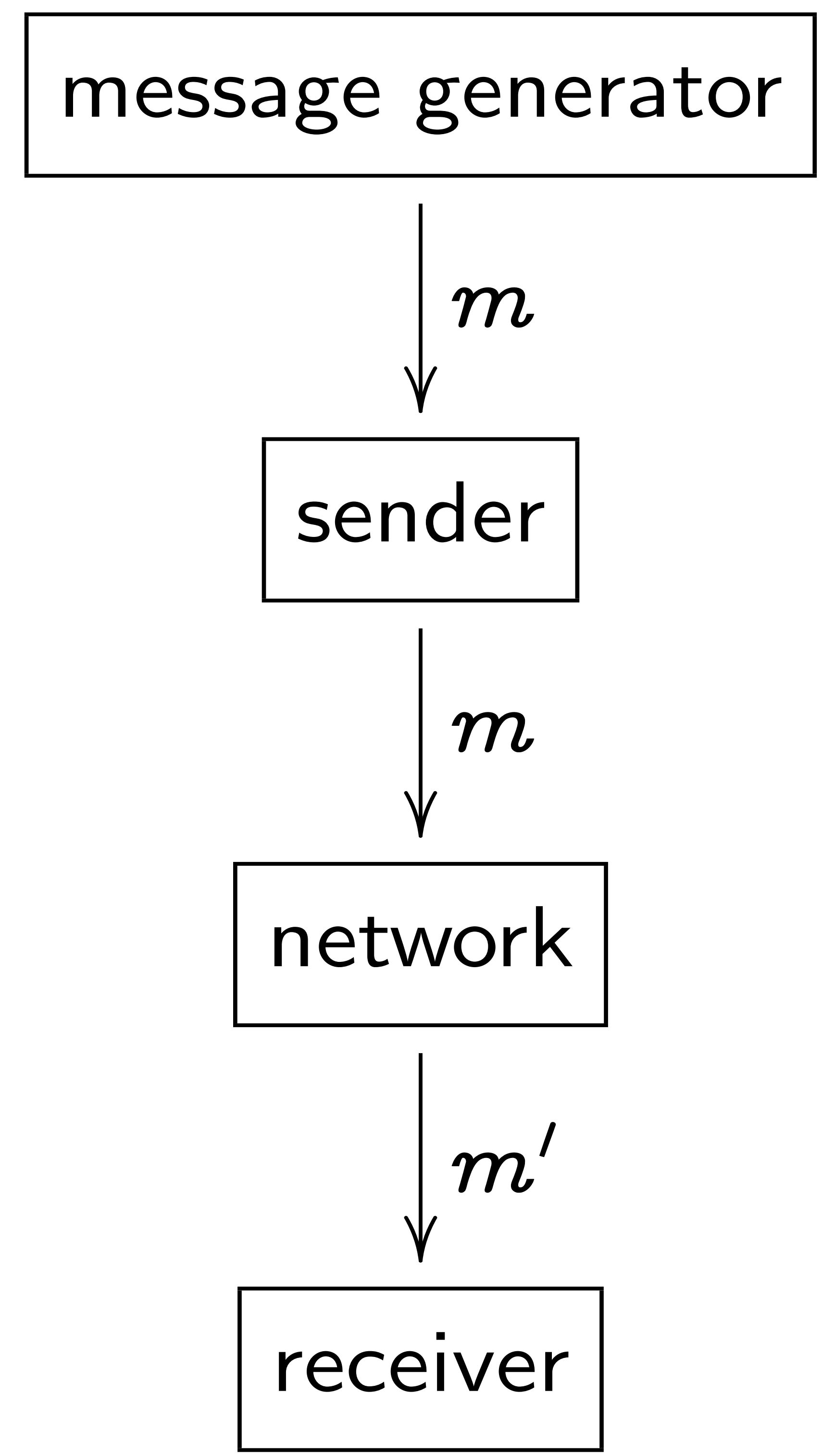
High-speed cryptographic functions

D. J. Bernstein

Typical Internet protocol:



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$$\begin{tikzcd}
\boxed{\text{message generator}} \arrow[d, "m"] \\
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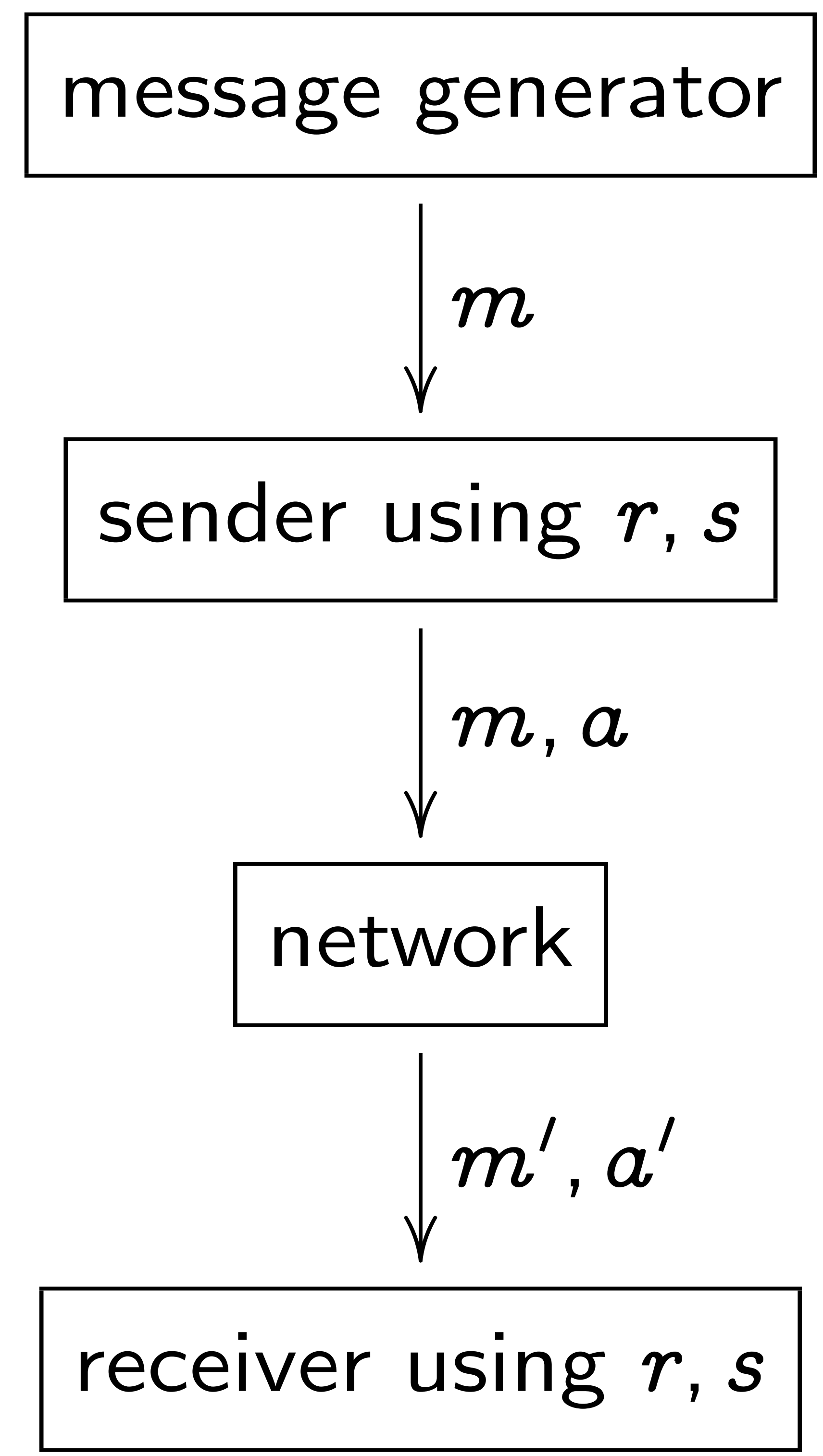

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Fix a finite field k.
Typically $\#k \approx 2^{128}$.

Sender, receiver share a **secret**: uniform random $(r, s) \in k \times k$.
Network's function $m, a \mapsto m', a'$ is independent of (r, s).

Sender encodes message m as polynomial $\underline{m} \in xk[x]$.
Sender then computes **authenticator** $a = \underline{m}(r) + s$.

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If $m' \neq m$ then
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e.g. $\Pr \leq 2^{-98}$ if $\#k = 2^{128}$
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$\downarrow 1, m_1$

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$\downarrow n'_1, m$

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Many messages, unprotected:

$$\begin{tikzcd}
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\boxed{\text{sender}} \arrow[d, "1, m_1; 2, m_2; \ldots"] \\
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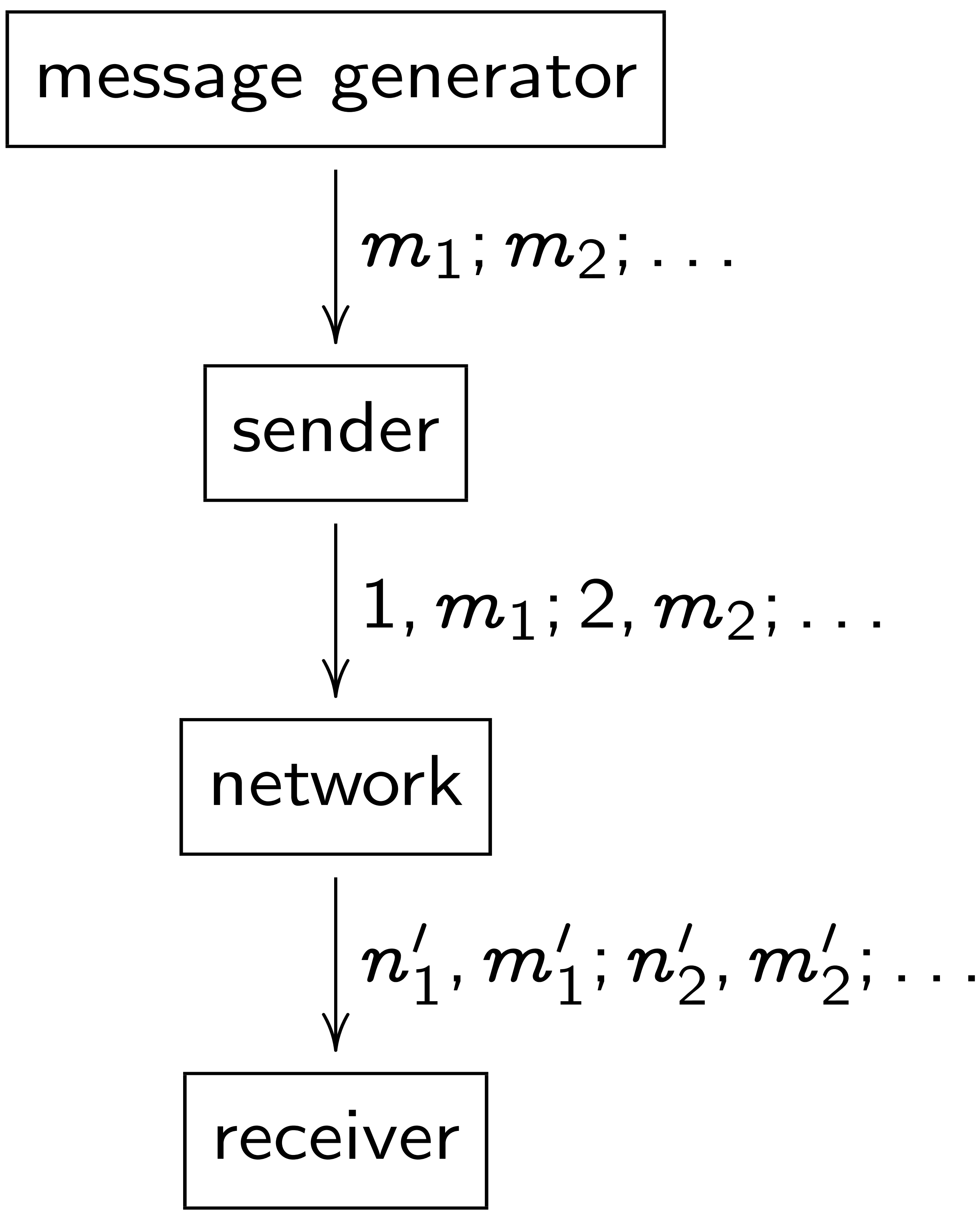

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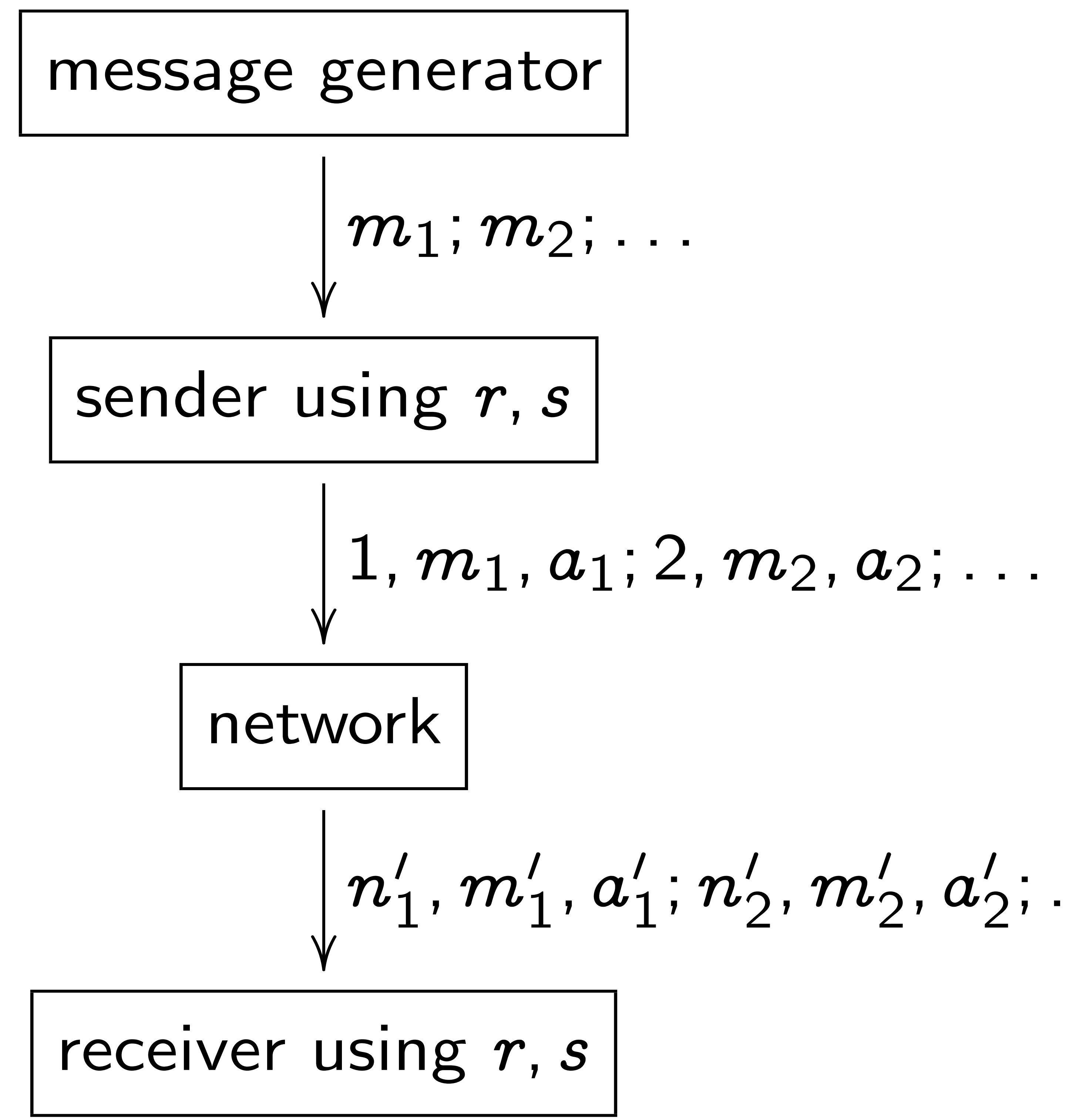

unprotected:

tor

$n_2; \ldots$

$; 2, m_2; \ldots$

$n_1'; n_2', m_2'; \ldots$

Many messages, protected:

message generator

$\downarrow$ $m_1; m_2; \ldots$

sender using r, s

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network

$\downarrow$ $n_1', m_1', a_1'; n_2', m_2', a_2'; \ldots$

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Secret here is un

$(r, s) \in k \times k^{\{1,2}$

i.e., $r \in k; s(1) \in$

e.g. 128000 secr

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e.g. 128000 secret bits to handle 999 messages if $\#k = 2^{128}$.

Sender transmits nth message m as $n, m, \underline{m}(r) + s(n)$.

Receiver discards n', m', a' if $a' \neq \underline{m'}(r) + s(n')$.

Forged n', m', a' has negligible chance of being accepted.

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$\boxed{, s}$

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$m'_1, a'_1; n'_2, m'_2, a'_2; \ldots$

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Why not use that channel for new messages?

Answer 1: Extend security through time. Previous channel can disappear after sending r, s. New channel sends new messages.

Answer 2: Expand bandwidth. Messages can be much longer than r, s.

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For b-bit security, $\lceil \lg \#k \rceil = b$, c messages, total length d:

transmit r, s through
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$$\Downarrow$$

transmit $m_1; m_2; \ldots$ through
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Speed records: P
See `cr.yp.to/m`
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128-bit coefficien
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128-bit coefficients of $\underline{m}$;
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etc., where $q = 2$

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How to reduce bandwidth?

Expand a short shared secret into a long shared secret.

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Conjecture: Hard to distinguish this expanded secret from a uniform random sequence of squares modulo q.

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For b bits of secu

Fix q with $q, (q -$

and with $\lg q \in b$

more precisely, w

$6.8 \ldots (\lg q)(\lg \mathrm{lo}$

Transmit short sh

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$t, u_0, u_1, \ldots \in \{1$

These sizes just

fastest discrete-lo

that we know.

Conjecture: Hard to distinguish this expanded secret from a uniform random sequence of squares modulo q.

Could try to do it by computing discrete logs modulo q: extract t from $4^t \bmod q$, tu_0 from $4^{tu_0} \bmod q$, etc., and see $(t)(tu_0u_1) = (tu_0)(tu_1)$. But discrete logs seem hard!

Thus (e.g.) bottom halves seem hard to distinguish from a uniform random string.

For b bits of security, $b \to \infty$:

Fix q with $q, (q-1)/2$ prime and with $\lg q \in b^{3+o(1)}$; more precisely, with $6.8\ldots(\lg q)(\lg \log q)^2 \approx b^3$.

Transmit short shared secret: independent uniform random $t, u_0, u_1, \ldots \in \{1, 2, \ldots, 2^{2b}\}$.

These sizes just barely resist fastest discrete-log methods that we know.

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Expand $t, u_0, u_1,$
$4^t \bmod q, \ldots, 4^{t}$

e.g. Expand t, u
into 2^{64} integers

Extract bottom
of each integer.

Compute results
Only $O(b)$ mults
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e.g. Expand $t, u_0, u_1, \ldots, u_{63}$ into 2^{64} integers modulo q.

Extract bottom $\lceil (1/2)\lg q \rceil$ bits of each integer.

Compute results sequentially. Only $O(b)$ mults mod q for each integer, so $b(\lg b)^{1+o(1)}$ bit ops per bit.

Random access is slow: $\geq b^{4+o(1)}$ bit ops.

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Do better by replacing $(\mathbf{Z}/q)^*$ with $E(\mathbf{Z}/q)$ for a safe elliptic curve E.

Discrete logs in $E(\mathbf{Z}/q)$ seem relatively difficult, so can take q smaller: specifically, $\lg q \approx 2b$.

Much faster random access: $b^{2+o(1)}$ bit ops.

Sequential access again takes $b(\lg b)^{1+o(1)}$ bit ops per bit.

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Many choices of expansion functions ("stream ciphers").
Fastest expansion functions *don't* have discrete-log structure.

Speed records: see eSTREAM, `www.ecrypt.eu.org/stream`.
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Receiver generates secret τ, sends public key $4^\tau \bmod q$ back through old channel.

Sender and receiver now compute $4^{\sigma\tau} \bmod q$,
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Speed records: Curve25519. See `cr.yp.to/ecdh.html` and `papers.html#curve25519`. Curve $y^2 = x^3 + 486662x^2 + x$ mod $2^{255} - 19$; eliminate y; $<$ 1000000 CPU cycles.

New records from Jacobians of genus-2 hyperelliptic curves? Come to ECC 2006 in Toronto. `www.fields.utoronto.ca` `/programs/scientific` `/06-07/crypto/`

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See PQCrypto 20
`postquantum.cr`

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Best to reuse one secret for many messages. Minimize public-key operations.

If large quantum computers are built then they will compute discrete logs quickly. Huge effects on cryptography!

Some public-key systems seem to survive quantum computers. See PQCrypto 2006 abstracts: `postquantum.cr.yp.to`.

Exactly how fast are RSA, DSA, ECDH, post-quantum cryptosystems, etc.? `www.ecrypt.eu.org/ebats` is benchmarking public-key systems.