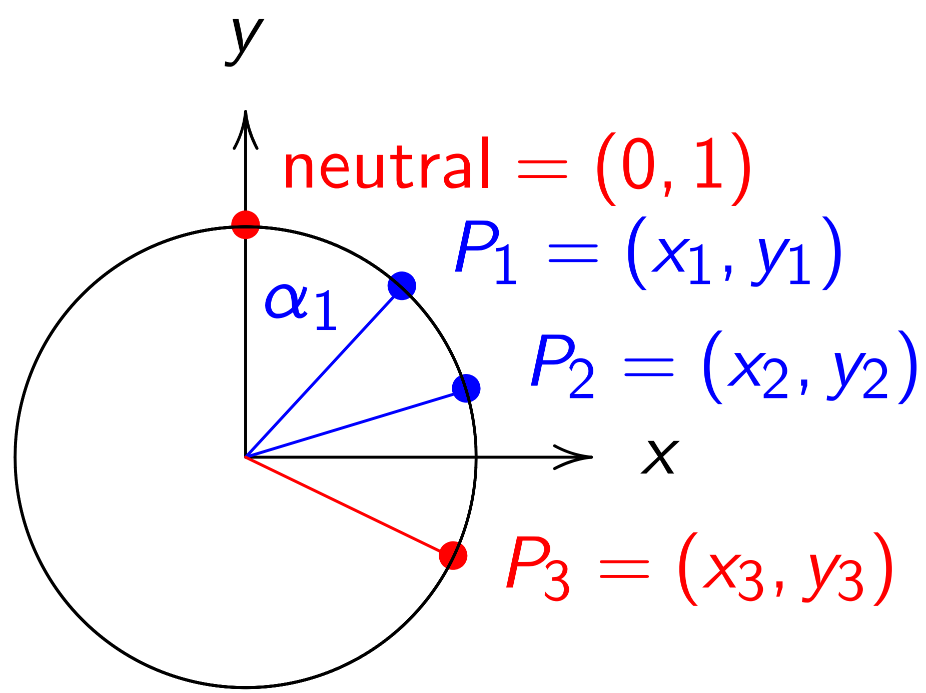

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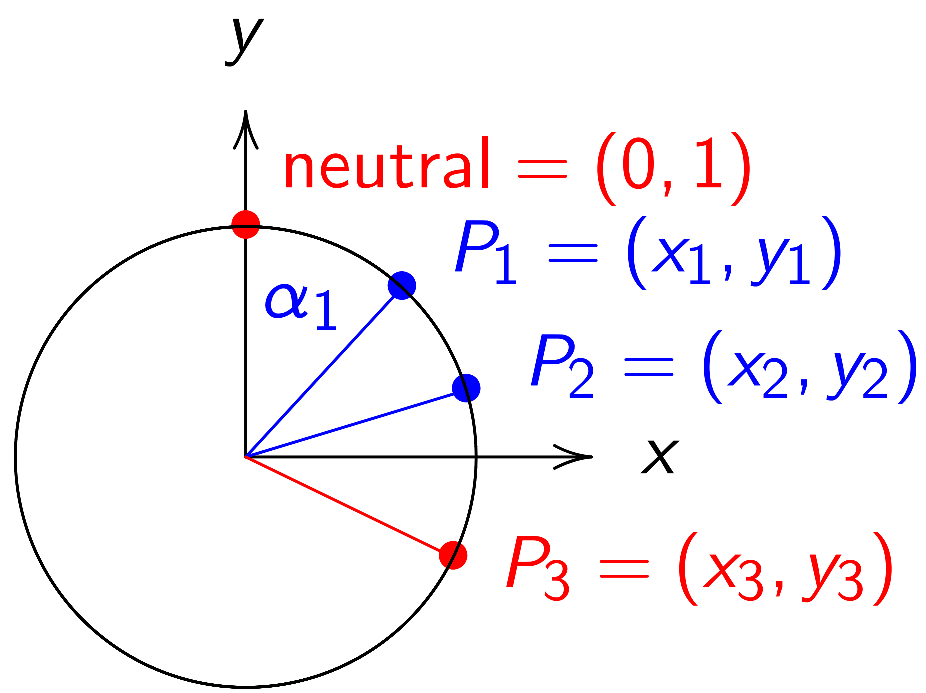

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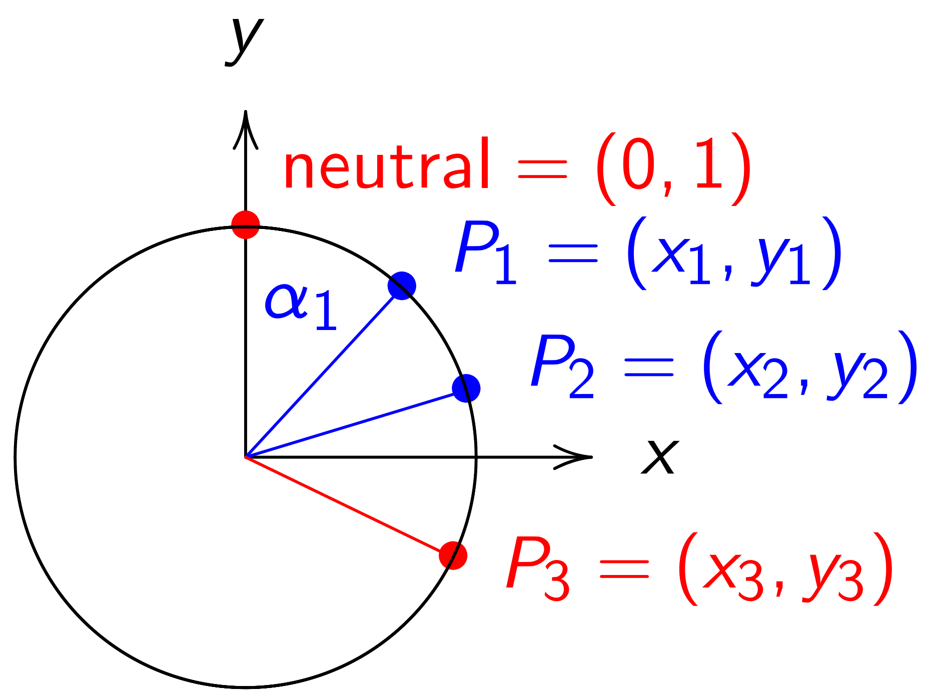

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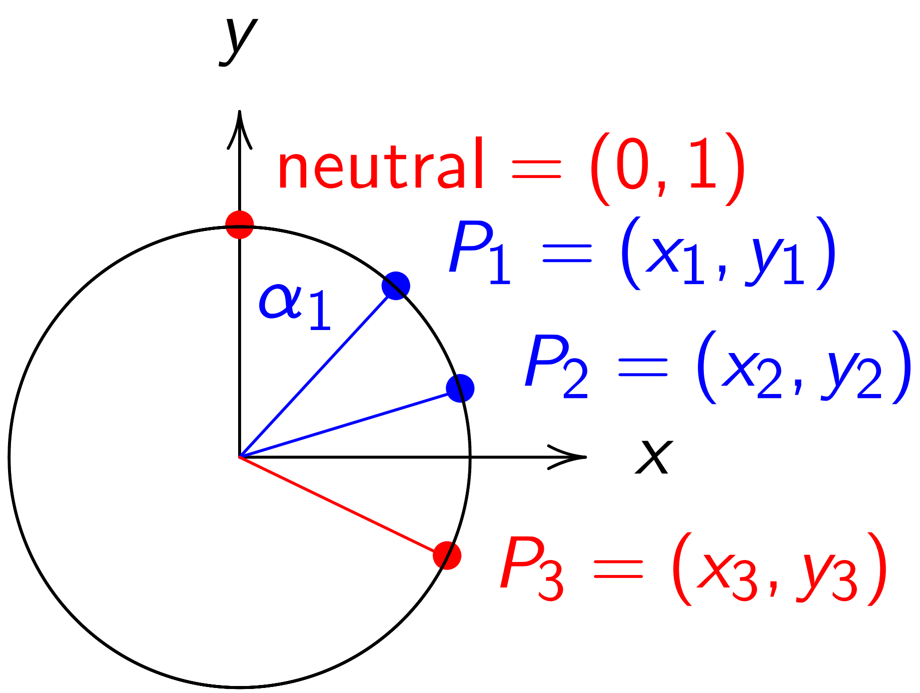

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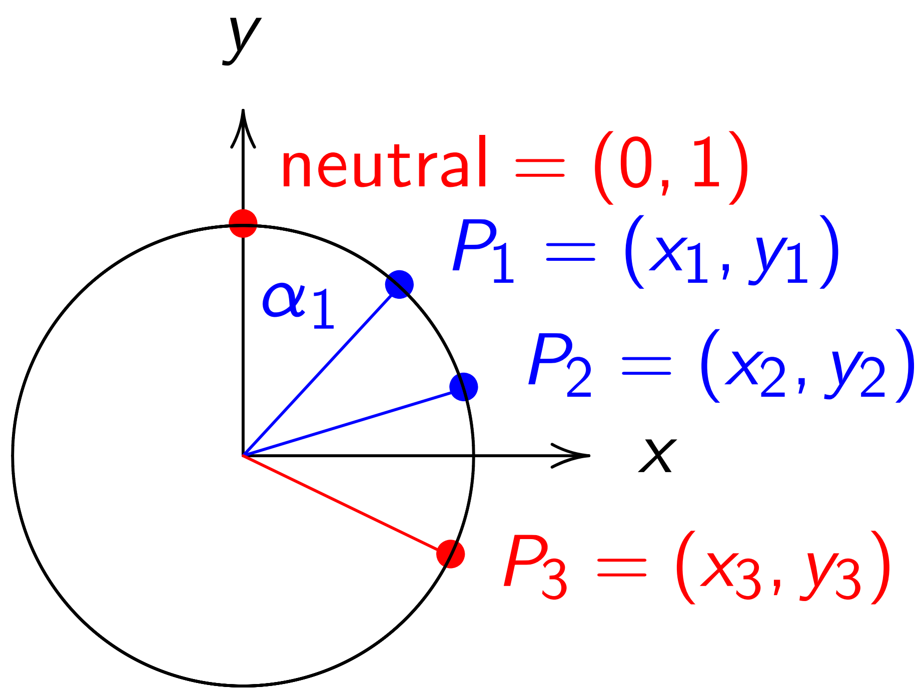

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Clock addi

Use Cartes

Addition fo

for the cloc

sum of $(x_1$

$(x_1 y_2 + y_1 x$

this curve:

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4/5).

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3/5).

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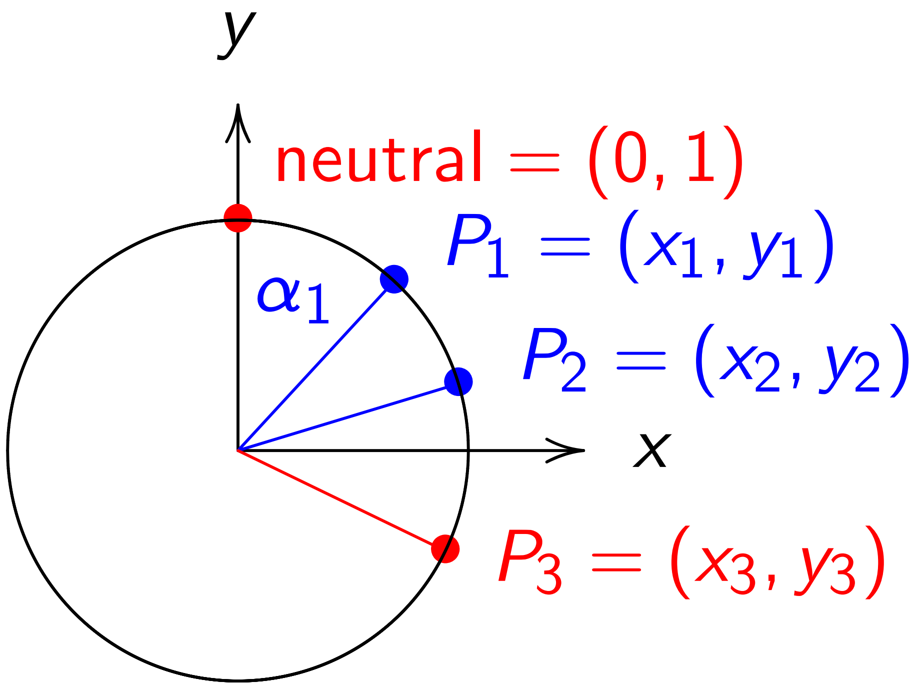

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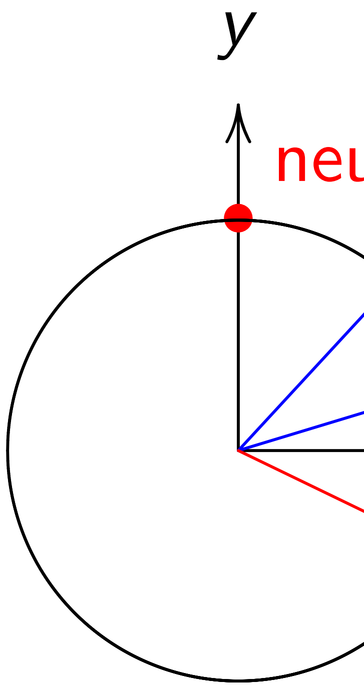
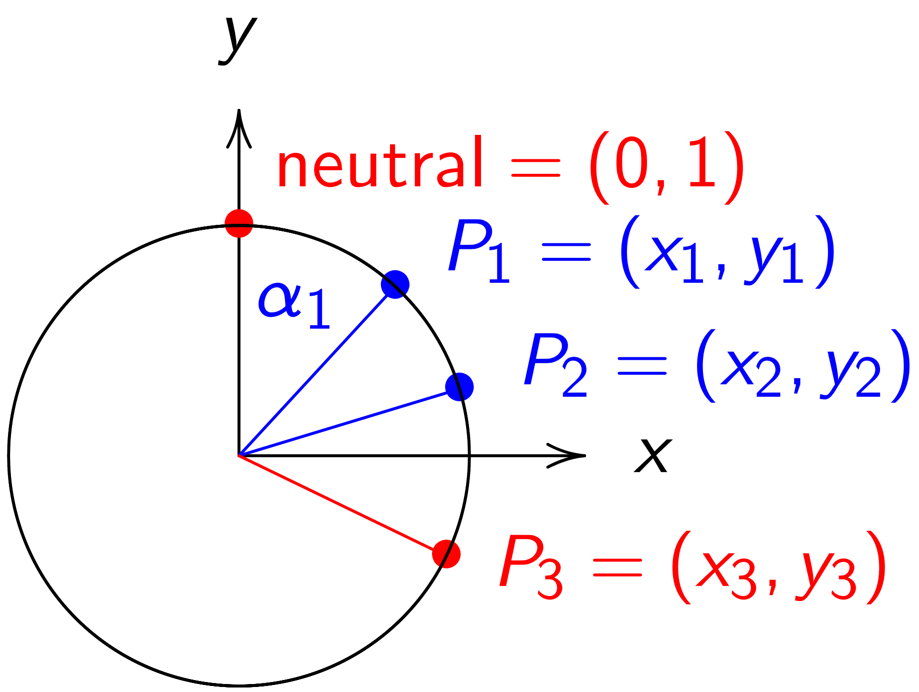
Clock addition without



Use Cartesian coordinat
Addition formula
for the clock $x^2 + y^2 =$
sum of (x_1, y_1) and $(x_2$
$(x_1 y_2 + y_1 x_2, y_1 y_2 - x_1$

Addition on the clock:



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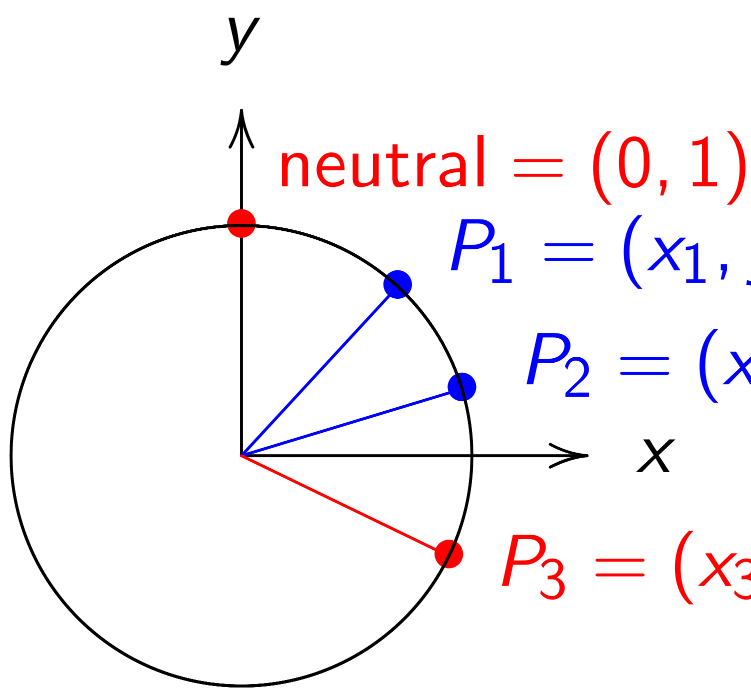

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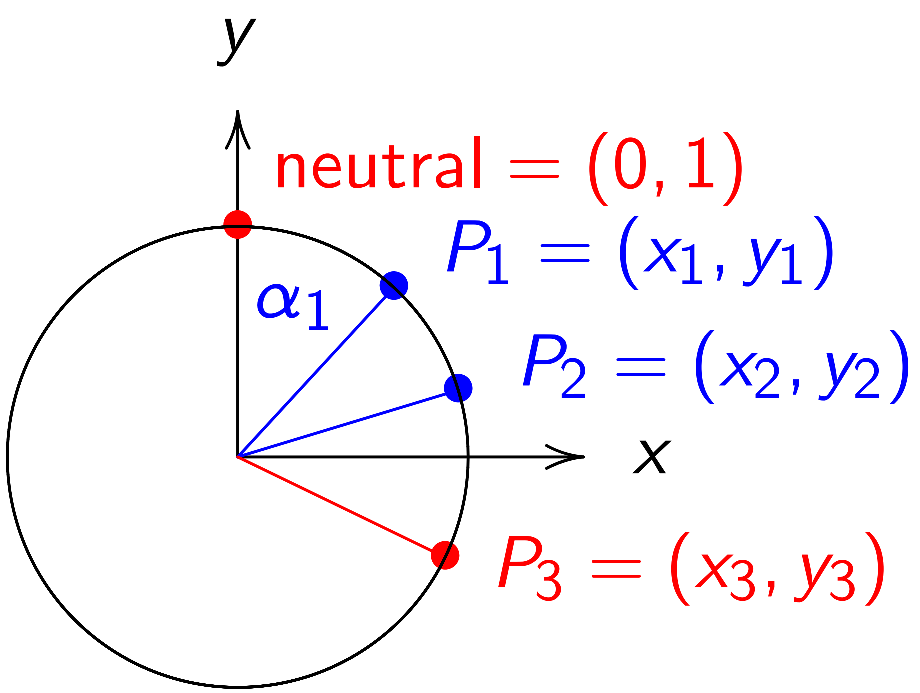

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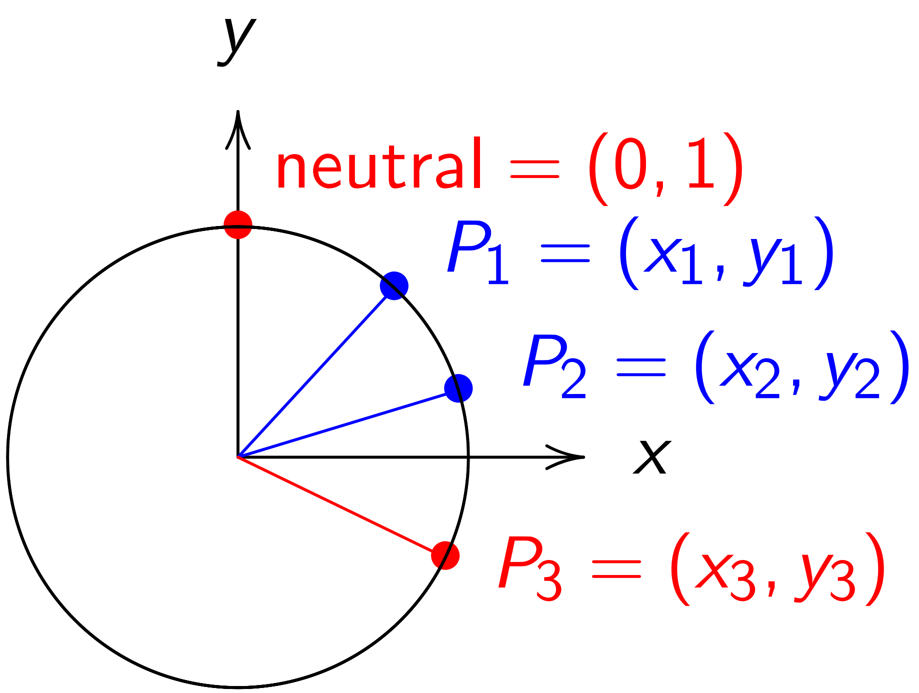

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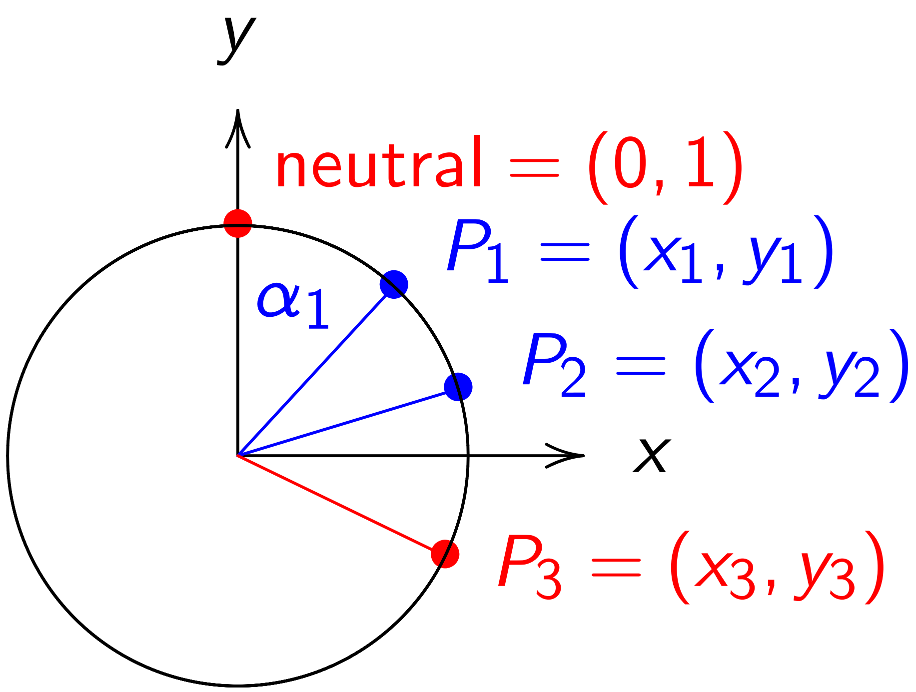

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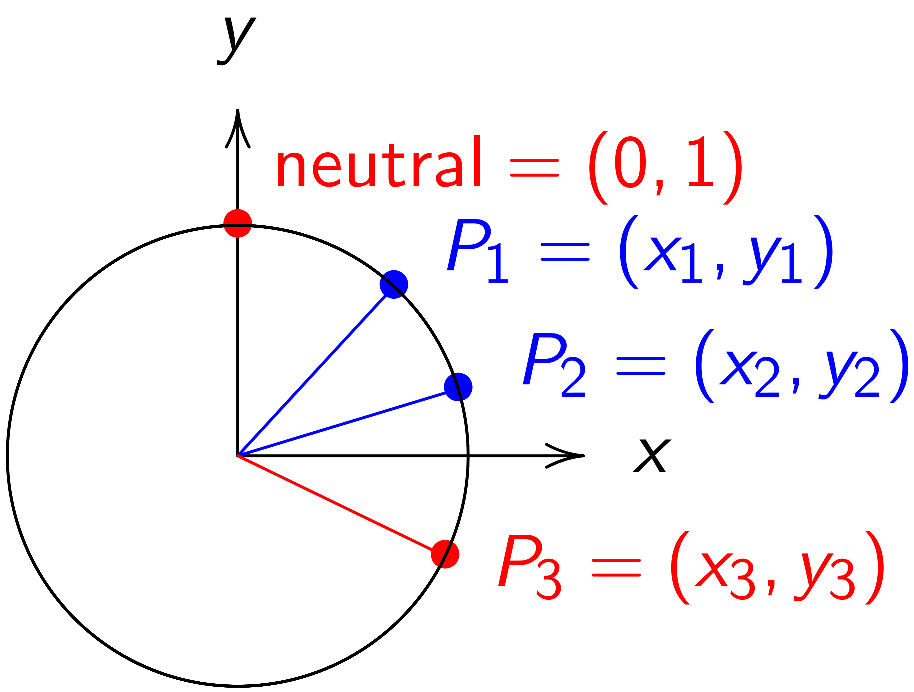

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Examples c

"2:00" + "

$= (\sqrt{3/4},$

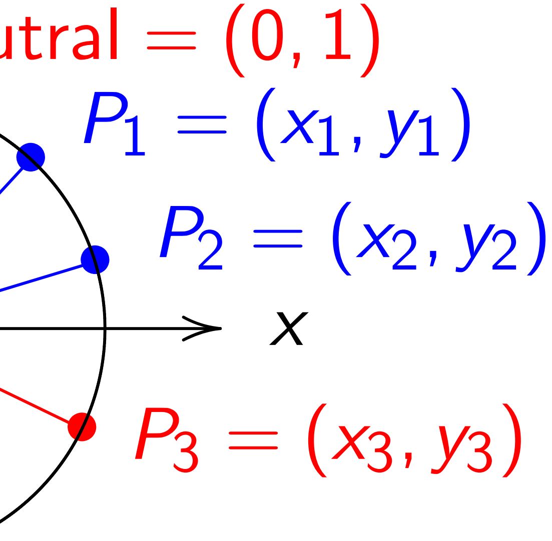
$= (-1/2, -$

"5:00" + "

$= (1/2, -$

$= (\sqrt{3/4},$

$2\left(\frac{3}{5}, \frac{4}{5}\right) =$



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Recall

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$\sin \alpha_2,$

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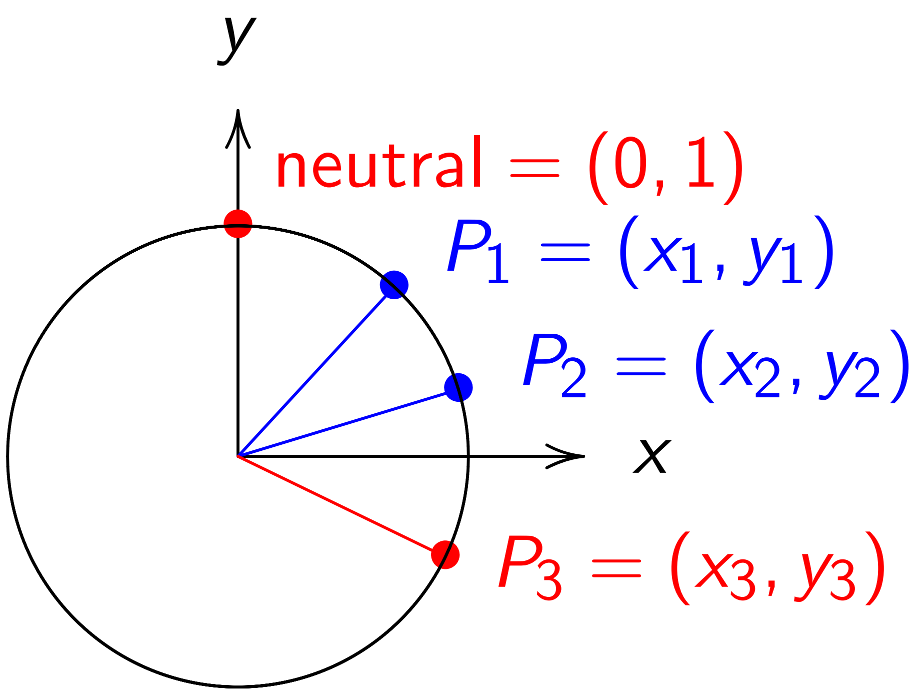

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$y_1)$

$x_2, y_2)$

$_3, y_3)$

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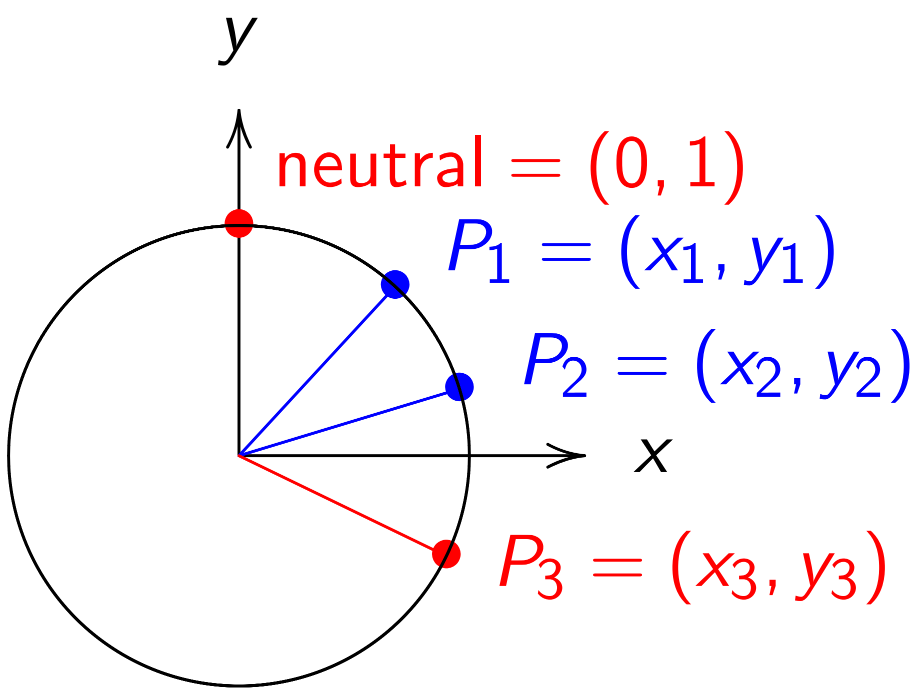

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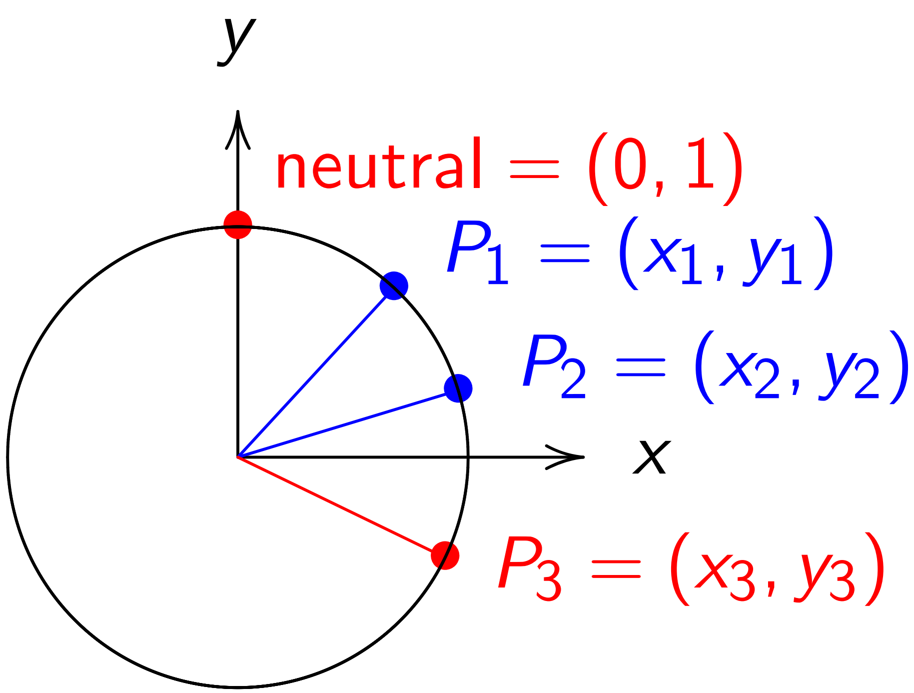

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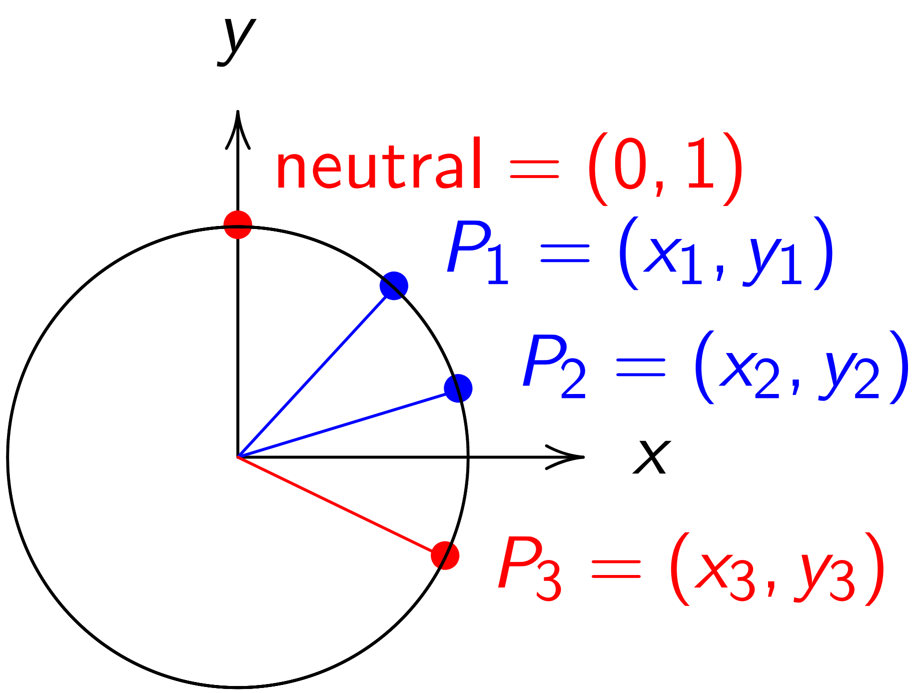

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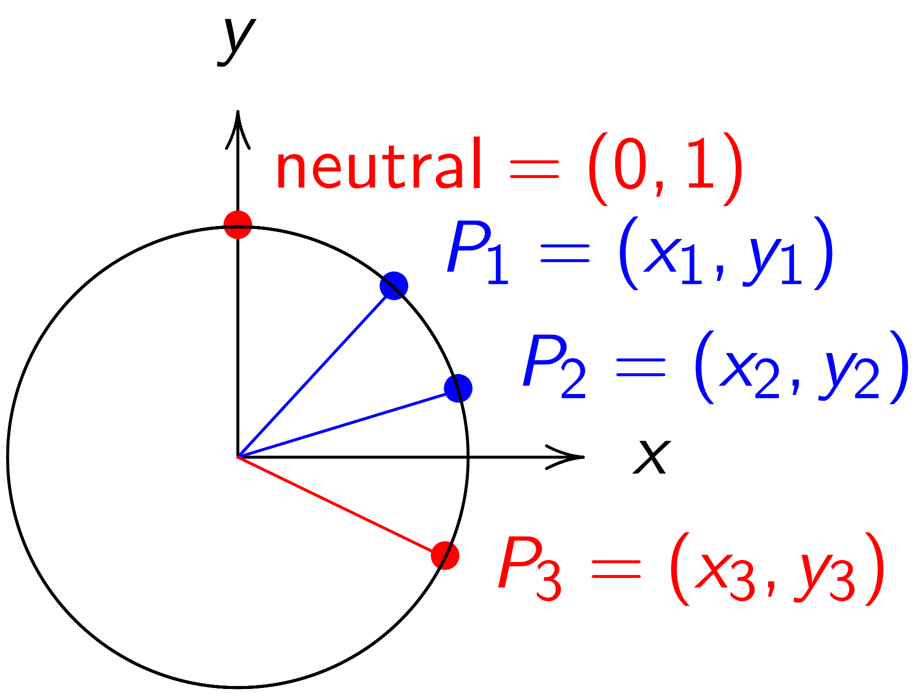

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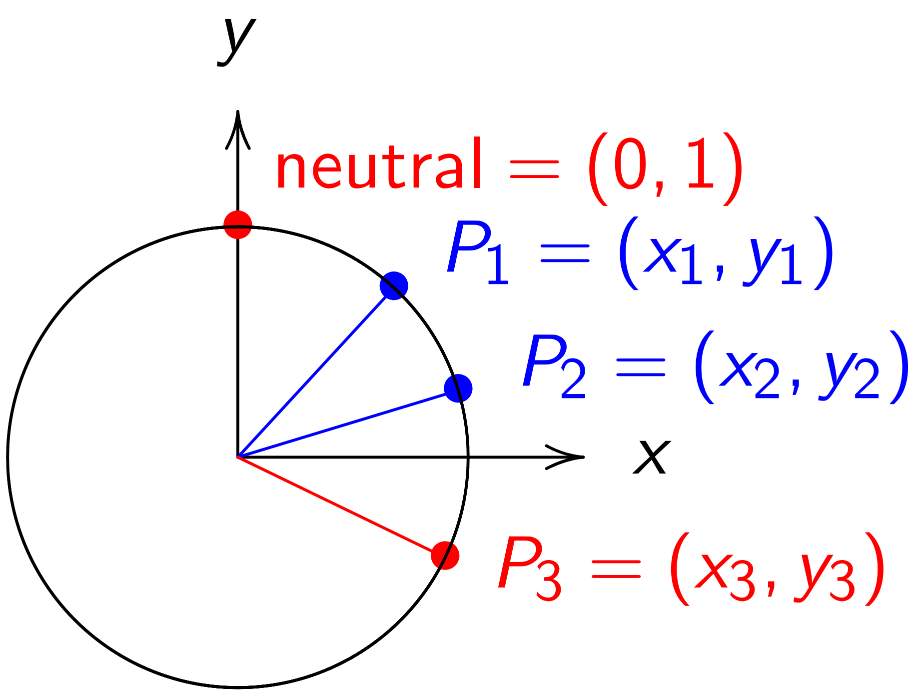

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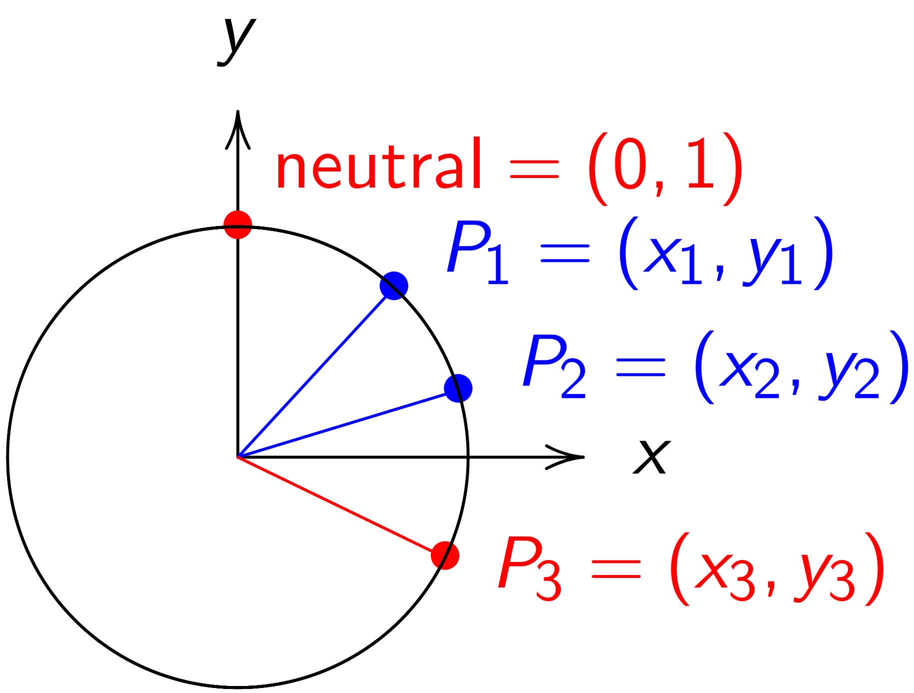

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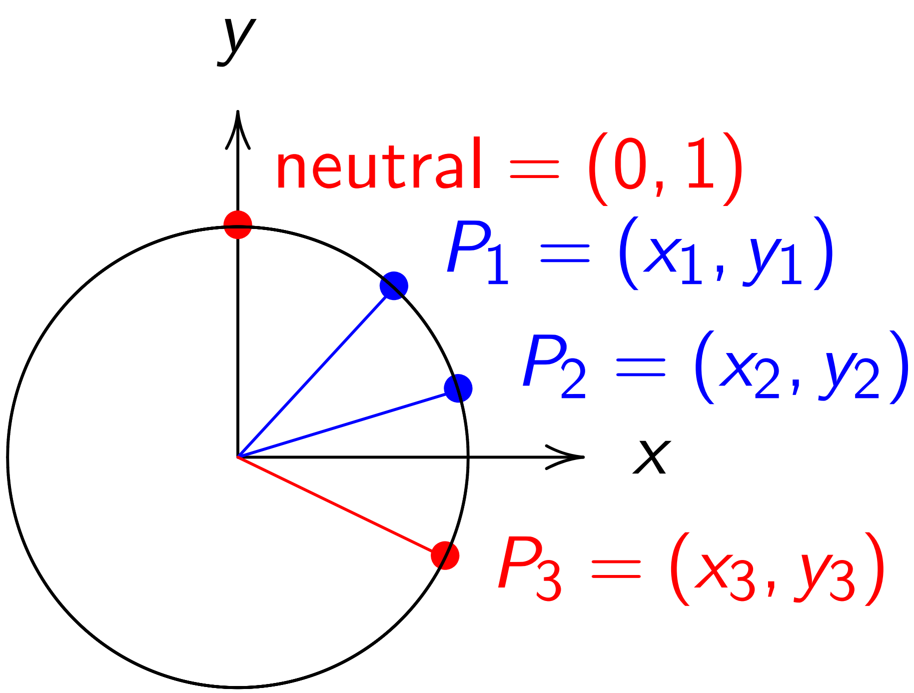

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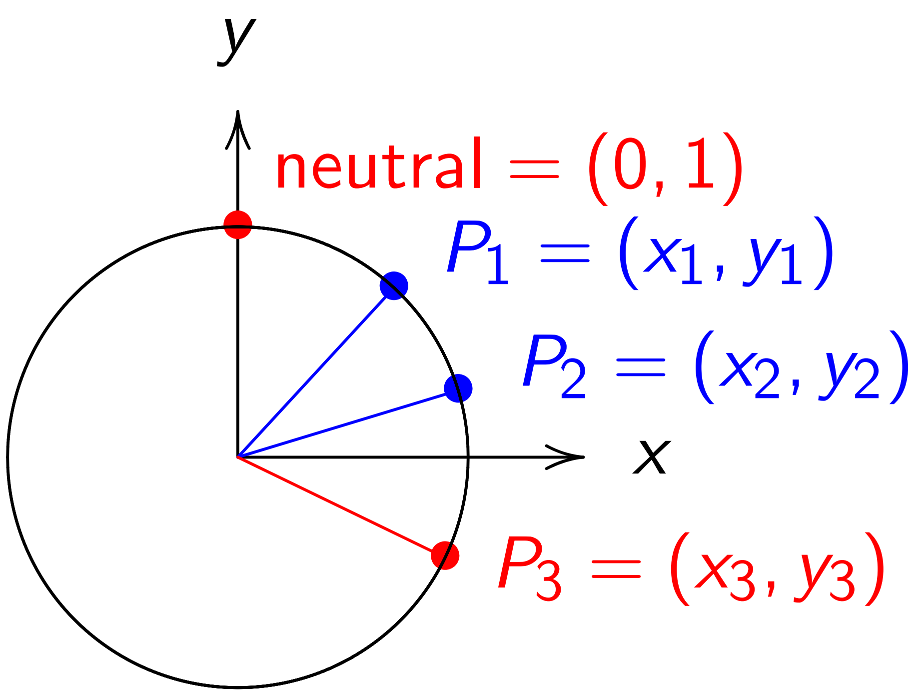

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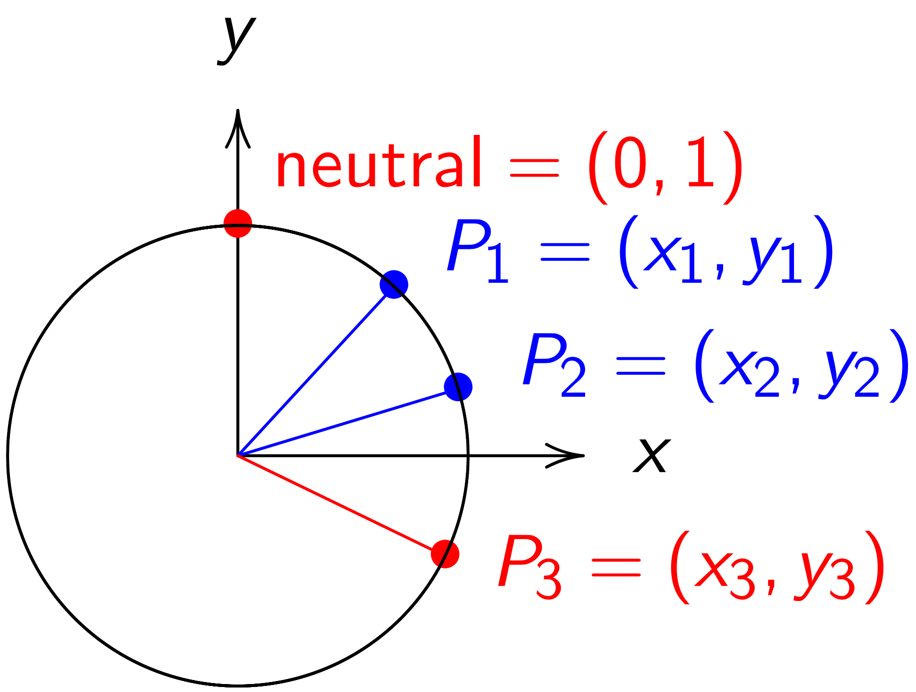
$$4\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{336}{625}, \frac{-527}{625}\right).$$

$(x_1, y_1) + (0, 1) = (x_1, y_1)$.

$(x_1, y_1) + (-x_1, y_1) = (0, 1)$.

tion without sin, cos:



ian coordinates for addition.

ormula

ck $x^2 + y^2 = 1$:

$, y_1)$ and (x_2, y_2) is

$x_2, y_1y_2 - x_1x_2)$.

Examples of clock addition:

"2:00" + "5:00"

$= (\sqrt{3/4}, 1/2) + (1/2, -\sqrt{3/4})$

$= (-1/2, -\sqrt{3/4}) =$ "7:00".

"5:00" + "9:00"

$= (1/2, -\sqrt{3/4}) + (-1, 0)$

$= (\sqrt{3/4}, 1/2) =$ "2:00".

$$2\left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{24}{25}, \frac{7}{25}\right).$$

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$(x_1, y_1) + (-x_1, y_1) = (0, 1)$.

Clocks ove

Clock($\mathbf{F}_7$)

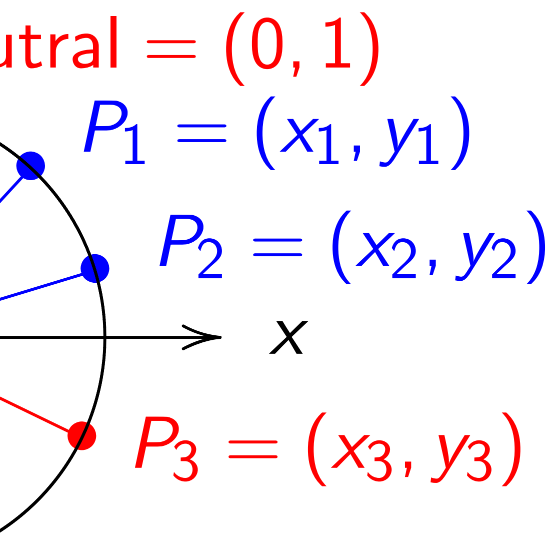
Here $\mathbf{F}_7 =$

=

with arithm

e.g. $2 \cdot 5 =$

sin, cos:



tes for addition.

= 1:

, y_2) is

x_2).

Examples of clock addition:

$$\text{“2:00”} + \text{“5:00”}$$
$$= (\sqrt{3/4}, 1/2) + (1/2, -\sqrt{3/4})$$
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$$\text{“5:00”} + \text{“9:00”}$$
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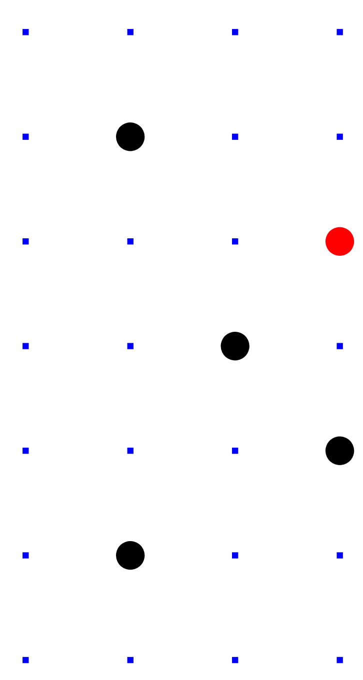
Clocks over finite fields



$\text{Clock}(\mathbf{F}_7) = \{(x, y) \in$

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4,$

$= \{0, 1, 2, 3, -$

with arithmetic modulo

e.g. $2 \cdot 5 = 3$ and $3/2 =$

$y_1)$

$x_2, y_2)$

$_3, y_3)$

on.

Examples of clock addition:

$$\text{“2:00”} + \text{“5:00”}$$
$$= (\sqrt{3/4}, 1/2) + (1/2, -\sqrt{3/4})$$
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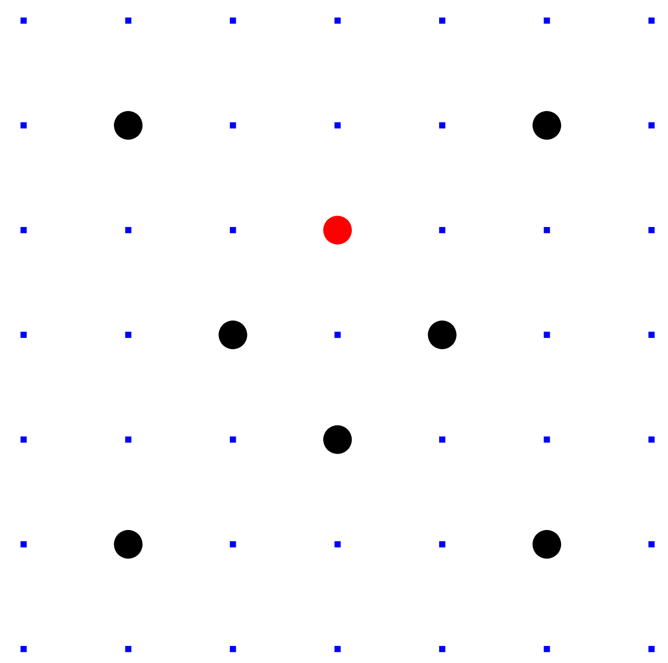
Clocks over finite fields



$\text{Clock}(\mathbf{F}_7) = \{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2$

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$= \{0, 1, 2, 3, -3, -2, -1\}$

with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

Examples of clock addition:

"2:00" + "5:00"

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"5:00" + "9:00"

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$= (\sqrt{3/4}, 1/2) =$ "2:00".

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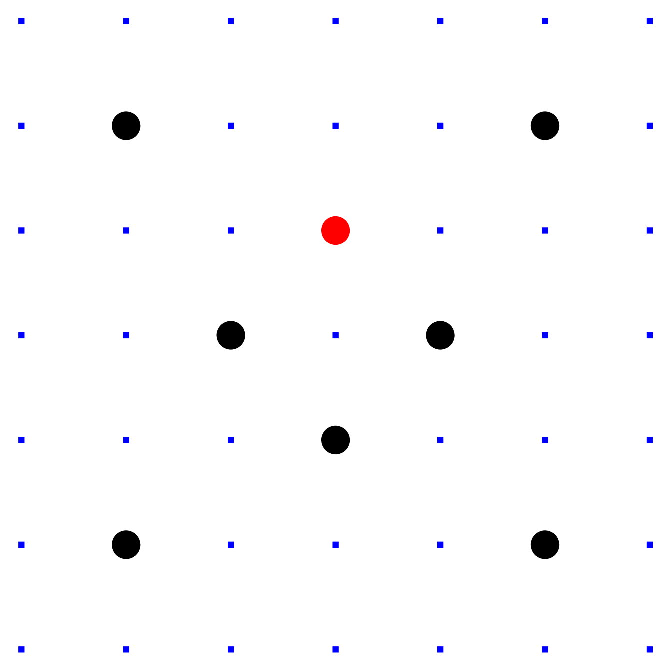
Clocks over finite fields



$\text{Clock}(\mathbf{F}_7) = \{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2+y^2=1\}$.

Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

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with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

of clock addition:

5:00”

$1/2) + (1/2, -\sqrt{3/4})$

$-\sqrt{3/4}) =$ “7:00”.

9:00”

$\sqrt{3/4}) + (-1, 0)$

$1/2) =$ “2:00”.

$= \left(\frac{24}{25}, \frac{7}{25}\right).$

$= \left(\frac{117}{125}, \frac{-44}{125}\right).$

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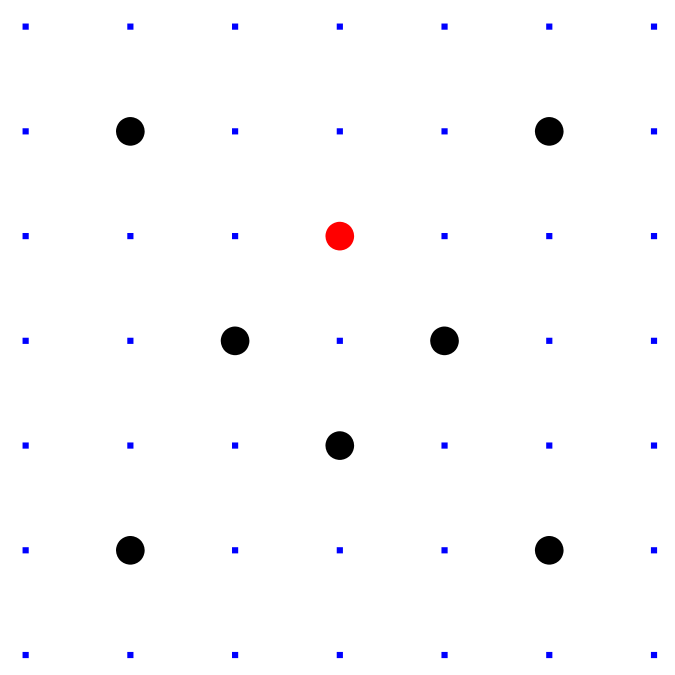
Clocks over finite fields



$\text{Clock}(\mathbf{F}_7) = \{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2+y^2=1\}.$

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$= \{0, 1, 2, 3, -3, -2, -1\}$

with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

```
>>> for x
...   for
...     if
...
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

tion:

$-\sqrt{3/4})$

7:00".

1, 0)

0".

.

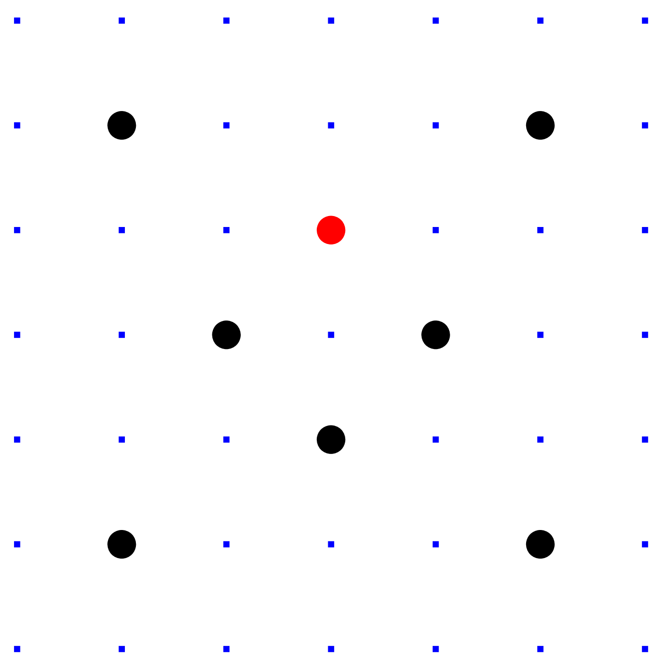
4 / 5).

27 / 5).

y_1).

(0, 1).

Clocks over finite fields



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Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

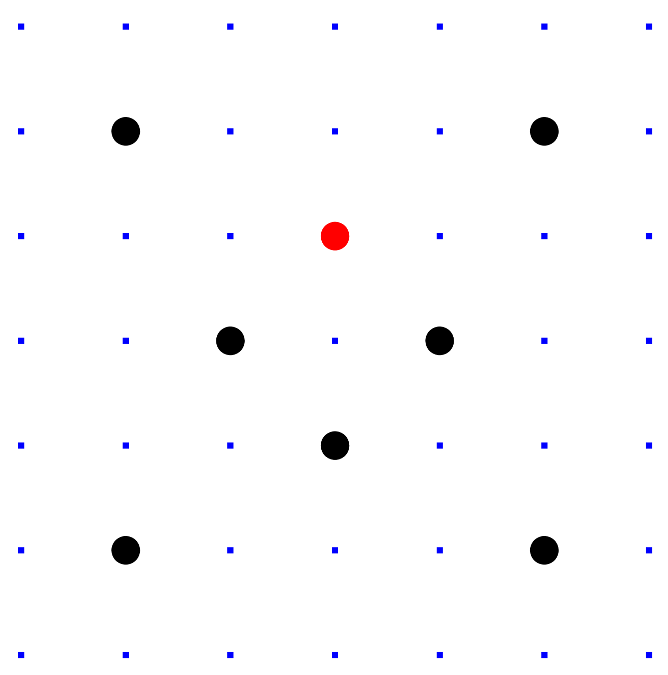
$= \{0, 1, 2, 3, -3, -2, -1\}$

with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

```
>>> for x in range(7)
...   for y in range(
...     if (x*x+y*y)
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

Clocks over finite fields



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Here $\mathbf{F}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

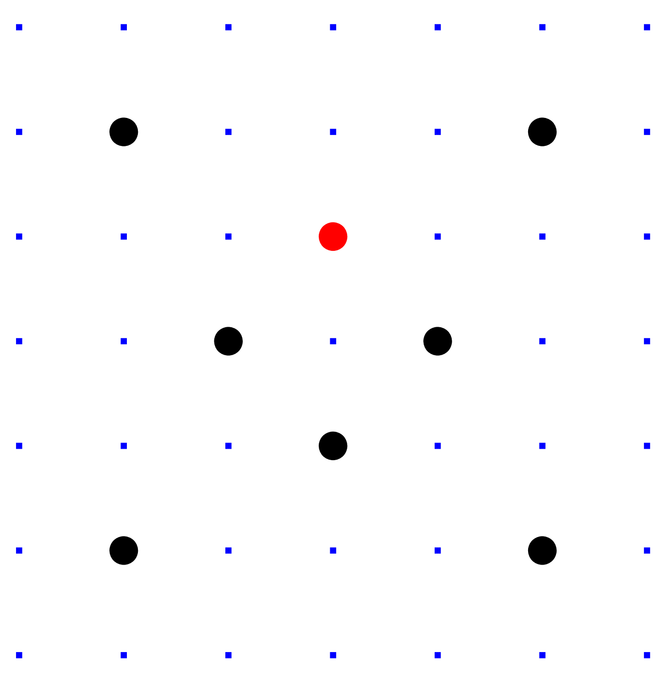
$= \{0, 1, 2, 3, -3, -2, -1\}$

with arithmetic modulo 7.

e.g. $2 \cdot 5 = 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

Clocks over finite fields



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with arithmetic modulo 7.

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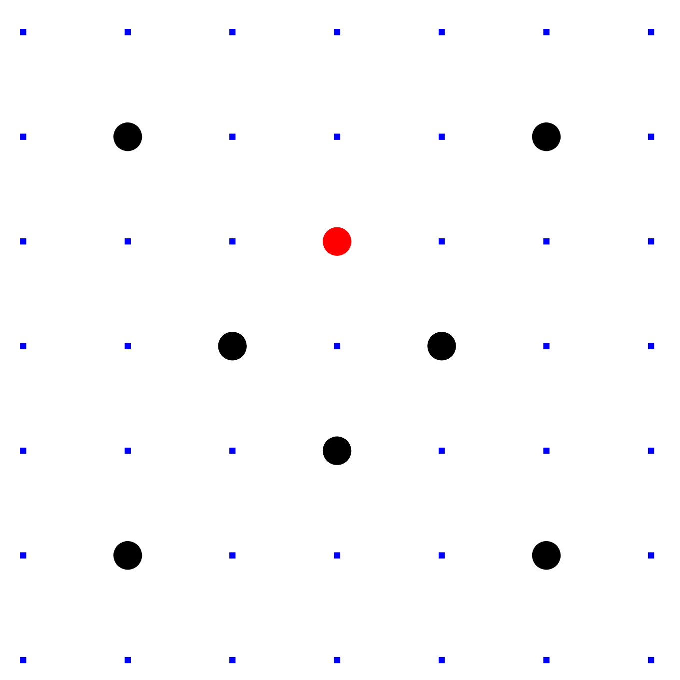
```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

r finite fields



$= \{(x, y) \in \mathbf{F}_7 \times \mathbf{F}_7 : x^2+y^2=1\}.$

$\{0, 1, 2, 3, 4, 5, 6\}$

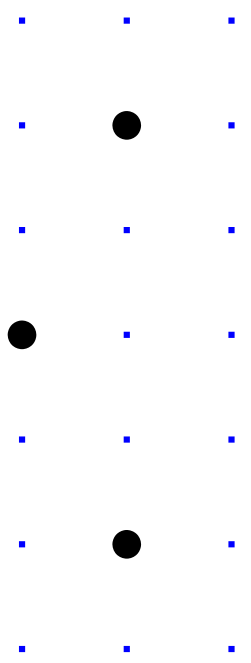
$\{0, 1, 2, 3, -3, -2, -1\}$

netic modulo 7.

$= 3$ and $3/2 = 5$ in $\mathbf{F}_7$.

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

```
>>> class
...   def
...     se
...   def
...     re
...   __re
...
>>> print
2
>>> print
6
>>> print
0
>>> print
3
```



$\mathbf{F}_7 \times \mathbf{F}_7 : x^2{+}y^2{=}1\}$.

$5, 6\}$

$3, -2, -1\}$

7.

$= 5$ in $\mathbf{F}_7$.

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

```
>>> class F7:
...   def __init__(se
...     self.int = x
...   def __str__(sel
...     return str(se
...   __repr__ = __st
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

$+y^2=1\}$.

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> for x in range(7):
...   for y in range(7):
...     if (x*x+y*y) % 7 == 1:
...       print (x,y)
...
(0, 1)
(0, 6)
(1, 0)
(2, 2)
(2, 5)
(5, 2)
(5, 5)
(6, 0)
>>>
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
 in range(7):
 y in range(7):
f (x*x+y*y) % 7 == 1:
 print (x,y)
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> F7.__e
...   lamb
>>>
>>> print
True
>>> print
True
>>> print
True
>>> print
False
>>> print
False
>>> print
False
```

```
):
(7):
 % 7 == 1:
)
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.i
>>>
>>> print F7(7) == F7
True
>>> print F7(10) == F
True
>>> print F7(-3) == F
True
>>> print F7(0) == F7
False
>>> print F7(0) == F7
False
>>> print F7(0) == F7
False
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> class F7:
...   def __init__(self,x):
...     self.int = x % 7
...   def __str__(self):
...     return str(self.int)
...   __repr__ = __str__
...
>>> print F7(2)
2
>>> print F7(6)
6
>>> print F7(7)
0
>>> print F7(10)
3
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
 F7:
 __init__(self,x):
elf.int = x % 7
 __str__(self):
eturn str(self.int)
epr__ = __str__

 F7(2)

 F7(6)

 F7(7)

 F7(10)
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> F7.__a
...   lamb
>>> F7.__s
...   lamb
>>> F7.__m
...   lamb
>>>
>>> print
0
>>> print
4
>>> print
3
>>>
```

```
elf,x):
 % 7
lf):
elf.int)
tr__
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(
>>> F7.__sub__ = \
...   lambda a,b: F7(
>>> F7.__mul__ = \
...   lambda a,b: F7(
>>>
>>> print F7(2) + F7(
0
>>> print F7(2) - F7(
4
>>> print F7(2) * F7(
3
>>>
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.i
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.i
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.i
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

```
>>> F7.__eq__ = \
...   lambda a,b: a.int == b.int
>>>
>>> print F7(7) == F7(0)
True
>>> print F7(10) == F7(3)
True
>>> print F7(-3) == F7(4)
True
>>> print F7(0) == F7(1)
False
>>> print F7(0) == F7(2)
False
>>> print F7(0) == F7(3)
False
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.int)
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.int)
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

```
eq__ = \
bda a,b: a.int == b.int

 F7(7) == F7(0)

 F7(10) == F7(3)

 F7(-3) == F7(4)

 F7(0) == F7(1)

 F7(0) == F7(2)

 F7(0) == F7(3)
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.int)
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.int)
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

Larger exa

```
p = 10000
class Fp:
  ...

def clocka
  x1,y1 =
  x2,y2 =
  x3 = x1*
  y3 = y1*
  return x
```

```
int == b.int

7(0)

F7(3)

F7(4)

7(1)

7(2)

7(3)
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.int)
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.int)
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

Larger example: Clock(

```
p = 1000003
class Fp:
  ...

def clockadd(P1,P2):
  x1,y1 = P1
  x2,y2 = P2
  x3 = x1*y2+y1*x2
  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.int)
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.int)
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

Larger example: Clock($\mathbf{F}_{1000003}$).

```
p = 1000003
class Fp:
  ...

def clockadd(P1,P2):
  x1,y1 = P1
  x2,y2 = P2
  x3 = x1*y2+y1*x2
  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> F7.__add__ = \
...   lambda a,b: F7(a.int + b.int)
>>> F7.__sub__ = \
...   lambda a,b: F7(a.int - b.int)
>>> F7.__mul__ = \
...   lambda a,b: F7(a.int * b.int)
>>>
>>> print F7(2) + F7(5)
0
>>> print F7(2) - F7(5)
4
>>> print F7(2) * F7(5)
3
>>>
```

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  y3 = y1*y2-x1*x2
  return x3,y3
```

```
add__ = \
bda a,b: F7(a.int + b.int)
sub__ = \
bda a,b: F7(a.int - b.int)
mul__ = \
bda a,b: F7(a.int * b.int)

 F7(2) + F7(5)

 F7(2) - F7(5)

 F7(2) * F7(5)
```

Larger example: Clock($\mathbf{F}_{1000003}$).

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  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> P = (F
>>> P2 = c
>>> print
(4000, 7)
>>> P3 = c
>>> print
(15000, 26
>>> P4 = c
>>> P5 = c
>>> P6 = c
>>> print
(780000, 1
>>> print
(780000, 1
>>>
```

```
(a.int + b.int)

(a.int - b.int)

(a.int * b.int)

(5)

(5)

(5)
```

Larger example: Clock($\mathbf{F}_{1000003}$).

```
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class Fp:
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  x2,y2 = P2
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  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> P = (Fp(1000),Fp(
>>> P2 = clockadd(P,P
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,
>>> P5 = clockadd(P4,
>>> P6 = clockadd(P5,
>>> print P6
(780000, 1351)
>>> print clockadd(P3
(780000, 1351)
>>>
```

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Larger example: Clock($\mathbf{F}_{1000003}$).

```
p = 1000003
class Fp:
  ...

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  x2,y2 = P2
  x3 = x1*y2+y1*x2
  y3 = y1*y2-x1*x2
  return x3,y3
```

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

Larger example: Clock($\mathbf{F}_{1000003}$).

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```

```
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>>> P3 = clockadd(P2,P)
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(15000, 26)
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>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

mple: Clock($\mathbf{F}_{1000003}$).

```
03
```

```
add(P1,P2):
 P1
 P2
*y2+y1*x2
*y2-x1*x2
x3,y3
```

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

```
>>> def sc
...   if n
...   if n
...   Q =
...   Q =
...   if n
...   retu
...
>>> n = ou
>>> scalar
(947472, 7
>>>
```

Can you fig

$(\mathbf{F}_{1000003})$.

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

```
>>> def scalarmult(n,
...   if n == 0: retu
...   if n == 1: retu
...   Q = scalarmult(
...   Q = clockadd(Q,
...   if n % 2: Q = c
...   return Q
...
>>> n = oursixdigitse
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

```
>>> def scalarmult(n,P):
...   if n == 0: return (Fp(0),F
...   if n == 1: return P
...   Q = scalarmult(n//2,P)
...   Q = clockadd(Q,Q)
...   if n % 2: Q = clockadd(P,Q
...   return Q
...
>>> n = oursixdigitsecret
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret *n*?

```
>>> P = (Fp(1000),Fp(2))
>>> P2 = clockadd(P,P)
>>> print P2
(4000, 7)
>>> P3 = clockadd(P2,P)
>>> print P3
(15000, 26)
>>> P4 = clockadd(P3,P)
>>> P5 = clockadd(P4,P)
>>> P6 = clockadd(P5,P)
>>> print P6
(780000, 1351)
>>> print clockadd(P3,P3)
(780000, 1351)
>>>
```

```
>>> def scalarmult(n,P):
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...   return Q
...
>>> n = oursixdigitsecret
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret *n*?

```
Fp(1000),Fp(2))
clockadd(P,P)
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clockadd(P2,P)
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6)
clockadd(P3,P)
clockadd(P4,P)
clockadd(P5,P)
 P6
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 clockadd(P3,P3)
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```

```
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...
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>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret *n*?

Clock cryp

The "Clock

Standardize

and **base p**

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Bob choose

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Alice comp

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to encrypt

```
(2))
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,P)
,P)
,P)
3,P3)
```

```
>>> def scalarmult(n,P):
...   if n == 0: return (Fp(0),Fp(1))
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...   return Q
...
>>> n = oursixdigitsecret
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret n?

Clock cryptography

The "Clock Diffie–Hell

Standardize a large prin
and **base point** (x, y)

Alice chooses big secret
Alice computes her pub

Bob chooses big secret
Bob computes his publi

Alice computes $a(b(x, y$
Bob computes $b(a(x, y$
They use this shared se
to encrypt with AES-G

```
>>> def scalarmult(n,P):
...   if n == 0: return (Fp(0),Fp(1))
...   if n == 1: return P
...   Q = scalarmult(n//2,P)
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...   return Q
...
>>> n = oursixdigitsecret
>>> scalarmult(n,P)
(947472, 736284)
>>>
```

Can you figure out our secret n?

Clock cryptography

The "Clock Diffie–Hellman protocol

Standardize a large prime p
and **base point** $(x, y) \in \text{Clock}(\mathbf{F}_p)$.

Alice chooses big secret a.
Alice computes her public key $a(x, y$

Bob chooses big secret b.
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>>> def scalarmult(n,P):
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(947472, 736284)
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```

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```
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n == 1: return P
 scalarmult(n//2,P)
 clockadd(Q,Q)
n % 2: Q = clockadd(P,Q)
urn Q

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rmult(n,P)
736284)
```

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```
,P):
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urn P
(n//2,P)
,Q)
clockadd(P,Q)
```

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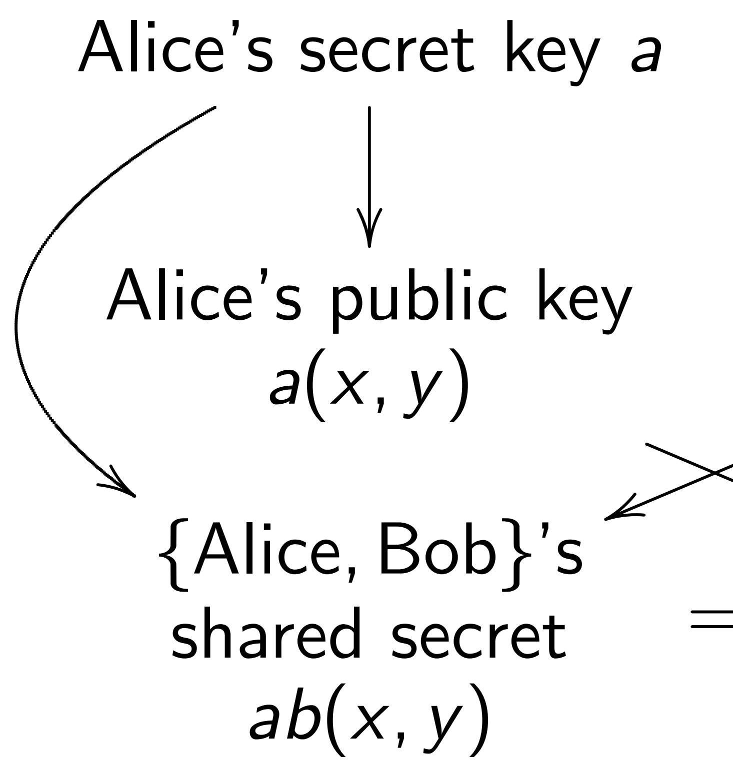

```
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```

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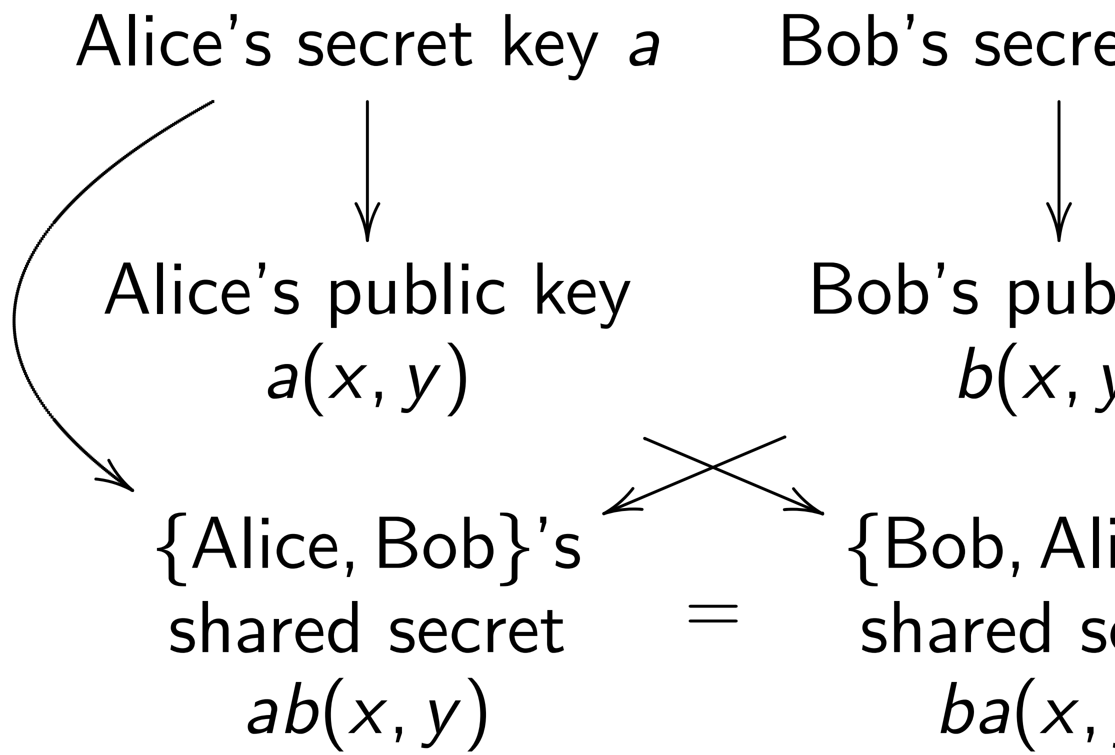

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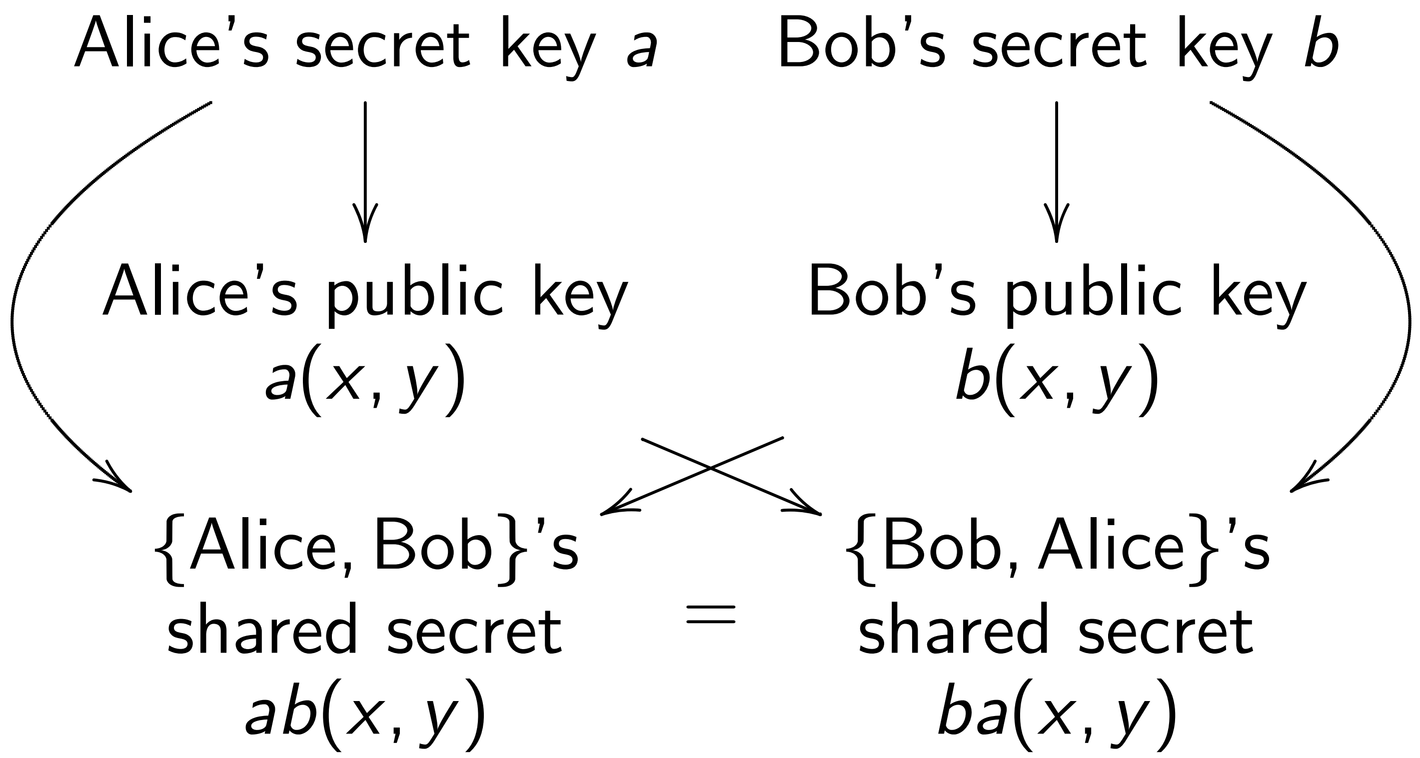

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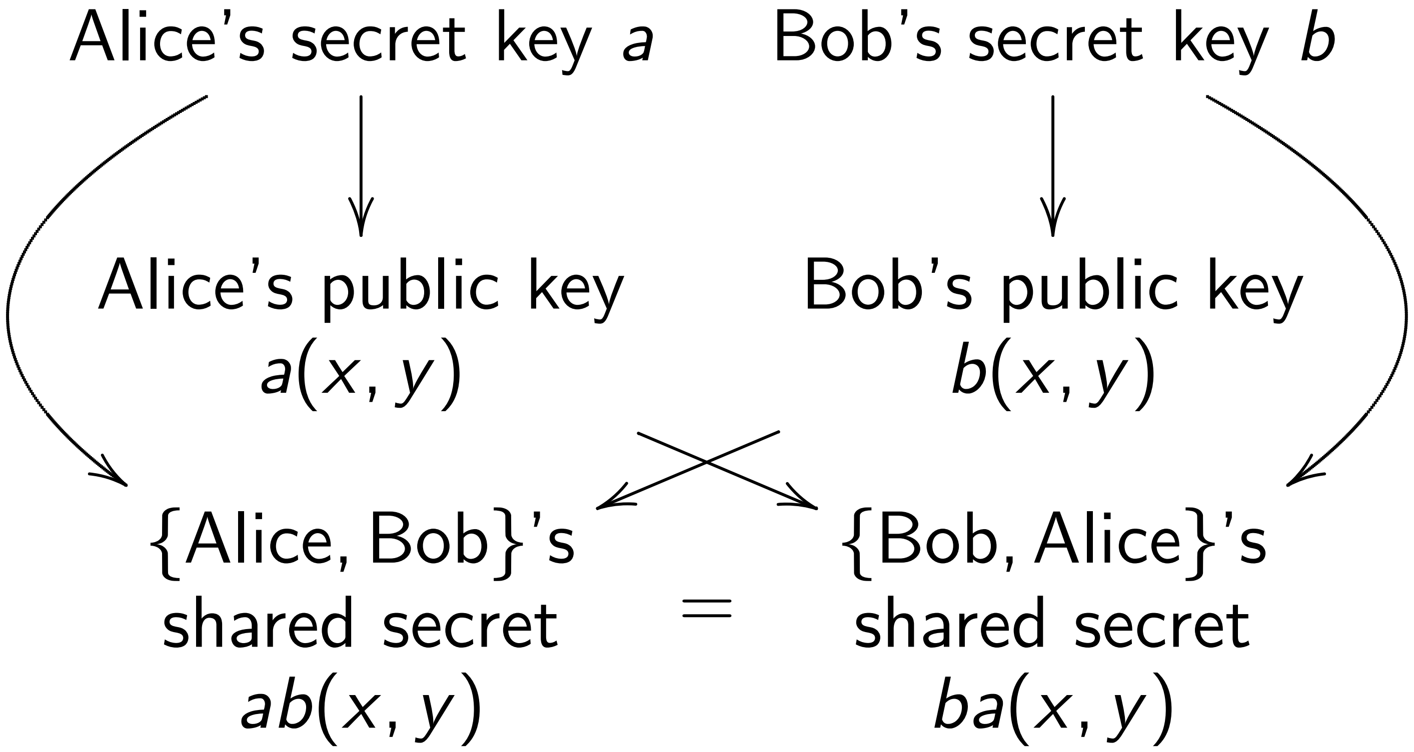

Warning #1: Many choices of p are unsafe!

Warning #2: Clocks aren't elliptic!
Can use index calculus
to attack clock cryptography.
To match RSA-3072 security
need $p \approx 2^{1536}$.

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ses big secret a.
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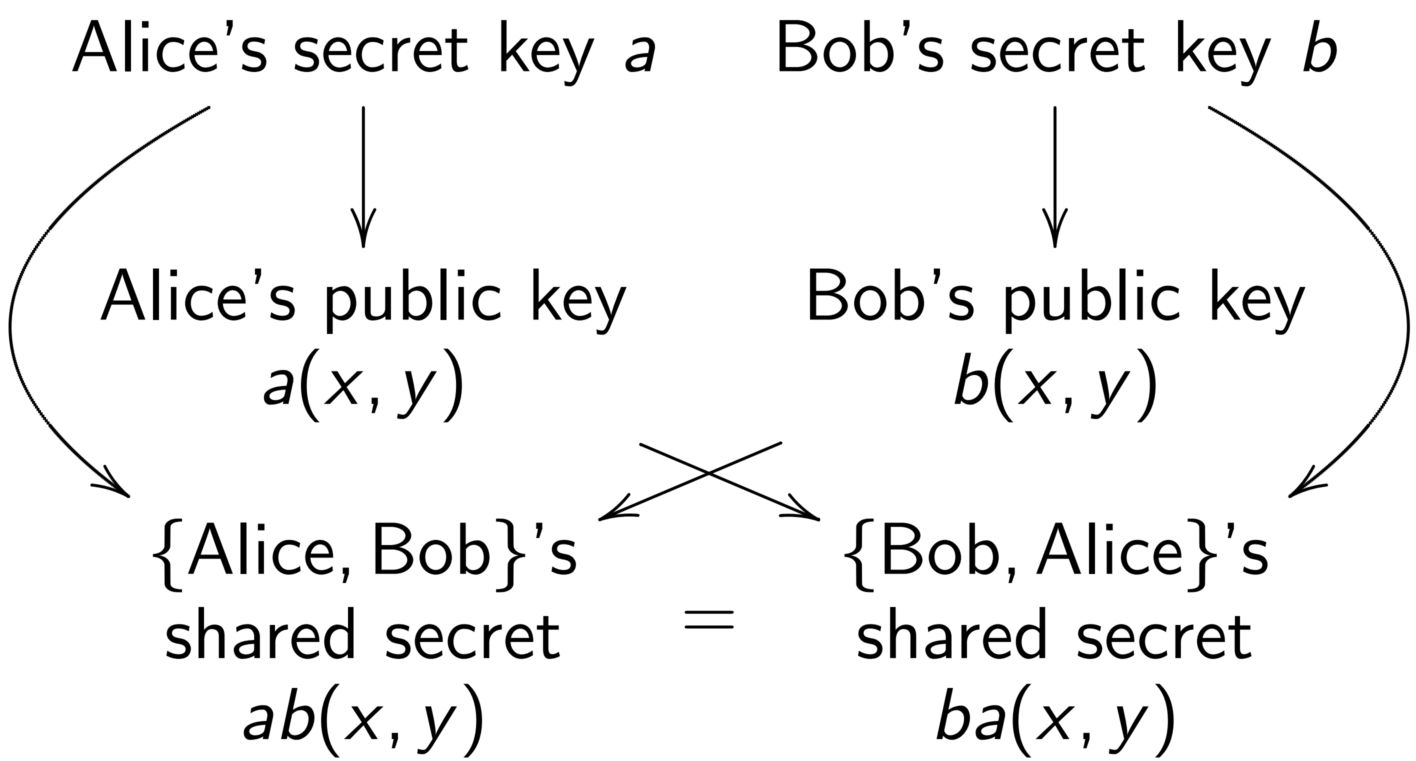

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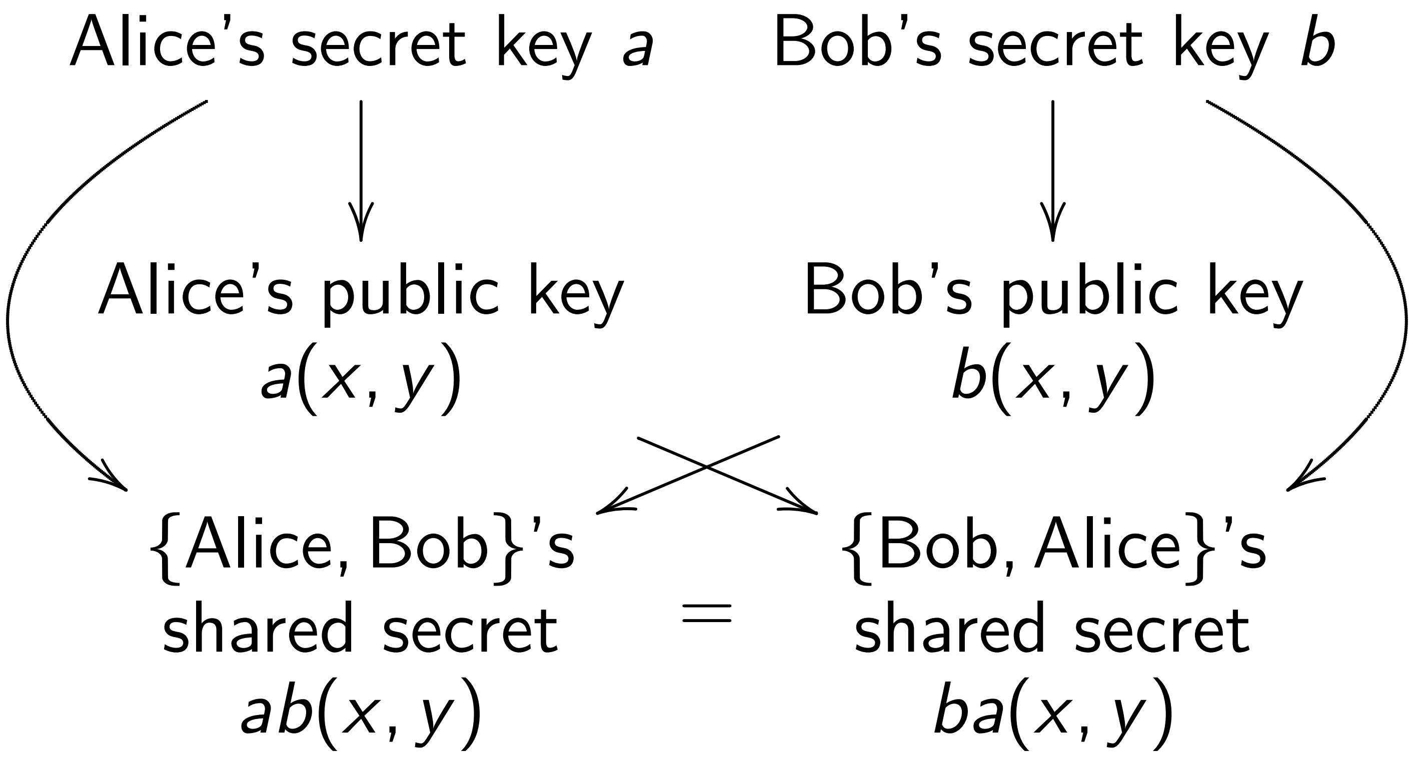

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Attacker sees how much
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Often attacker can see
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This reveals secret scal

Some timing attacks: 2
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2014 Benger–van de Po

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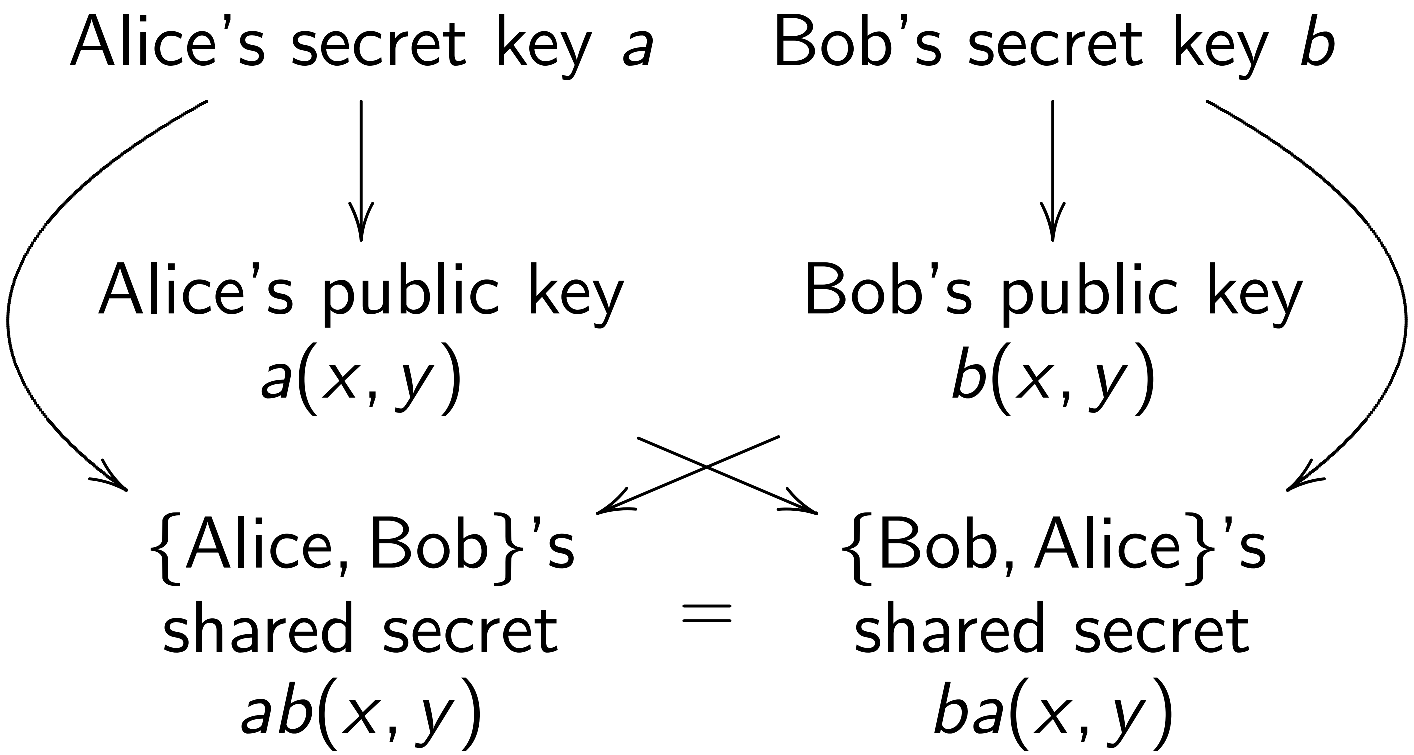

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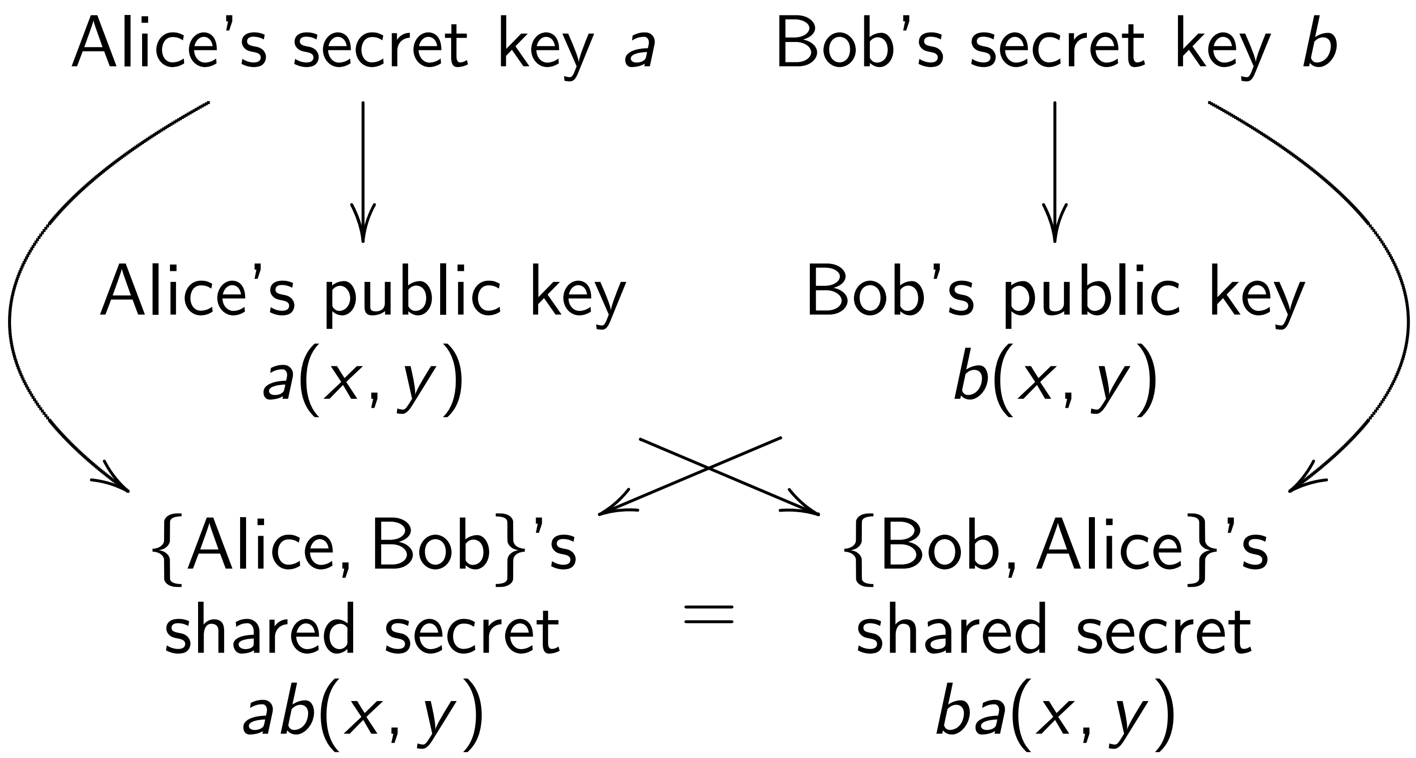

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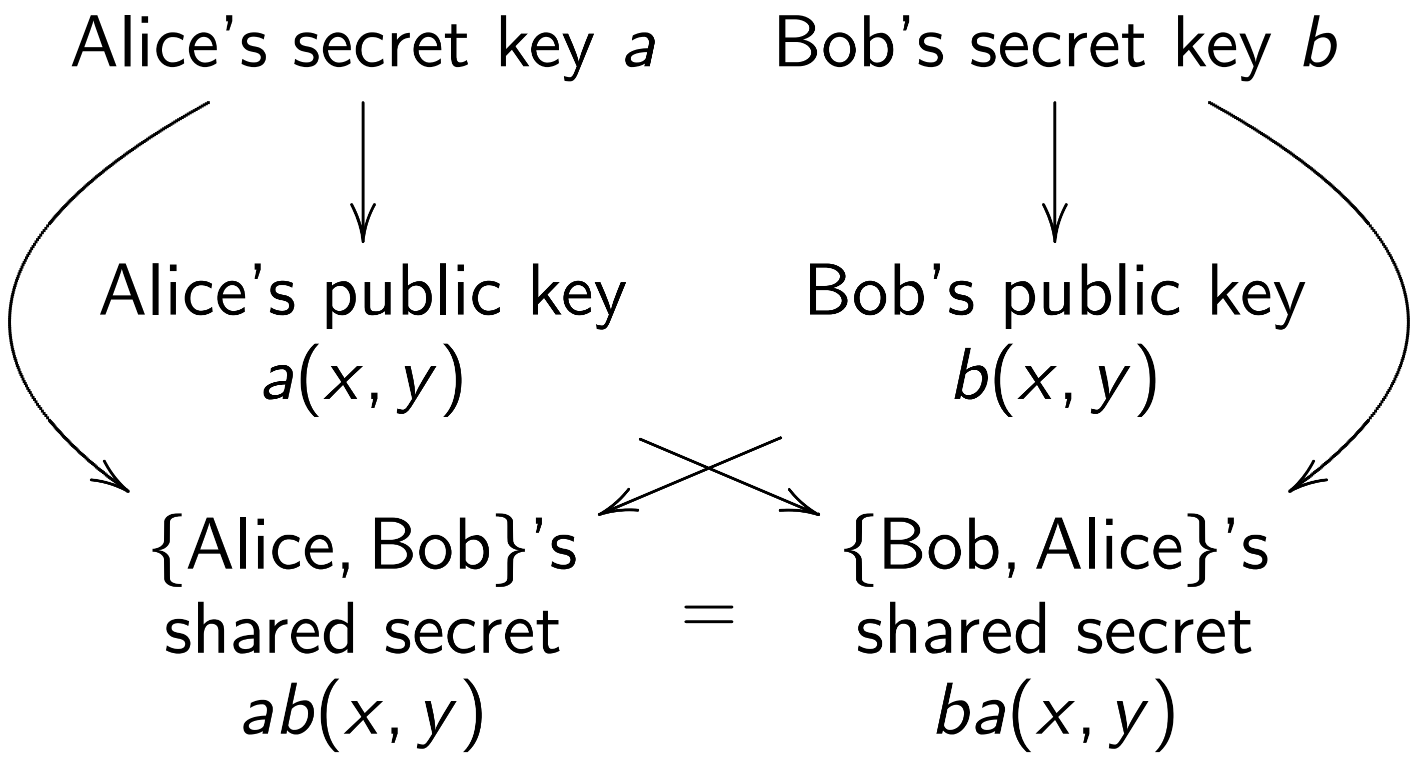

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Fix: **constant-time** code,
performing same operations
no matter what scalar is.

ecret key a Bob's secret key b

ublic key Bob's public key

$x, y)$ $b(x, y)$

, Bob}'s {Bob, Alice}'s

d secret = shared secret

$x, y)$ $ba(x, y)$

1: Many choices of p are unsafe!

2: Clocks aren't elliptic!

dex calculus

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RSA-3072 security

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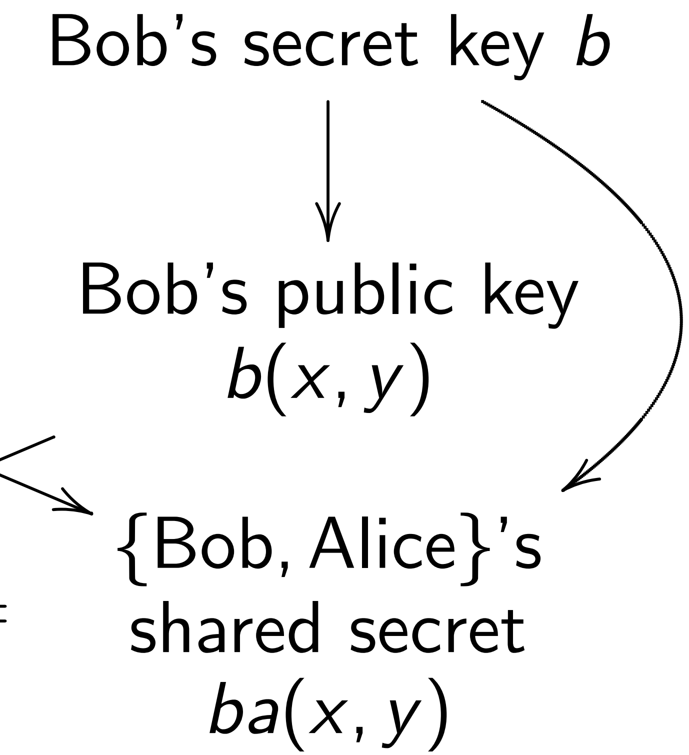
Addition on

$x^2 + y^2 =$

Sum of $(x_1$

$((x_1y_2+y_1x$

$(y_1y_2-x_1x$



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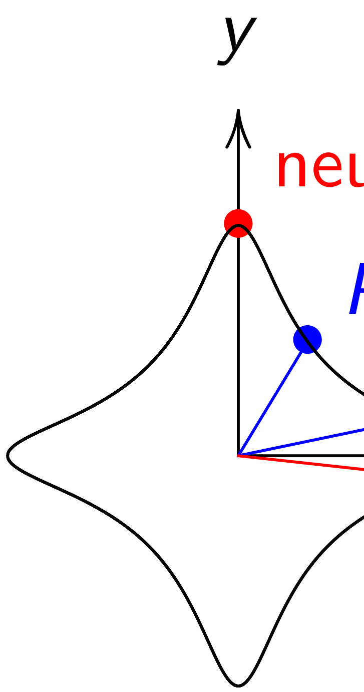

$x^2 + y^2 = 1 - 30x^2y^2$.
Sum of (x_1, y_1) and $(x_2$
$((x_1y_2+y_1x_2)/(1-30x_1$
$(y_1y_2-x_1x_2)/(1+30x_1$

et key b

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$y)$

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e unsafe!

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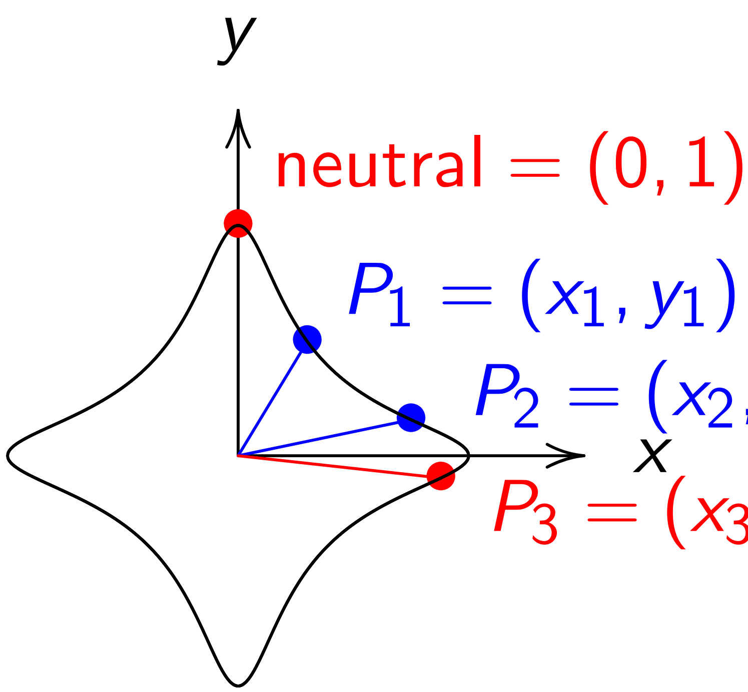
Fix: **constant-time** code, performing same operations no matter what scalar is.

Addition on an elliptic curve



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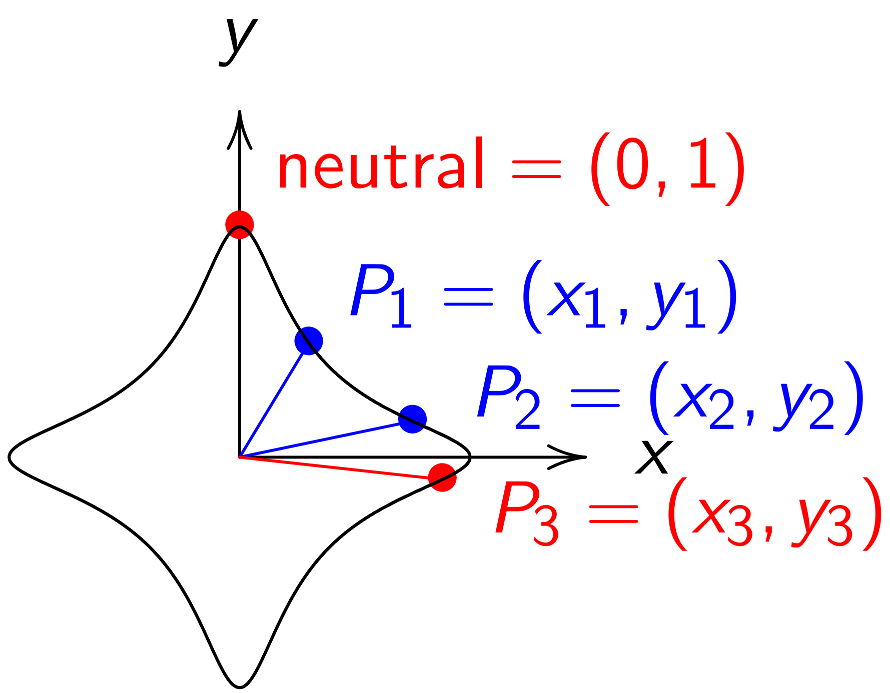

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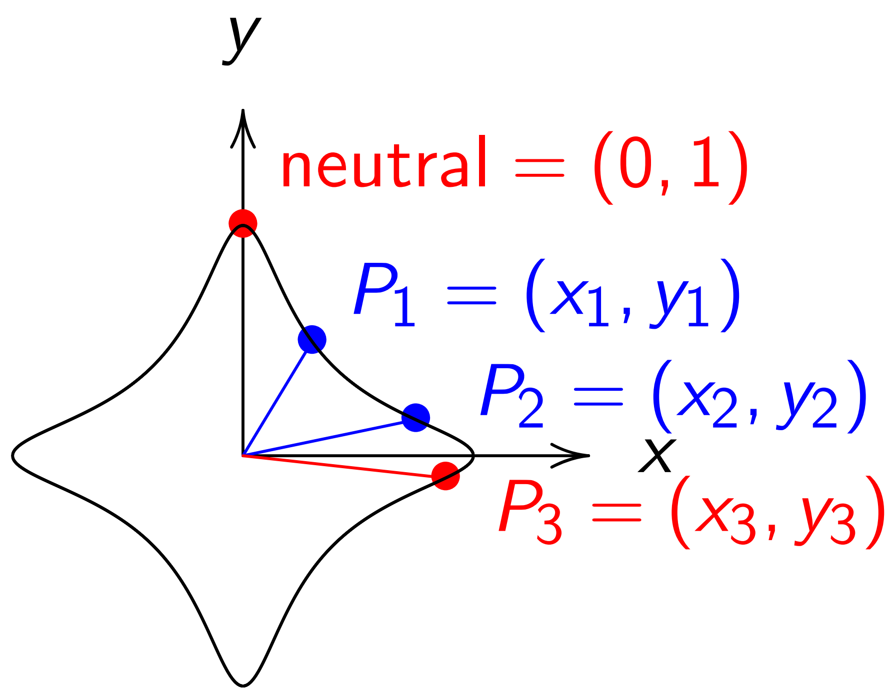

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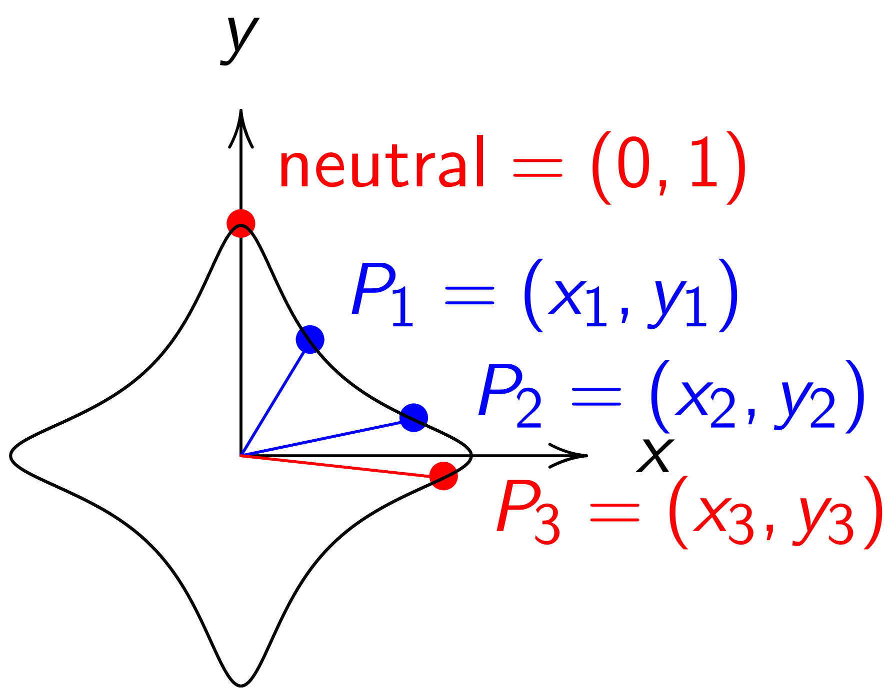

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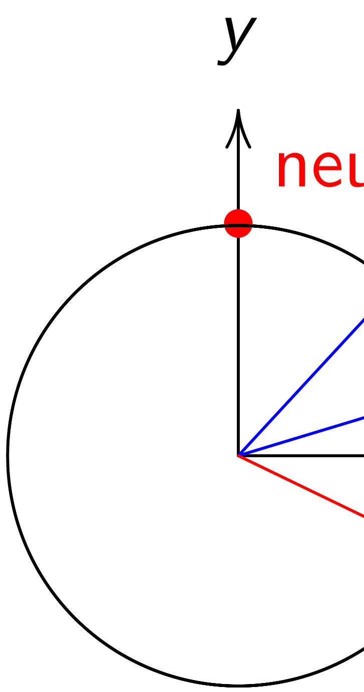

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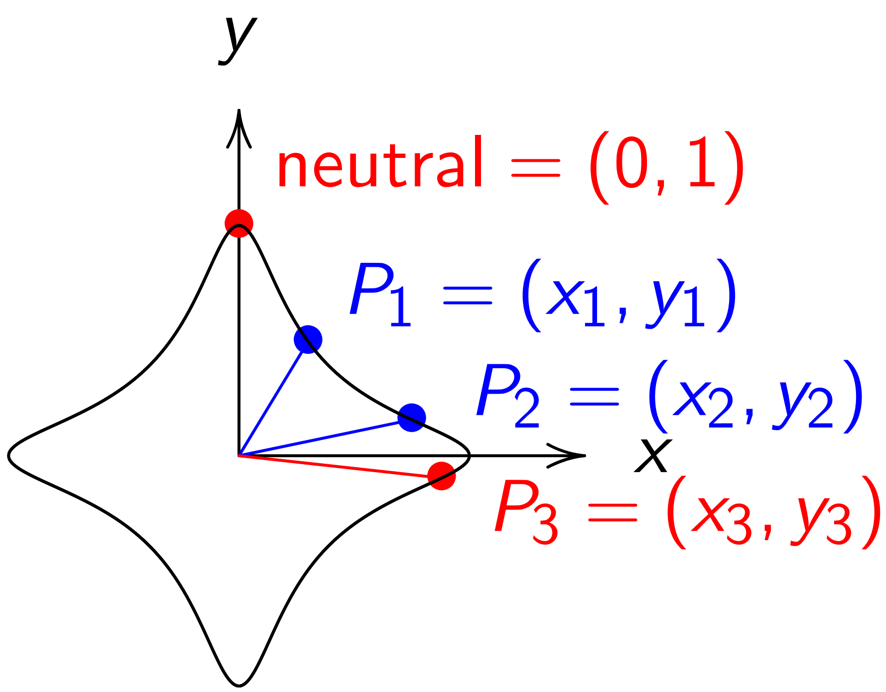

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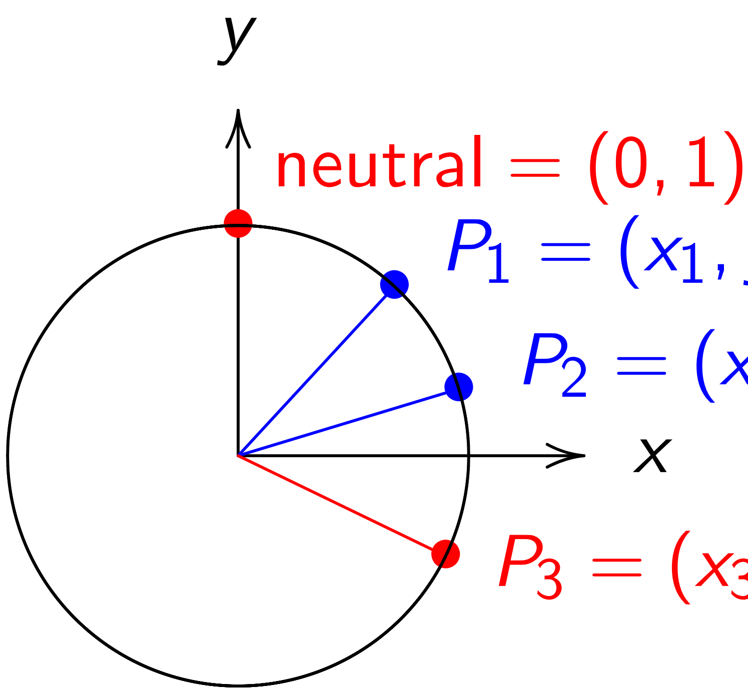

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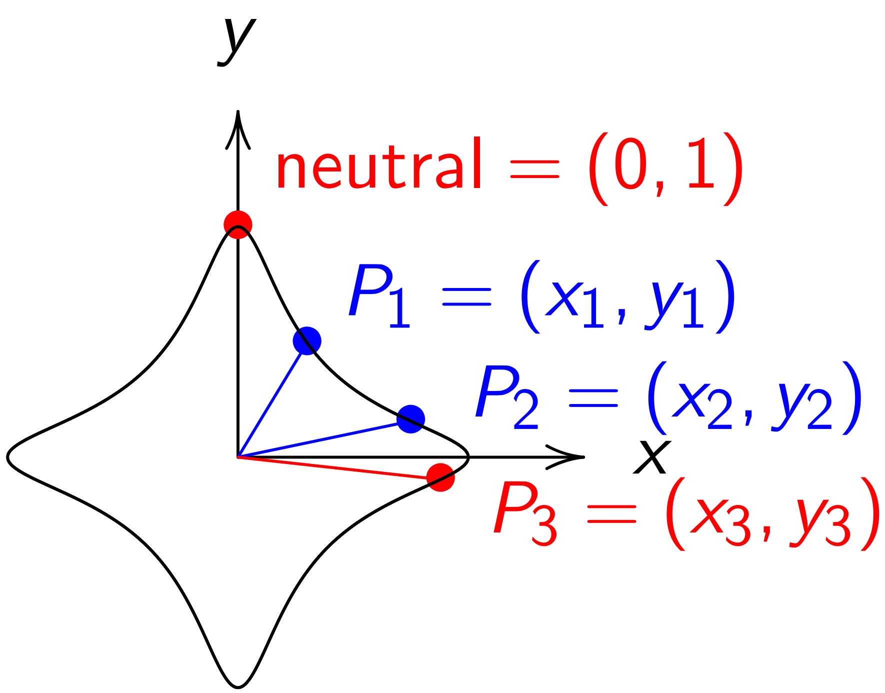

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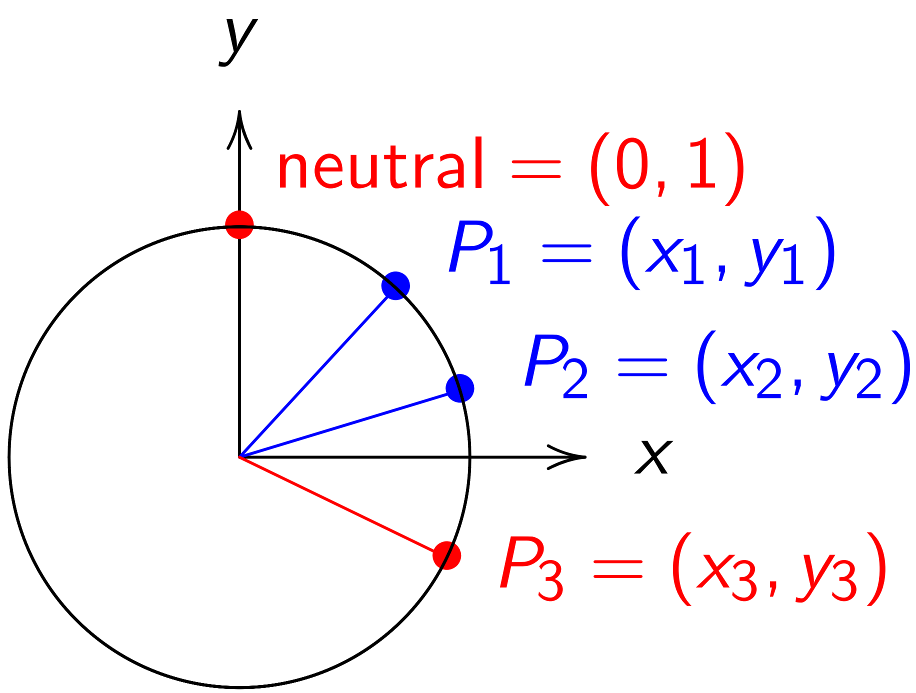

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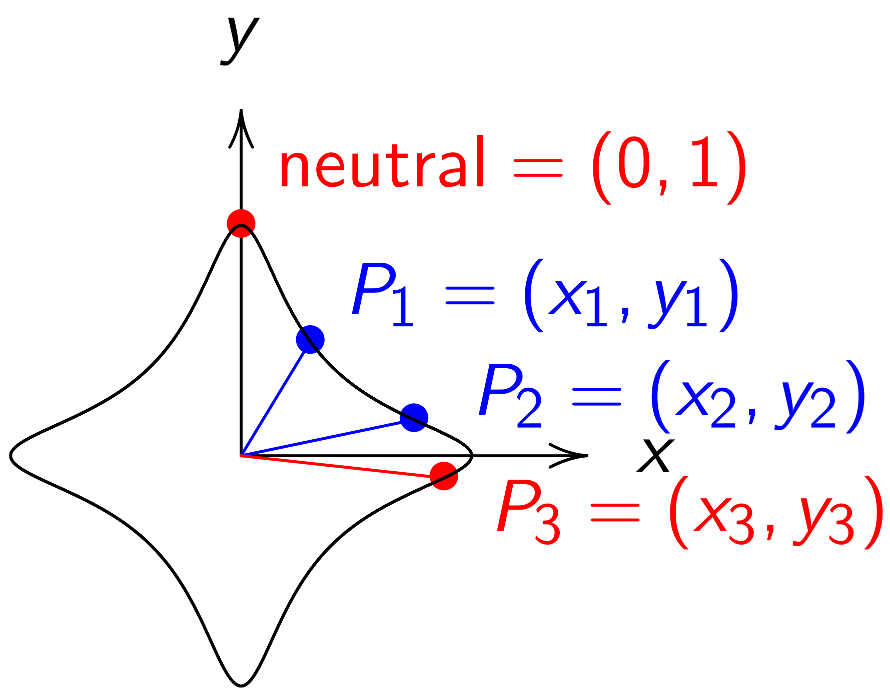

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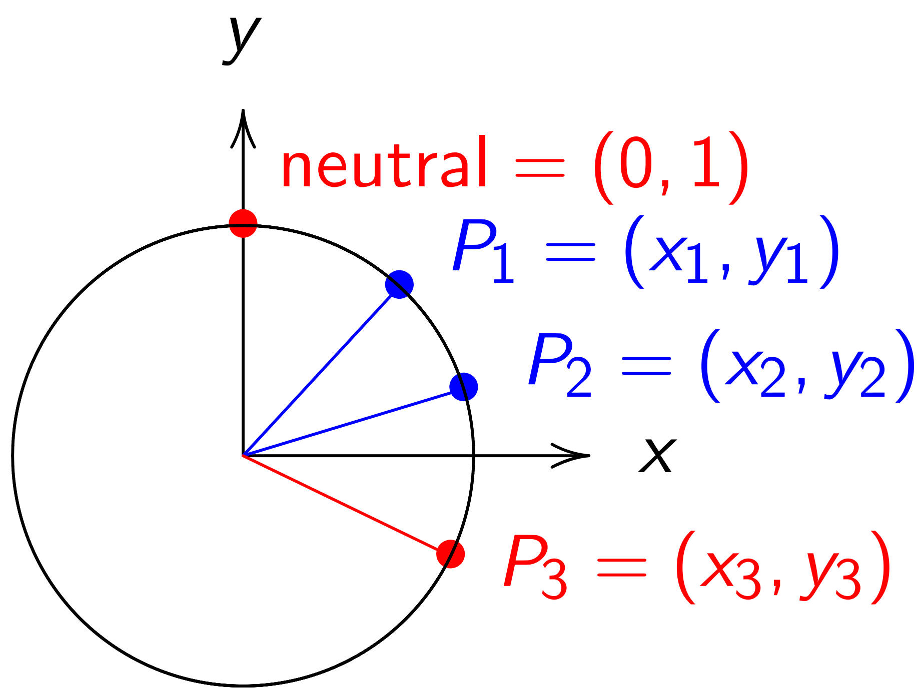

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  x2,y2 =
  x3 = (x1
  y3 = (y1
  return x
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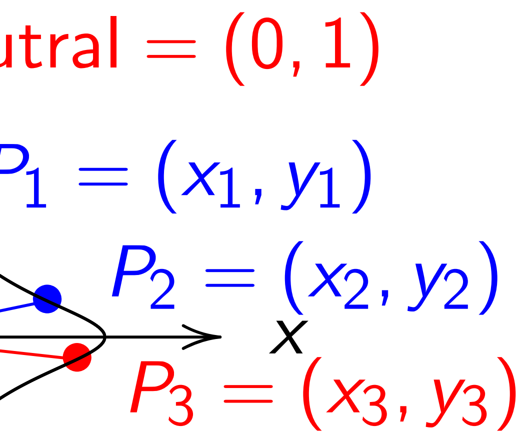

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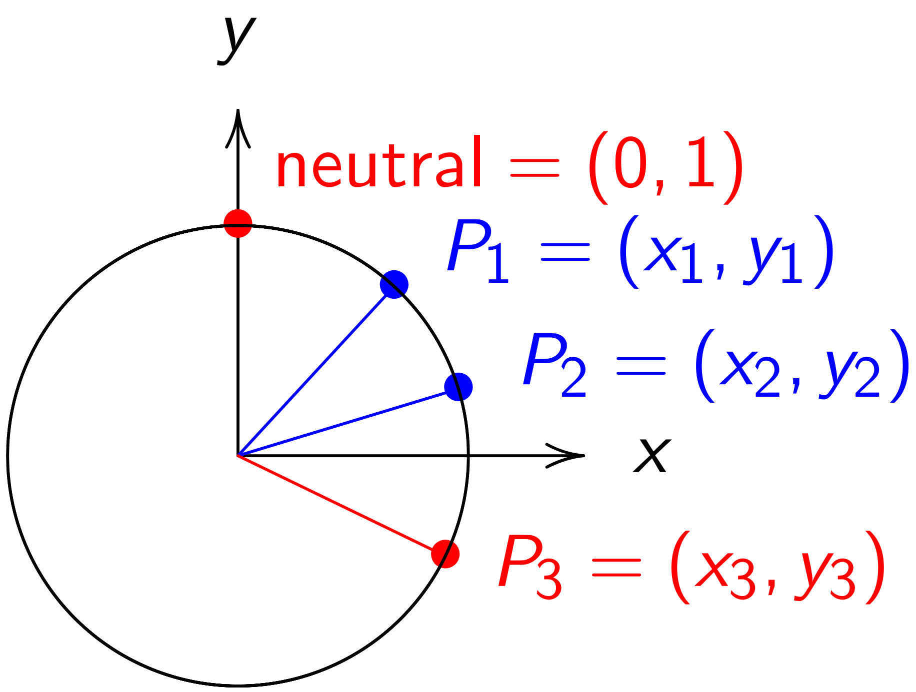

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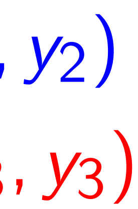

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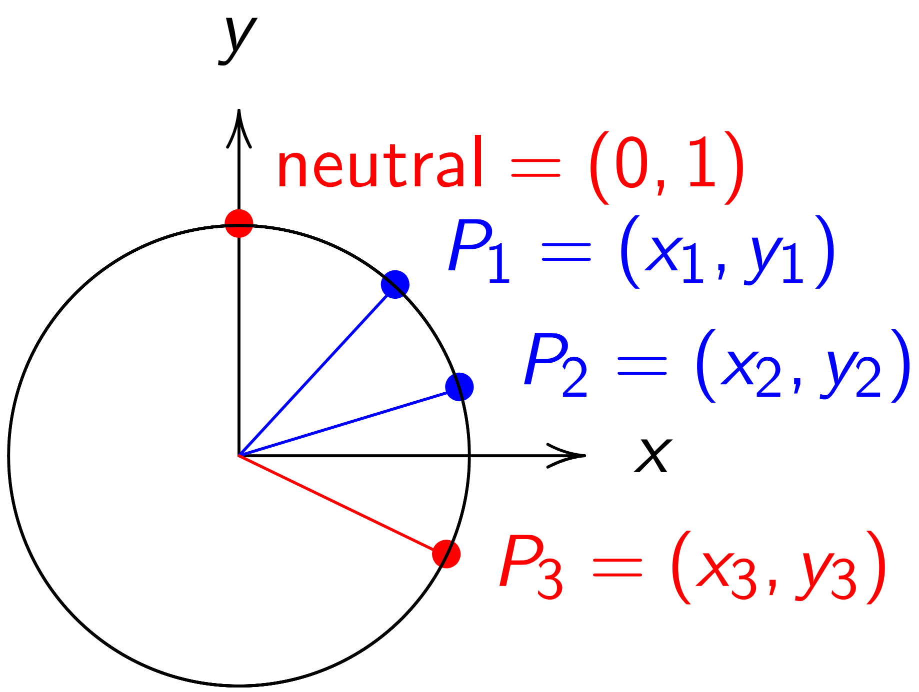

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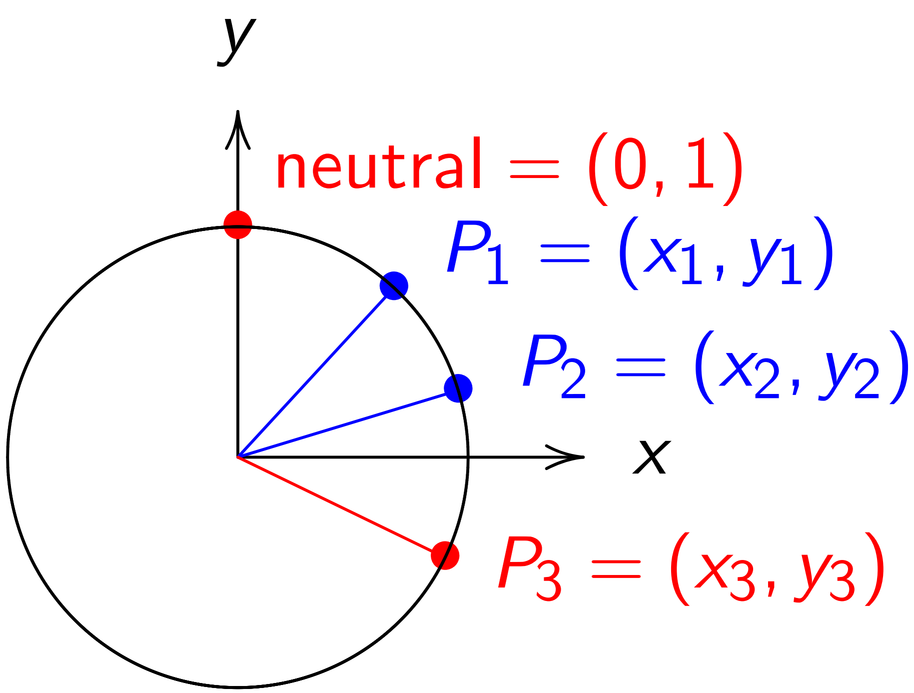

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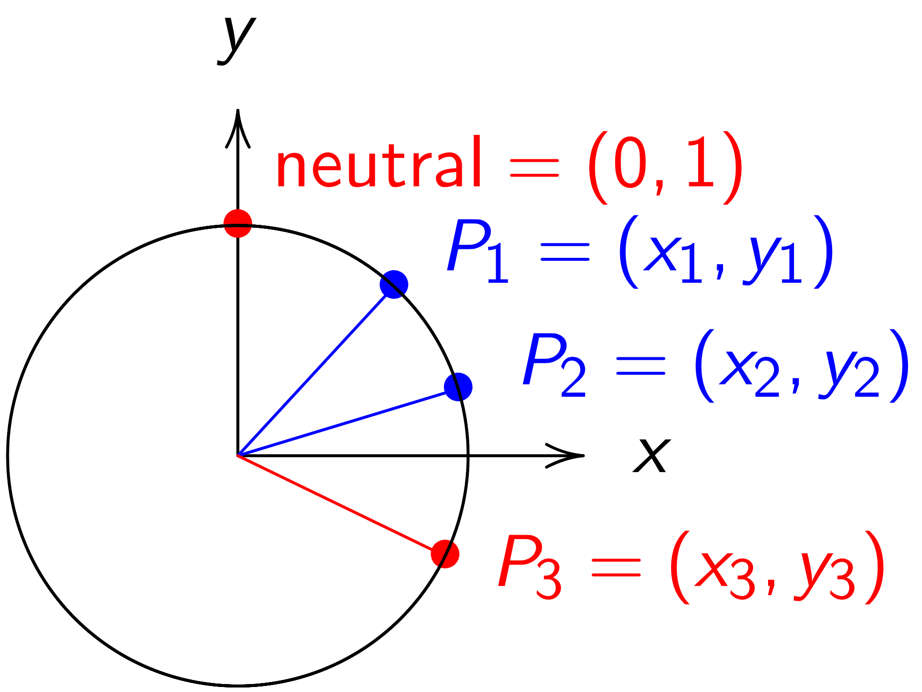

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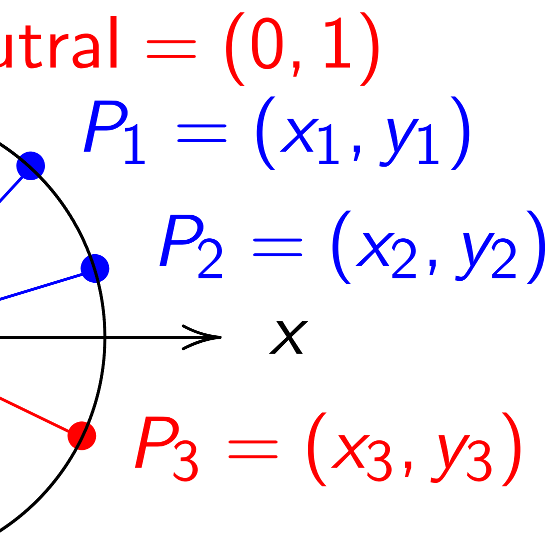

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See "Explicit Formulas Database" for many more options and speedups:
`hyperelliptic.org/EFD`

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Elliptic-curve cryptogra

Standardize prime p, sa
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Alice knows her secret
and Bob’s public key b(
Alice computes (and ca
shared secret $ab(x, y)$.

Alice uses shared secret
and authenticate packe

Packet overhead at hig
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Elliptic-curve cryptography

Standardize prime p, safe non-squar base point (x, y) on elliptic curve.

Alice knows her secret key a and Bob's public key $b(x, y)$. Alice computes (and caches) shared secret $ab(x, y)$.

Alice uses shared secret to encrypt and authenticate packet for Bob.

Packet overhead at high security lev
32 bytes for Alice's public key,
24 bytes for nonce,
16 bytes for authenticator.

“Hey, divisions are really slow!”

Instead of dividing a by b,
store fraction a/b as pair (a, b).
Remember arithmetic on fractions?

One option: “projective coordinates”.
Store (X, Y, Z) representing $(X/Z, Y/Z)$.

Another option: “extended coordinates”.
Store projective (X, Y, Z) and $T = XY/Z$.

See “Explicit Formulas Database”
for many more options and speedups:
hyperelliptic.org/EFD

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A safe example

Choose $p = 2^{255} - 19$.
Choose $d = 121665/12$
this is non-square in $\mathbf{F}_p$

$x^2 + y^2 = 1 + dx^2y^2$
is a safe curve for ECC.

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Much nice

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```
def scala
  x2,z2,x
  for i i
    bit =
    x2,x3
    z2,z3
    x3,z3

    x2,z2


    x2,x3
    z2,z3
  return 
```

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Much nicer than Weiers
curves with the "Montg

```
def scalarmult(n,x1):
  x2,z2,x3,z3 = 1,0,x
  for i in reversed(r
    bit = 1 & (n >> i
    x2,x3 = cswap(x2,
    z2,z3 = cswap(z2,
    x3,z3 = ((x2*x3-z
              x1*(x2*z3
    x2,z2 = ((x2^2-z2
              4*x2*z2*(
    x2,x3 = cswap(x2,
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  return x2*z2^(p-2)
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Much nicer than Weierstrass: Mont
curves with the "Montgomery ladde

```
def scalarmult(n,x1):
  x2,z2,x3,z3 = 1,0,x1,1
  for i in reversed(range(maxnbi
    bit = 1 & (n >> i)
    x2,x3 = cswap(x2,x3,bit)
    z2,z3 = cswap(z2,z3,bit)
    x3,z3 = ((x2*x3-z2*z3)^2,
             x1*(x2*z3-z2*x3)^2)
    x2,z2 = ((x2^2-z2^2)^2,
             4*x2*z2*(x2^2+A*x2*z
    x2,x3 = cswap(x2,x3,bit)
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Much nicer than Weierstrass: Montgomery curves with the "Montgomery ladder".

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def scalarmult(n,x1):
  x2,z2,x3,z3 = 1,0,x1,1
  for i in reversed(range(maxnbits)):
    bit = 1 & (n >> i)
    x2,x3 = cswap(x2,x3,bit)
    z2,z3 = cswap(z2,z3,bit)
    x3,z3 = ((x2*x3-z2*z3)^2,
             x1*(x2*z3-z2*x3)^2)
    x2,z2 = ((x2^2-z2^2)^2,
             4*x2*z2*(x2^2+A*x2*z2+z2^2))
    x2,x3 = cswap(x2,x3,bit)
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  return x2*z2^(p-2)
```

n Weierstrass curves

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mplement and test.

Much nicer than Weierstrass: Montgomery curves with the "Montgomery ladder".

```
def scalarmult(n,x1):
  x2,z2,x3,z3 = 1,0,x1,1
  for i in reversed(range(maxnbits)):
    bit = 1 & (n >> i)
    x2,x3 = cswap(x2,x3,bit)
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You can pick any of these standards

What your chosen standard achieves
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(“Elliptic-curve discrete-log problem

Example of criterion in all standards
Standard base point (x, y)
has huge prime “order” ℓ,
i.e., exactly ℓ different multiples.

All criteria are computer-verifiable.
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The attacker sent you (

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`1e86bec6c6bac120535e`

$y' =$ `12ace5eeae9a5b0bca8e`
`d123d55f68100099b65a`

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What you never noticed: (x', y') isn't his public key $b(x, y)$; it isn't even a point on `brainpoolP256t1`; it's a point on $y^2 = x^3 - 3x + 5$ of order only 4999.

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for $x_1 \neq x_2$, $(x_1, y_1) + (x_2, y_2)$
(x_3, y_3) with $x_3 = \lambda^2 - x_1 - x$
$y_3 = \lambda(x_1 - x_3) - y_1$,
$\lambda = (y_2 - y_1)/(x_2 - x_1)$;
for $y_1 \neq 0$, $(x_1, y_1) + (x_1, y_1) =$
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$(x_1, y_1) + \infty = (x_1, y_1)$;
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for $x_1 \neq x_2$, $(x_1, y_1) + (x_2, y_2) = (x_3, y_3)$ with $x_3 = \lambda^2 - x_1 - x_2$, $y_3 = \lambda(x_1 - x_3) - y_1$, $\lambda = (y_2 - y_1)/(x_2 - x_1)$;
for $y_1 \neq 0$, $(x_1, y_1) + (x_1, y_1) = (x_3, y_3)$ with $x_3 = \lambda^2 - x_1 - x_2$, $y_3 = \lambda(x_1 - x_3) - y_1$, $\lambda = (3x_1^2 + a_4)/2y_1$;
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`5abab9150d86770f6bda12f8ec`
`c6c6bac120535e4134fea87831` and
`eeae9a5b0bca8ed1c0f9540d05`
`5f68100099b65a99ac358e3a75`.

uted "shared secret" $a(x', y')$
Weierstrass formulas.
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Your formulas worked for $y^2 = x^3 - 3x + 5$ because they work for any $y^2 = x^3 - 3x + a_6$:

Addition on Weierstrass curves
$y^2 = x^3 + a_4x + a_6$:
for $x_1 \neq x_2$, $(x_1, y_1) + (x_2, y_2) = (x_3, y_3)$ with $x_3 = \lambda^2 - x_1 - x_2$, $y_3 = \lambda(x_1 - x_3) - y_1$, $\lambda = (y_2 - y_1)/(x_2 - x_1)$;
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$y' =$ `0d124e9e94dcede52aa0e3bcac1852cf` `ed28eb86039c0d8e0cfaa4ae703eac07`,

a point of order 19559 on $y^2 = x^3 - 3x + 211$; learns your secret a mod 19559.

Etc. Uses "Chinese remainder theorem" to combine this information.

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But it's much better to *design the system without traps*.

Never send uncompressed (x, y)**.**
Design protocols to compress one coordinate down to 1 bit, or 0 bits!
Drastically limits possibilities for attacker to choose points.

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Design DH protocols to multiply by

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Montgomery formulas use only A, but modifying B gives only *two* diff curve orders. Require both of these to be large primes times small cofac

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2013.09 Douglas Stebila: Reasons to support Curve25519 are "efficiency and resistance to side-channel attacks" rather than concerns about backdoors.

2013.09 Nick Mathewson: "In the FOSS cryptography world nowadays, I see many more new users of curve25519 than of the NIST curves, because of efficiency and ease-of-implementation issues."

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Sage scripts to verify criteria for ECDLP security and ECC security:
`safecurves.cr.yp.to`

Analysis of manipulability of various curve-generation methods:
`safecurves.cr.yp.to/bada55.html`

Many computer-verified addition formulas:
`hyperelliptic.org/EFD/`

Python scripts for this talk:
`ecchacks.cr.yp.to`